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ESSENTIALS OF
BUSINESS ARITHMETIC

BY

GEORGE H. VAN TUYL

TEACHER OF BUSINESS ARITHMETIC
HIGH SCHOOL OF COMMERCE, NEW YORK
FORMERLY TEACHER OF ARITHMETIC IN THE ALBANY
BUSINESS COLLEGE, ALBANY, N.Y. AND IN THE
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ESSEN. BUS. ARITH.

W. P. 2

PREFACE

This book has been prepared to meet the needs of business schools and academic high schools where an intensive study of the most important topics of arithmetic is required, with a view to developing skill in the fundamental operations. This requirement is met by the omission of some of the more advanced topics found in the author's "Complete Arithmetic" and by the inclusion of a number of new features; such as problems in Domestic Parcel Post, farm problems, problems in estimating, and a comparison of common food products, accompanied by a series of exercises to determine the value of given foods as tissue builders and as energy producers.

The book is adapted also for use in the upper grades of elementary schools where a business training in arithmetic is desired. The clear and full explanations, the practical problems, the short methods of solution, and the numerous exercises for drills and reviews render this book available for such courses in grammar schools.

Many suggestions have been made for rapid calculation. The Speed and Written Tests of the larger volume have been retained with such changes in the problems as have been necessary to meet the requirements of the smaller text.

Attention is directed also to the following features:

The chapter on aliquot parts, as applied to billing, trade discount, and simple interest, is placed early in the text.

Common and decimal fractions are treated together, as is the case in business.

A great many problems are provided for mental work.

Many of the problems are taken from the business affairs of corporations, cities, states, and nations of the world.

Many calculation tables are illustrated and applied to the solution of problems.

There are five sets of examinations (each set consisting of a speed test and a written test) to determine the student's mastery of the subject at various stages of the work.

The author gratefully acknowledges his indebtedness to his many friends who, by their advice and suggestions, have aided him in preparing this volume.

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ESSENTIALS
OF
BUSINESS ARITHMETIC

READING AND WRITING NUMBERS

1. Numbers are expressed in three ways: (1) by written words, (2) by letters (Roman system), and (3) by figures (Arabic system).

ROMAN NOTATION

2. In the **Roman system** of writing numbers seven letters are used. They are:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

3. Writing numbers by the Roman system is based on the following principles:

PRINCIPLES: **1.** *Repeating a letter repeats its value.*

2. *The value of a letter written after a letter of greater value is to be added to the larger value.*

3. *The value of a letter written before a letter of greater value is to be subtracted from the greater value.*

4. *A horizontal bar over a letter increases its value one thousand times.*

NOTES. (1) Only whole numbers can be written in the Roman system of notation.

(2) Roman notation is of little practical value, being used only to record dates and chapters in books, dates on buildings, numbers on a watch face, etc.

4. ROMAN NOTATION ILLUSTRATED

II	= 2	XIV	= 14	LIX	= 59	CCCLXVI	= 366
III	= 3	XVI	= 16	LXXIV	= 74	DCXCVI	= 696
IV or IIII*	= 4	XIX	= 19	LXXXIX	= 89	MCXIX	= 1119
VI	= 6	XXIII	= 23	XC	= 90	MCM	= 1900
IX	= 9	XXXIV	= 34	CX	= 110	$\overline{\text{MCMX}}$	= 1,910,000

5. 1. Write in Roman notation:

12
18
21
47
63
85
94
111
128
247
336
472
505
666
872
598
1077
1384
1527
1898
10,000
15,000
100,000
150,000
1,000,000

2. Read the following:

XVIII
XXIX
XXXI
XLIV
XLIX
LXIX
XCIX
CIX
CLXXII
CCCLX
CDLIX
$\overline{\text{D}}$CD
DCCCLXXXIX
CM
MCMIX
$\overline{\text{MC}}$
MDCCCXCIX
MDCCXXVIII
$\overline{\text{V}}$CCLXXX
$\overline{\text{L}}$

ARABIC NOTATION

6. The **Arabic system** of notation makes use of ten characters or figures, and a period or decimal point. They are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and .

7. With these figures and the decimal point any required number can be written.

8. For convenience in reading numbers, the figures are sometimes divided into groups of three, in this way:

2,564,879,598.

The reason for this grouping is best understood by observing the following table:

---

* IIII is used on timepieces.

TABLE SHOWING HOW TO READ NUMBERS

ten-trillions	trillions	hundred-billions	ten-billions	billions	hundred-millions	ten-millions	millions	hundred-thousands	ten-thousands	thousands	hundreds	tens	units
8	9,	7	2	4,	5	6	3,	9	0	0,	1	0	6.

The number is read, "eighty-nine trillion, seven hundred twenty-four billion, five hundred sixty-three million, nine hundred thousand, one hundred six."

9. In like manner read the following numbers:

1. 21,460
2. 34,829
3. 127,724
4. 306,200
5. 425,002
6. 800,005
7. 962,024
8. 872,463
9. 900,047
10. 1,456,789
11. 3,248,764
12. 5,002,005
13. 27,250,820
14. 20,300,600
15. 65,897,249
16. 185,496,728
17. 593,972,847
18. 15,647,989,721

10. Write the following numbers, using the Arabic system:

1. Seven hundred forty-three.
2. Nine thousand fourteen.
3. Eleven thousand one hundred four.
4. Fifteen thousand twenty.
5. Seventy-six thousand ninety-nine.
6. Fourteen hundred eighty-eight.
7. One hundred sixty thousand four hundred.
8. Eight hundred thousand nine hundred ninety.
9. Two million sixteen thousand seventeen.
10. Six hundred eleven million fourteen thousand eighty-six.

UNITED STATES MONEY

11. Any number of cents less than one hundred is read as cents: thus, $.45 or 45¢ is read "forty-five cents," but any value equal to or greater than one hundred cents is read as dollars, or as dollars and cents: thus, $8.75 or 875¢ would be read "eight dollars, seventy-five cents." A period, or point, separates the number of dollars from the number of cents.

12. Read the following:

$3.17
$4.28
$9.15
$10.20
$12.40
$16.50
$85.90
$104.26
$826.52
$1624.43*
$1837.16
$2487.49
$1476.83
$1982.76
$14,728.41
$146,789.09

13. Write the following in figures:

1. One hundred sixteen dollars, fifty-nine cents.
2. Three hundred nine dollars, seventy cents.
3. Twelve hundred twenty-one dollars, eighty-eight cents.
4. Twenty-four hundred forty-three dollars, nine cents.
5. Fifty-seven hundred six dollars, thirty-three cents.

14. Money is the generally accepted measure of value.

15. Each nation has its own system of money or coinage.

The principal nations of the world have adopted gold as the standard of monetary value.

16. **United States money** consists of the coins, notes, and certificates authorized by Congress to be used as money. It is a decimal system.

TABLE

10 mills = 1 cent, ¢
10 cents = 1 dime, d.
10 dimes = 1 dollar, $
10 dollars = 1 eagle
(The mill is not a coin.)

17. There are ten different kinds of money in circulation in the United States: gold coins, silver dollars, subsidiary silver, nickels, pennies, gold certificates, silver certificates, Treasury notes, United States notes (greenbacks), and national bank notes.

---

* In reading $1624.43 say "sixteen hundred twenty-four dollars, forty-three cents," not "one thousand six hundred twenty-four dollars, forty-three cents." One reason for saying it and writing it as suggested is that it takes less time and space.

18. The gold dollar, containing 25.8 grains of standard gold .9 fine, is the unit of value.

19. The silver dollar weighs 15.988 times as much as the gold dollar. That is, the ratio of gold to silver is 15.988 to 1 (commonly called 16 to 1).

20. DENOMINATIONS, WEIGHT, AND FINENESS OF THE COINS OF THE UNITED STATES

GOLD			
DENOMINATION	FINE GOLD CONTAINED (GRAINS)	ALLOY CONTAINED (GRAINS)	WEIGHT (GRAINS)
One dollar (not coined since 1890)	23.22	2.58	25.80
Quarter eagle ($2.50)	58.05	6.45	64.5
Half eagle ($5.00)	116.10	12.90	129.
Eagle ($10.00)	232.20	25.80	258.
Double eagle ($20.00)	464.40	51.60	516.
	SILVER		
	PURE SILVER CONTAINED		
One dollar	371.25	41.25	412.5
Half dollar	173.61	19.29	192.9
Quarter dollar	86.705	9.645	96.45
Dime	34.722	3.858	38.58
	MINOR		
	PURE COPPER CONTAINED		
Five cents* (nickel)	57.87	19.29	77.16
One cent † (copper)	45.60	2.40	48.

NOTE. The alloy in gold and silver coins neither adds to nor detracts from the value of the coins. Its purpose is to harden the coin, thus reducing the loss by abrasion.

*75% copper and 25% nickel. †95% copper and 5% tin and zinc.

ALIQUOT PARTS

21. An **aliquot part** of a number is a number that is contained in it an integral number of times.

Thus, 2, 4, 5, 10, 20, 25, and 50 are aliquot parts of 100; that is, $2 = \frac{1}{50}$ of 100, $25 = \frac{1}{4}$ of 100, etc.

Note. Other fractional parts of 100 are called **aliquant parts** of 100. For convenience, all fractional parts of 100 will be treated as aliquot parts.

Halves and Quarters

22. 1. What part of 100 cents, or $1.00, is 25¢? 50¢? 75¢?

2. At 25¢ a yard, how many yards can be bought for $1.00?

3. If 4 yd. cost $1.00, how much will 8 yd. cost? 12 yd.? 16 yd.?

4. How much will 20 yd. cost at 25¢ a yard? at 50¢ a yard? at 75¢ a yard?

5. Compare 25¢ with 50¢; with 75¢. 50¢ with 75¢; with 25¢. 75¢ with 25¢; with 50¢.

6. Compare the number of yards that can be bought for 50¢ and for 25¢; for 25¢ and for 75¢; for 75¢ and for 50¢.

7. State a rule for finding the cost of any number of articles at 25¢ each; at 50¢ each; at 75¢ each.

23. Make all extensions mentally, and find the total value of each of the following:

1.

16 yd. @ 25¢.
24 yd. @ 25¢.
30 yd. @ 50¢.
16 yd. @ 75¢.
40 yd. @ 50¢.

2.

32 yd. @ 50¢.
48 yd. @ 25¢.
54 yd. @ 50¢.
60 yd. @ 75¢.
80 yd. @ 75¢.

3.

120 yd. @ 25¢.
140 yd. @ 50¢.
150 yd. @ 25¢.
160 yd. @ 75¢.
126 yd. @ 25¢.

4.	5.	6.
13 yd. @ 25¢.	3486 lb. @ 25¢.	128 lb. @ 75¢.
21 yd. @ 50¢.	1984 lb. @ 50¢.	324 lb. @ 50¢.
39 yd. @ 25¢.	4821 lb. @ 25¢.	233 lb. @ 25¢.
31 yd. @ 75¢.	8448 lb. @ 75¢.	199 lb. @ 75¢.
48 yd. @ 75¢.	1331 lb. @ 50¢.	247 lb. @ 25¢.

Make up and find the total value of each of two or more groups of quantities like the above as directed by the teacher.

EIGHTHS

24. 1. 12½¢ is what part of 25¢? 2. ½ of ¼ is how much?

3. 12½¢ is what part of $1?

4. 25¢ is how many eighths of a dollar?

5. 25¢ + 12½¢ = how many cents?

6. 37½¢ is what part of a dollar?

7. What is the sum of ½ and ⅛? 50¢ + 12½¢ = ?

8. 62½¢ is what part of a dollar? 9. 75¢ + 12½¢ = ?

10. $⅞ = how many cents?

11. What part of a dollar is 12½¢? 37½¢? 62½¢? 87½¢?

12. What is the cost of 16 lb. of raisins at 12½¢ a pound? of 16 baskets of potatoes at 37½¢ a basket? of 16 yd. of cloth at 62½¢ per yard? of 16 bu. of apples at 87½¢ a bushel?

25. Using the aliquot-part method, find the total value of each of the following:

1.	2.	3.
24 lb. @ 37½¢.	48 lb. @ 12½¢.	120 yd. @ 12½¢.
32 lb. @ 12½¢.	64 lb. @ 25¢.	164 yd. @ 12½¢.
40 lb. @ 62½¢.	72 lb. @ 37½¢.	200 yd. @ 37½¢.
56 lb. @ 37½¢.	88 lb. @ 87½¢.	240 yd. @ 12½¢.

Make two or more groups like the above and find the total value of each.

SIXTEENTHS

26. 1. $6\frac{1}{4}$¢ is what part of $12\frac{1}{2}$¢? of 25¢? of 50¢? of $1?

2. 3 times $6\frac{1}{4}$¢ is how many cents?

3. $18\frac{3}{4}$¢ is what part of a dollar?

4. 25¢ or $\$\frac{1}{4}$ is how many sixteenths of a dollar?

5. 25¢ + $6\frac{1}{4}$¢ are how many cents?

6. $31\frac{1}{4}$¢ is what part of a dollar?

7. $\frac{1}{2}-\frac{1}{16}=?$ $43\frac{3}{4}$¢ is what part of a dollar?

8. $\frac{1}{2}+\frac{1}{16}=?$ $56\frac{1}{4}$¢ = what part of a dollar?

9. In like manner find how many cents there are in $\$\frac{11}{16}$, $\$\frac{13}{16}$, and $\$\frac{15}{16}$ by comparing $6\frac{1}{4}$¢ ($\$\frac{1}{16}$) with 75¢ and $1.

27. Complete the following table of sixteenths, eighths, quarters, and halves.

Compare one part with another.

	Eighths	Quarters	Halves	Cents		Eighths	Quarters	Halves	Cents
$\$\frac{1}{16}$				$6\frac{1}{4}$	$\$\frac{9}{16}$				
$\$\frac{2}{16}$	$\$\frac{1}{8}$			$12\frac{1}{2}$	$\$\frac{10}{16}$				
$\$\frac{3}{16}$				$18\frac{3}{4}$	$\$\frac{11}{16}$				
$\$\frac{4}{16}$	$\$\frac{2}{8}$	$\$\frac{1}{4}$		25	$\$\frac{12}{16}$				
$\$\frac{5}{16}$					$\$\frac{13}{16}$				
$\$\frac{6}{16}$					$\$\frac{14}{16}$				
$\$\frac{7}{16}$					$\$\frac{15}{16}$				
$\$\frac{8}{16}$					$\$\frac{16}{16}$				

28. Make the extensions mentally, and find the total value of each of the following:

1.

32 yd. @ $6\frac{1}{4}$¢.
48 yd. @ $12\frac{1}{2}$¢.
64 yd. @ $18\frac{3}{4}$¢.
96 yd. @ $6\frac{1}{4}$¢.
160 yd. @ $43\frac{3}{4}$¢.

2.

80 yd. @ $31\frac{1}{4}$¢.
128 yd. @ $37\frac{1}{2}$¢.
144 yd. @ $56\frac{1}{4}$¢.
96 yd. @ $93\frac{3}{4}$¢.
72 yd. @ $37\frac{1}{2}$¢.

3.

47 yd. @ 25¢.
53 yd. @ $12\frac{1}{2}$¢.
58 yd. @ $6\frac{1}{4}$¢.
63 yd. @ 50¢.
75 yd. @ $37\frac{1}{2}$¢.

Note. In the second item, in No. 3, 53 yd. @ $12\frac{1}{2}$¢ = \$$6\frac{5}{8}$ = \$$6.62\frac{1}{2}$; write the extension as \$6.63.

Write two or more groups like the above and find the total value of each.

Thirds and Sixths

29. 1. What part of a dollar is $33\frac{1}{3}$¢? $66\frac{2}{3}$¢?

2. What is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{2}$ of $33\frac{1}{3}$¢ is how many cents?

3. $16\frac{2}{3}$¢ = what part of a dollar?

4. 3 times $16\frac{2}{3}$¢ = how many cents? what part of a dollar?

5. 4 times $16\frac{2}{3}$¢ = how many cents? what part of a dollar?

6. \$1 − $16\frac{2}{3}$¢ = how many cents? what part of a dollar?

30. Complete the following table. State the relation of each part to the parts that follow and precede it.

	Thirds	Halves	Cents
\$$\frac{1}{6}$			$16\frac{2}{3}$
\$$\frac{2}{6}$	\$$\frac{1}{3}$		$33\frac{1}{3}$
\$$\frac{3}{6}$		\$$\frac{1}{2}$	50
\$$\frac{4}{6}$			
\$$\frac{5}{6}$			
\$$\frac{6}{6}$			

31. Find total value of each group:

1. 48 lb. @ $16\frac{2}{3}$¢.
54 lb. @ $66\frac{2}{3}$¢.
30 lb. @ $83\frac{1}{3}$¢.

2. 66 yd. @ $33\frac{1}{3}$¢.
76 yd. @ 25¢.
76 yd. @ $12\frac{1}{2}$¢.

3.
64 lb. @ $6\frac{1}{4}$¢.
96 lb. @ $33\frac{1}{3}$¢.
88 lb. @ $37\frac{1}{2}$¢.
91 lb. @ $66\frac{2}{3}$¢.

4.
48 qt. @ $33\frac{1}{3}$¢.
72 qt. @ $16\frac{2}{3}$¢.
96 qt. @ $12\frac{1}{2}$¢.
60 qt. @ $6\frac{1}{4}$¢.

5.
120 yd. @ $37\frac{1}{2}$¢.
126 yd. @ $16\frac{2}{3}$¢.
132 yd. @ $83\frac{1}{3}$¢.
144 yd. @ $33\frac{1}{3}$¢.

Write and find the total value of two or more groups like the above.

Twelfths

32. 1. $\frac{1}{3}$ of 25¢ = how many cents?

2. $\frac{1}{3}$ of $\frac{1}{4}$ = ? $8\frac{1}{3}$¢ = what part of a dollar?

3. 50¢ − $8\frac{1}{3}$¢ = how many cents?

4. $\frac{1}{2} - \frac{1}{12}$ = ? \$ $41\frac{2}{3}$¢ = what part of a dollar?

5. 50¢ + $8\frac{1}{3}$¢ = how many cents? what part of a dollar?

6. \$1 − $8\frac{1}{3}$¢ = how many cents? what part of a dollar?

33. Complete the following table. Compare each part with the parts that follow and precede it.

34. Find total value of each of the following groups, making all extensions mentally:

	Sixths	Thirds	Quarters	Cents
\$ $\frac{1}{12}$				$8\frac{1}{3}$
\$ $\frac{2}{12}$	\$ $\frac{1}{6}$			$16\frac{2}{3}$
\$ $\frac{3}{12}$			\$ $\frac{1}{4}$	25
\$ $\frac{4}{12}$		\$ $\frac{1}{3}$		$33\frac{1}{3}$
\$ $\frac{5}{12}$				
\$ $\frac{6}{12}$				
\$ $\frac{7}{12}$				
\$ $\frac{8}{12}$				
\$ $\frac{9}{12}$				
\$ $\frac{10}{12}$				
\$ $\frac{11}{12}$				
\$ $\frac{12}{12}$				

1.

36 yd. @ $8\frac{1}{3}$¢.
48 yd. @ $16\frac{2}{3}$¢.
24 yd. @ $41\frac{2}{3}$¢.
60 yd. @ $58\frac{1}{3}$¢.
72 yd. @ $83\frac{1}{3}$¢.

2.

66 yd. @ $33\frac{1}{3}$¢.
84 yd. @ $41\frac{2}{3}$¢.
120 yd. @ $91\frac{2}{3}$¢.
42 yd. @ $58\frac{1}{3}$¢.
54 yd. @ $16\frac{2}{3}$¢.

Note. Many articles are bought by the dozen and sold by the piece. A knowledge of the "twelfths" is essential in determining the price of one.

3.

75 lb. @ $8\frac{1}{3}$¢.
88 lb. @ $16\frac{2}{3}$¢.
90 lb. @ $41\frac{2}{3}$¢.
96 lb. @ $58\frac{1}{3}$¢.
84 lb. @ $66\frac{2}{3}$¢.

4.

24 yd. @ $58\frac{1}{3}$¢.
36 yd. @ $41\frac{2}{3}$¢.
60 yd. @ $91\frac{2}{3}$¢.
48 yd. @ $16\frac{2}{3}$¢.
72 yd. @ $33\frac{1}{3}$¢.

5.

140 lb. @ $33\frac{1}{3}$¢.
160 lb. @ $91\frac{2}{3}$¢.
240 lb. @ $58\frac{1}{3}$¢.
360 lb. @ $41\frac{2}{3}$¢.
118 lb. @ $8\frac{1}{3}$¢.

Write two or more groups like the above and find the total value of each. Endeavor to use the shortest method of solution in each exercise. Omit actual multiplication wherever possible.

35. OTHER SUGGESTIONS FOR SHORTENING THE WORK

1. Find the cost of 45 lb. at 18¢.

18¢ = \$ $\frac{1}{5}$ − ($\frac{1}{10}$ of \$ $\frac{1}{5}$).

$$\begin{array}{rl} \frac{1}{5} \text{ of } 45 = & 9 \\ \frac{1}{10} \text{ of } 9 = & \underline{.90} \\ & 8.10\text{; Cost} = \$8.10. \end{array}$$

2. Find the cost of 72 yd. at $17\frac{1}{2}$¢.

$17\frac{1}{2}$¢ = \$ $\frac{1}{10}$ + ($\frac{1}{2}$ of \$ $\frac{1}{10}$) + $\frac{1}{2}$($\frac{1}{2}$ of \$ $\frac{1}{10}$) or, 10¢ + 5¢ + $2\frac{1}{2}$¢. Hence 72 yd. @ $17\frac{1}{2}$¢ cost \$7.20 + \$3.60 + \$1.80, or \$12.60.

3. Formulate a short method for finding the cost of articles @ 24¢; @ 27½¢; @ 45¢.

24¢ = $¼ − 1¢.
27½¢ = $¼ + (1/10 of $¼).
45¢ = $½ − (1/10 of $½).

The student is urged to devise short methods for himself, and to use them on all possible occasions.

36. Find total value of each group:

1.
48 yd. @ 7½¢.
56 yd. @ 9¢.
64 yd. @ 13½¢.
80 yd. @ 11½¢.
88 yd. @ 17½¢.

2.
90 yd. @ 18¢.
96 yd. @ 22¢.
110 yd. @ 22½¢.
116 yd. @ 27½¢.
124 yd. @ 35¢.

3.
7½ yd. @ 36¢.
11½ yd. @ 48¢.
16 yd. @ 45¢.
17½ yd. @ 62¢.
18 yd. @ 65¢.

4.
55 yd. @ 44¢.
55 yd. @ 36¢.
65 yd. @ 33¢.
51 yd. @ 27¢.
39 yd. @ 55¢.

5.
132 yd. @ 22¢.
141 yd. @ 22½¢.
123 yd. @ 17½¢.
163 yd. @ 27½¢.
172 yd. @ 27¢.

6.
219½ yd. @ 33¢.
313¼ yd. @ 35¢.
317½ yd. @ 44¢.
273½ yd. @ 45¢.
124¾ yd. @ 55¢.

How did you calculate by 13½¢ in No. 1? by 22½¢ in No. 2? by 48¢ in No. 3? by 36¢ in No. 4? etc.

REVIEW

37. Using the shortest solution, find the total cost of:

1.
54 yd. @ 16⅔¢.
40 yd. @ 25¢.
36 yd. @ 33⅓¢.
84 yd. @ 50¢.
75 yd. @ 66⅔¢.
96 yd. @ 12½¢.
48 yd. @ 8⅓¢.
36 yd. @ 16⅔¢.
42 yd. @ 25¢.

2.
120 yd. @ 62½¢.
240 yd. @ 87½¢.
160 yd. @ 66⅔¢.
360 yd. @ 91⅔¢.
164 yd. @ 37½¢.
66 yd. @ 41⅔¢.
68 yd. @ 43¾¢.
75 yd. @ 58⅓¢.
111 yd. @ 33⅓¢.

3.
33 yd. @ 12½¢.
41 yd. @ 8⅓¢.
52 yd. @ 6¼¢.
63 yd. @ 12½¢.
89 yd. @ 33⅓¢.
113 yd. @ 75¢.
97 yd. @ 62½¢.
116 yd. @ 87½¢.
121 yd. @ 56¼¢.

4.

76 yd. @ 37½¢.
92 yd. @ 31¼¢.
83 yd. @ 43¾¢.
78 yd. @ 41⅔¢.
107 yd. @ 6¼¢.
113 yd. @ 62½¢.
114 yd. @ 83⅓¢.
131 yd. @ 16⅔¢.
125 yd. @ 25¢.
143 yd. @ 6¼¢.

5.

78 lb. @ 18¾¢.
93 lb. @ 25¢.
82 lb. @ 33⅓¢.
13 lb. @ 31¼¢.
9 lb. @ 37½¢.
27 lb. @ 41⅔¢.
32 lb. @ 43¾¢.
84 lb. @ 50¢.
79 lb. @ 56¼¢.
97 lb. @ 58⅓¢.

6.

87 lb. @ 6¼¢.
94 lb. @ 8⅓¢.
104 lb. @ 12½¢.
231 lb. @ 16⅔¢.
321 lb. @ 18¾¢.
88 lb. @ 93¾¢.
136 lb. @ 91⅔¢.
316 lb. @ 83⅓¢.
28 lb. @ 81¼¢.
7 lb. @ 68¾¢.

7.

117 lb. @ 93¾¢.
104 lb. @ 91⅔¢.
116 lb. @ 87½¢.
203 lb. @ 83⅓¢.
308 lb. @ 81¼¢.
27 lb. @ 75¢.
96 lb. @ 68¾¢.
84 lb. @ 66⅔¢.
119 lb. @ 62½¢.
17 lb. @ 58⅓¢.
45 lb. @ 12½¢.

8.

13 lb. @ 56¼¢.
27 lb. @ 50¢.
4 lb. @ 43¾¢.
9 lb. @ 41⅔¢.
17 lb. @ 37½¢.
37 lb. @ 33⅓¢.
45 lb. @ 31¼¢.
83 lb. @ 25¢.
107 lb. @ 18¾¢.
119 lb. @ 16⅔¢.
128 lb. @ 18¾¢.

9.

67 lb. @ 6¼¢.
74 lb. @ 8⅓¢.
83 lb. @ 12½¢.
99 lb. @ 16⅔¢.
199 lb. @ 18¾¢.
127 lb. @ 25¢.
217 lb. @ 31¼¢.
134 lb. @ 33⅓¢.
229 lb. @ 37½¢.
168 lb. @ 41⅔¢.
39 lb. @ 31¼¢.

SEVENTHS, NINTHS, AND FIFTEENTHS

38. In addition to the aliquot parts of a dollar already found, there are a few others of which use is occasionally made: viz. sevenths, ninths, and fifteenths.

1. What is $\frac{1}{7}$ of a dollar? $\frac{2}{7}$? $\frac{3}{7}$?
2. What is $\frac{1}{9}$ of a dollar? $\frac{2}{9}$? $\frac{4}{9}$?
3. What is $\frac{1}{15}$ of a dollar? $\frac{2}{15}$?

NOTE. These lists will not be made complete, as there is seldom any use for others than those given above.

39. Make the extensions mentally, and find total value of each group:

1.

21 lb. @ $14\frac{2}{7}$¢.
28 lb. @ $28\frac{4}{7}$¢.
36 lb. @ $11\frac{1}{9}$¢.
45 lb. @ $6\frac{2}{3}$¢.
48 lb. @ $6\frac{1}{4}$¢.

2.

42 lb. @ $42\frac{6}{7}$¢.
54 lb. @ $44\frac{4}{9}$¢.
60 lb. @ $13\frac{1}{3}$¢.
84 lb. @ $14\frac{2}{7}$¢.
91 lb. @ $28\frac{4}{7}$¢.

3.

33 lb. @ $14\frac{2}{7}$¢.
45 lb. @ $28\frac{4}{7}$¢.
83 lb. @ $6\frac{2}{3}$¢.
96 lb. @ $11\frac{1}{9}$¢.
87 lb. @ $12\frac{1}{2}$¢.

Write two or more groups and find total value of each.

40. In the preceding exercises the price has been an aliquot part of $1. The same principle applies when the quantity is an aliquot part of 100 units of the article involved.

The cost of 48 yd. @ $12\frac{1}{2}$¢ = the cost of $12\frac{1}{2}$ yd. @ 48¢.

(*a*) **48 yd. @ $12\frac{1}{2}$¢ = $\frac{1}{8}$ of 48 = 6, hence $6, cost.**

(*b*) **100 yd. @ 48¢ = $48.**
$12\frac{1}{2}$ yd. = $\frac{1}{8}$ of 100 yd.
Hence, $12\frac{1}{2}$ yd. @ 48¢ cost $\frac{1}{8}$ of $48 = $6, cost.

41. Find total value of each of the following groups:

1.

25 yd. @ 32¢.
$33\frac{1}{3}$ yd. @ 45¢.
50 yd. @ 78¢.
$37\frac{1}{2}$ yd. @ 48¢.
75 yd. @ 80¢.

2.

$16\frac{2}{3}$ yd. @ 42¢.
$18\frac{3}{4}$ yd. @ 56¢.
$31\frac{1}{4}$ yd. @ 80¢.
$62\frac{1}{2}$ yd. @ 72¢.
$93\frac{3}{4}$ yd. @ 96¢.

3.

$6\frac{1}{4}$ lb. @ 16¢.
$8\frac{1}{3}$ lb. @ 24¢.
$12\frac{1}{2}$ lb. @ 32¢.
$16\frac{2}{3}$ lb. @ 42¢.
$18\frac{3}{4}$ lb. @ 48¢.

4.

25 lb. @ 43¢.
$31\frac{1}{4}$ lb. @ 56¢.
$33\frac{1}{3}$ lb. @ 58¢.
$37\frac{1}{2}$ lb. @ 68¢.
$43\frac{3}{4}$ lb. @ 96¢.

5.

78 yd. @ $33\frac{1}{3}$¢.
$16\frac{2}{3}$ yd. @ 33¢.
$56\frac{1}{4}$ yd. @ 44¢.
$58\frac{1}{3}$ yd. @ 66¢.
38 yd. @ $62\frac{1}{2}$¢.

6.

43 yd. @ $66\frac{2}{3}$¢.
$68\frac{3}{4}$ yd. @ 32¢.
$81\frac{1}{4}$ yd. @ 16¢.
$83\frac{1}{3}$ yd. @ 72¢.
80 yd. @ $93\frac{3}{4}$¢.

7.

89 lb. @ $6\frac{2}{3}$¢.
93 lb. @ $6\frac{1}{4}$¢.
87 lb. @ $12\frac{1}{2}$¢.
76 lb. @ $16\frac{2}{3}$¢.
116 lb. @ 25¢.

8.

160 yd. @ 75¢.
200 yd. @ $87\frac{1}{2}$¢.
240 yd. @ $93\frac{3}{4}$¢.
25 yd. @ $1.20.
50 yd. @ $1.60.

9.

1148 yd. @ $33\frac{1}{3}$¢.
2682 yd. @ $37\frac{1}{2}$¢.
$81\frac{1}{4}$ yd. @ 80¢.
$68\frac{3}{4}$ yd. @ 96¢.
75 yd. @ 72¢.

APPLICATION OF THE PRINCIPLES OF ALIQUOT PARTS

42. 1. Multiply 48 by 10; by 100; by 1000.

2. What did you do to multiply by 10? by 100? by 1000?

3. 2½ is what part of 10? 25 is what part of 100? 250 is what part of 1000?

4. Multiply 48 by 2½; by 25; by 250.

First multiply 48 by 10: that is, annex one cipher, thus, 480. Since the given multiplier is ¼ of 10, the required product is ¼ of 480, or 120.

In like manner to multiply by 25, first annex two ciphers to 48, thus, 4800, and divide by 4, because 25 is ¼ of 100, etc.

5. 3⅓ is what part of 10?
33⅓ is what part of 100?
333⅓ is what part of 1000?

6. Multiply 144 by 3⅓.

Since 3⅓ is ⅓ of 10, first annex one cipher (multiply by 10), thus, 1440, and divide by 3. The required product is 480.

7.
1¼ is what part of 10?
12½ is what part of 100?
125 is what part of 1000?

8.
1⅔ is what part of 10?
16⅔ is what part of 100?
166⅔ is what part of 1000?

43. Copy and complete the following table:

	1	10	100	1000
⅛'s	.12½	1¼	12½	125
¼'s				
⅙'s				
⅓'s				

44. Make the extensions mentally and find the total value of the items in each group:

1. 24 A. @ $25.
18 cows @ $25.
72 sheep @ $2½.
16 plows @ $12½.
32 horses @ $125.
160 bu. @ $1¼.
330 bu. @ $1.66⅔.
48 wagons @ $125.

2. 25 A. @ $96.
33⅓ bu. @ $1.08.
166⅔ bu. @ $.84.
1250 bu. @ $.64.
2500 bu. @ $.78.
1600 bu. @ $1.25.
16⅔ bu. @ $1.44.
3800 lb. @ $.12½.

ALIQUOT PARTS APPLIED TO DIVISION

45. **1.** At 25¢ a yard, how many yards of cloth can be bought for \$1? for \$2? for \$10? for \$25?

2. At $37\frac{1}{2}$¢ a yard, how many yards can be bought for \$1? for \$2? for \$5? for \$15?

$37\frac{1}{2}$¢ = \$$\frac{3}{8}$. \$1 ÷ \$$\frac{3}{8} = \frac{8}{3}$; $\frac{8}{3}$ yd., or $2\frac{2}{3}$ yd., can be bought for \$1.

Since $\frac{8}{3}$ yd., or $2\frac{2}{3}$ yd., can be bought for \$1, 2 times $2\frac{2}{3}$ yd., or $5\frac{1}{3}$ yd., can be bought for \$2, and 5 times $2\frac{2}{3}$ yd., or $13\frac{1}{3}$ yd., can be bought for \$5, etc.

NOTE. $1 \div \frac{3}{8} = \frac{8}{3}$. $\frac{8}{3}$ is called the **reciprocal** of $\frac{3}{8}$. The reciprocal of any fraction is the fraction inverted.

Hence, to divide by an aliquot part, multiply by the reciprocal of the fractional equivalent.

46. Calculate the number of yards that can be bought in each case:

TOTAL COST	COST OF ONE YARD	TOTAL COST	COST OF ONE YARD
1. \$15.00	\$$.33\frac{1}{3}$	**16.** \$91.00	\$$.58\frac{1}{3}$
2. 36.00	.75	**17.** 100.00	$.62\frac{1}{2}$
3. 75.00	$.62\frac{1}{2}$	**18.** 110.00	$.66\frac{2}{3}$
4. 25.00	$.31\frac{1}{4}$	**19.** 121.00	$.68\frac{3}{4}$
5. 13.00	$.14\frac{2}{7}$	**20.** 129.00	.75
6. 16.00	$.08\frac{1}{3}$	**21.** 130.00	$.81\frac{1}{4}$
7. 27.00	$.18\frac{3}{4}$	**22.** 135.00	$.83\frac{1}{3}$
8. 29.00	.25	**23.** 59.50	$.87\frac{1}{2}$
9. 35.00	$.31\frac{1}{4}$	**24.** 66.00	$.91\frac{2}{3}$
10. 41.00	$.33\frac{1}{3}$	**25.** 75.00	$.93\frac{3}{4}$
11 42.00	$.37\frac{1}{2}$	**26.** 45.00	1.25*
12. 47.50	$.41\frac{2}{3}$	**27.** 93.00	1.50
13. 49.00	$.43\frac{3}{4}$	**28.** 90.00	$1.87\frac{1}{2}$
14. 57.00	.50	**29.** 105.00	2.50
15. 81.00	$.56\frac{1}{4}$	**30.** 440.00	$3.33\frac{1}{3}$

TRADE DISCOUNT

47. All the information about aliquot parts in the preceding pages is directly applicable to Trade Discount.

48. The expression "per cent" (symbol %) means hundredths. That is, $25\% = \frac{25}{100} = \frac{1}{4}$. It is also written .25, in its decimal form, the figures being the same as are used to write 25¢ decimally. It follows, that for every price quoted in the problems in Aliquot Parts, the fractional part would be the same if the sign % were substituted for "cents."

---

* $1.25 = \frac{5}{4}$, the reciprocal of which is $\frac{4}{5}$. Hence the number of yards $= \frac{4}{5}$ of 45.

Therefore, $12\frac{1}{2}\%$, $33\frac{1}{3}\%$, $56\frac{1}{4}\%$, etc. represent the same common fractions, or aliquot parts, as do $12\frac{1}{2}$¢, $33\frac{1}{3}$¢, $56\frac{1}{4}$¢, etc.

49. Find the net cost of the following, allowing a discount of $37\frac{1}{2}\%$:

66 lb. @ $66\frac{2}{3}$¢ = $44
80 lb. @ 40¢ = $32
120 lb. @ 75¢ = $90
240 lb. @ $37\frac{1}{2}$¢ = $90 $256

Less $37\frac{1}{2}\%$ $(\frac{1}{4}+\frac{1}{2}$ of $\frac{1}{4})$ 96 $160

The items are extended mentally by taking $\frac{2}{3}$ of 66; $\frac{2}{5}$ of 80; $\frac{3}{4}$ of 120; and $\frac{3}{8}$ of 240. By adding, the gross cost is found to be $256. $37\frac{1}{2}\%$ discount = a discount of $\frac{3}{8}(\frac{1}{4}+\frac{1}{8})$. $\frac{3}{8}$ of $256 = $96, which, deducted, leaves $160, the net cost.

50. Find the net cost of each of the following:

1.
72 lb. @ $16\frac{2}{3}$¢.
48 lb. @ $6\frac{1}{4}$¢.
32 lb. @ $12\frac{1}{2}$¢.
56 lb. @ $37\frac{1}{2}$¢.
Less $12\frac{1}{2}\%$.

2.
72 lb. @ 25¢.
96 lb. @ $33\frac{1}{3}$¢.
90 lb. @ $44\frac{4}{9}$¢.
80 lb. @ $87\frac{1}{2}$¢.
Less $18\frac{3}{4}\%$.

3.
36 doz. @ 75¢.
48 doz. @ $12\frac{1}{2}$¢.
60 doz. @ 25¢.
72 doz. @ $37\frac{1}{2}$¢.
Less $16\frac{2}{3}\%$.

4.
48 doz. @ $6\frac{1}{4}$¢.
64 doz. @ $1.25.
24 doz. @ 84¢.
$37\frac{1}{2}$ doz. @ $2.40.
Less $33\frac{1}{3}\%$.

5.
64 lb. @ $12\frac{1}{2}$¢.
240 lb. @ $33\frac{1}{3}$¢.
480 lb. @ $56\frac{1}{4}$¢.
320 lb. @ $6\frac{1}{4}$¢.
Less 10%.

6.
125 lb. @ 48¢.
375 lb. @ 72¢.
320 lb. @ $62\frac{1}{2}$¢.
250 lb. @ 36¢.
Less $12\frac{1}{2}\%$.

7.
196 doz. @ $1.25.
26 doz. @ 75¢.
86 doz. @ $12\frac{1}{2}$¢.
50 doz. @ 74¢.
Less 25%.

8.
326 doz. @ $62\frac{1}{2}$¢.
112 doz. @ 75¢.
84 doz. @ $62\frac{1}{2}$¢.
2500 lb. @ $6\frac{1}{4}$¢.
Less $37\frac{1}{2}\%$.

9.
841 lb. @ $12\frac{1}{2}$¢.
963 lb. @ $16\frac{2}{3}$¢.
1047 lb. @ 25¢.
141 lb. @ $33\frac{1}{3}$¢.
Less 20%.

FURTHER SUGGESTIONS FOR SHORTENING MULTIPLICATION

51. In addition to the so-called Aliquot Parts of 100, the use and application of which have been explained, there are many other numbers that are a fractional part more or less than the aliquot parts. By making use of this fact the work of multiplying by numbers that are not aliquot parts of 100 may be shortened. For example, $13\frac{1}{8}$¢ is not an aliquot part of a dollar, yet to multiply by $.13\frac{1}{8}$ requires no actual multiplication.

1. Multiply 120 by $.13\frac{1}{8}$.

$$.13\tfrac{1}{8} = .10 + (\tfrac{1}{3} \text{ of } .10).$$

Therefore, to multiply 120 by $.13\frac{1}{8}$, point off one place in 120, thus, 12.0, and add $\frac{1}{3}$ of 12, which gives a result of 16.

2. Multiply 36 by $.23\frac{1}{8}$.

$$.23\tfrac{1}{3} = .20 + (\tfrac{1}{6} \text{ of } .20) = \tfrac{2}{10} + (\tfrac{1}{6} \text{ of } \tfrac{2}{10}).$$

$$\begin{array}{l} \underline{3.6 = \tfrac{1}{10} \text{ of } 36.} \\ 7.2 = \tfrac{2}{10} \text{ of } 36. \\ \underline{1.2} = \tfrac{1}{6} \text{ of } \tfrac{2}{10} \text{ of } 36. \\ 8.4 = .23\tfrac{1}{3} \text{ of } 36. \end{array}$$

Moving the point one place to the left in 36 gives $\frac{1}{10}$ of 36; multiplying by 2 gives $\frac{2}{10}$ of 36. To find the remaining part take $\frac{1}{6}$ of 7.2, which is 1.2. Add this to 7.2, giving 8.4, the desired result.

3. To multiply by $.02\frac{1}{2}$, point off 1 place and divide by 4.

4. To multiply by $.03\frac{1}{3}$, point off 1 place and divide by 3.

5. To multiply by .05, point off 1 place and divide by 2.

6. To multiply by $.07\frac{1}{2}$, point off 1 place and deduct $\frac{1}{4}$ of the result.

7. To multiply by $.11\frac{1}{4}$, point off 1 place and add $\frac{1}{8}$ of the result.

8. To multiply by $.13\frac{3}{4}$, point off 1 place and add $\frac{1}{4}$ of the result, plus $\frac{1}{2}$ of the $\frac{1}{4}$. Etc.

EXERCISE

52. Devise a short method of multiplying by each of the following:

1. .09
2. .10
3. .11
4. $.11\frac{1}{4}$
5. $.13\frac{1}{8}$
6. .15
7. $.17\frac{1}{2}$
8. .18
9. .20
10. .22
11. $.22\frac{1}{2}$
12. $.23\frac{1}{3}$
13. $.23\frac{3}{4}$
14. .24
15. .26
16. $.26\frac{2}{3}$
17. .27
18. $.27\frac{1}{2}$
19. $.28\frac{3}{4}$
20. $.31\frac{1}{4}$
21. .33
22. $.33\frac{3}{4}$
23. .35
24. .36
25. $.37\frac{1}{2}$
26. .44
27. .45
28. $112\frac{1}{2}$
29. 425
30. 375

53. Write by inspection the product of each of the following, and find the total of all the products in each group:

1.	2.	3.
24 lb. @ $2\frac{1}{2}$¢.	123 lb. @ 10¢.	84 lb. @ 15¢.
24 lb. @ $3\frac{1}{3}$¢.	113 lb. @ 11¢.	92 lb. @ $17\frac{1}{2}$¢.
48 lb. @ 5¢.	120 lb. @ $11\frac{1}{4}$¢.	64 lb. @ 22¢.
44 lb. @ $7\frac{1}{2}$¢.	140 lb. @ 12¢.	72 lb. @ $23\frac{3}{4}$¢.
56 lb. @ 9¢.	150 lb. @ $13\frac{1}{3}$¢.	44 lb. @ 24¢.

4.	5.	6.
54 lb. @ $26\frac{2}{3}$¢.	37 by 11.	140 by $31\frac{1}{4}$.
88 lb. @ $27\frac{1}{2}$¢.	28 by $13\frac{3}{4}$.	150 by $37\frac{1}{2}$.
144 lb. @ $31\frac{1}{4}$¢.	44 by $18\frac{3}{4}$.	160 by $43\frac{3}{4}$.
160 lb. @ $33\frac{3}{4}$¢.	48 by 22.	180 by 45.
176 lb. @ 35¢.	56 by $22\frac{1}{2}$.	190 by $47\frac{1}{2}$.

54. To find the cost of goods sold by the 100, 1000, or ton.

1. Find the cost of 168 lb. of feed @ $1.10 per cwt.

$1.68 = cost at $1 per cwt.
.17 = cost at .10 per cwt.
$1.85 = cost at 1.10 per cwt.

First find the number of hundredweight by pointing off 2 places in the number of pounds, thus, 168 lb. = 1.68 cwt. At $1 per hundredweight, the cost would be $1.68. At $.10 per hundredweight, the cost would be $.168, or $.17. Hence, at $1.10 per hundredweight, the cost would be the sum of $1.68 and $.17, or $1.85.

2. At $1.87 per C, find the cost of 634 bolts.

634 = 6.34 C.
6.34 × $1.87 = $11.86

634 bolts = 6.34 hundreds. Since 1 hundred costs $1.87, 6.34 hundreds cost 6.34 times $1.87, or $11.86.

3. At $15 per thousand, find the cost of 648 ft. of lumber.

$6.48 = cost at 1¢ per foot.
3.24 = cost at $\frac{1}{2}$¢ per foot.
$9.72 = cost at $15 per M.

$15 per M = $1\frac{1}{2}$¢ per foot. At 1¢ a foot, 648 ft. cost $6.48, and at $\frac{1}{2}$¢ a foot, the cost is $\frac{1}{2}$ of $6.48, or $3.24. Hence, at $1\frac{1}{2}$¢ a foot or $15 per M, the cost is $6.48 + $3.24, or $9.72.

4. At $16.40 per M, find the cost of 13,400 bricks.

13400 = 13.4 thousands.
13.4 × $16.40 = $219.76

The number of M is found by pointing off 3 places in the number 13,400, giving 13.4 M. At $16.40 per M, 13.4 M cost 13.4 × $16.40, or $219.76.

5. At $6.50 per ton, how much will 13,480 lb. of coal cost?

$6.50 per ton = $3.25 per 1000 lb.
13.48 × $3.25 = $43.81.

The price of 1000 lb. is $\frac{1}{2}$ the price of a ton, or $3.25. 13,480 lb. = 13.48 thousand pounds. Since 1 thousand pounds cost $3.25, 13.48 thousands cost 13.48 × $3.25, or $43.81.

6. What is the cost of 24,680 lb. of hay at $18 per ton?

$18 per ton = $9 per 1000 lb.
24680 = 24.68 thousands.
$246.80 = cost at $10 per 1000 lb.
24.68 = cost at 1 per 1000 lb.
$222.12 = cost at 9 per 1000 lb.

The cost of $\frac{1}{2}$ of ton, or 1000 lb., is $9. 24,680 lb. = 24.680 thousand pounds (point moved 3 places to the left). If the price were $10 per 1000 lb., the cost would be $246.80 (point moved 1 place to the right). Deducting the cost at $1 per 1000 lb., or $24.68, leaves $222.12, the required result.

SUGGESTIONS FOR RAPID WORK

1. $12.50 per 1000 ft. = $1\frac{1}{4}$¢ per foot.
Hence 4728 ft. at $12.50 per 1000 ft. =

$47.28 at 1¢ a foot
11.82 at $\frac{1}{4}$¢ a foot
$59.10 at $1\frac{1}{4}$¢ a foot, or $12.50 per 1000 ft.

2. $13.75 per 1000 ft. = ($12.50 + $\frac{1}{10}$ of $12.50) per 1000 ft.
Hence 3976 ft. at $13.75 per 1000 ft. =

$49.70 at $12.50 per 1000 ft.
4.97 at $1.25 per 1000 ft.
$54.67 at $13.75 per 1000 ft.

3. 3.37\frac{1}{2}$ per 100 lb. = ($3 + $\frac{1}{8}$ of $3) per 100 lb., or
($3 + $$\frac{1}{4}$ + $\frac{1}{2}$ of $$\frac{1}{4}$) per 100 lb.
Hence 760 lb. at 3.37\frac{1}{2}$ per 100 lb. =

$22.80 at $3 per 100 lb.
2.85 at $.37$\frac{1}{2}$ per 100 lb.
$25.65 at 3.37\frac{1}{2}$ per 100 lb.,

or

$22.80 at $3 per 100 lb.
1.90 at $$\frac{1}{4}$ per 100 lb.
.95 at $\frac{1}{2}$ of $$\frac{1}{4}$ per 100 lb.
$25.65 at 3.37\frac{1}{2}$ per 100 lb.

4. 5.62\frac{1}{2}$ per 100 lb. = ($5 + $\frac{1}{8}$ of $5) per 100 lb.

Hence 875 lb. at 5.62\frac{1}{2}$ per100 lb. =
$43.75 at $5 per 100 lb.
5.47 at $.62$\frac{1}{2}$ per 100 lb.
$49.22 at 5.62\frac{1}{2}$ per 100 lb.

5. \$13.50 per ton = (\$6 + $\frac{1}{8}$ of \$6) per 1000 lb.
Hence 2780 lb. at \$13.50 per ton =

\$16.68 at \$6 per 1000 lb.
2.09 at \$.75 per 1000 lb.
\$18.77 at \$6.75 per 1000 lb., or \$13.50 per ton.

6. \$1.35 per 100 lb. = (1¢ + $\frac{1}{4}$¢ + $\frac{1}{10}$¢) per 100 lb.

Hence 580 lb. at \$1.35 per 100 lb. =
\$5.80 at 1¢ per pound
1.45 at $\frac{1}{4}$¢ per pound
.58 at $\frac{1}{10}$¢ per pound
\$7.83 at \$1.35 per 100 lb.

7. \$31.25 per 1000 lb. = (\$25 + $\frac{1}{4}$ of \$25) per 1000 lb.
Hence 6720 lb. at \$31.25 per 1000 lb. =

\$168.00 at \$25 per 1000 lb.
42.00 at \$6.25 per 1000 lb.
\$210.00 at \$31.25 per 1000 lb.

Devise a short method of finding the value of:

8. 4370 lb. coal at \$6.75 per ton.
9. 5860 lb. hay at \$15.75 per ton.
10. 22,470 brick at \$11.25 per 1000.
11. 528 lb. meal at \$1.75 per 100 lb.
12. 3156 lb. feed at \$27 per ton.
13. 17,640 ft. lumber at \$23.75 per 1000 ft.
14. 13,260 ft. lumber at \$33.75 per 1000 ft.

55. Solve mentally, if possible, the following:

Find the cost of:

1.	220 lb. bran	@ \$	.90	per cwt.;	@ \$	1.10	per cwt.
2.	350 lb. feed	@	1.25	per cwt.;	@	1.15	per cwt.
3.	675 lb. nails	@	3.12$\frac{1}{2}$	per cwt.;	@	3.37$\frac{1}{2}$	per cwt.
4.	875 lb. salt	@	.80	per cwt.;	@	.88	per cwt.
5.	950 posts	@	6.25	per C;	@	6.75	per C.
6.	1100 posts	@	7.25	per C;	@	7.70	per C.
7.	1250 posts	@	8.00	per C;	@	7.20	per C.
8.	1550 lb. lead	@	2.50	per cwt.;	@	3.37$\frac{1}{2}$	per cwt.
9.	38,440 lb. pig iron	@	1.50	per cwt.;	@	1.62$\frac{1}{2}$	per cwt.
10.	24,680 lb. sugar	@	5.50	per cwt.;	@	5.75	per cwt.
11.	1875 ft. lumber	@	16.00	per M;	@	18.00	per M.

12.	1450 ft. lumber	@ $ 25.00 per M;	@ $ 22.00 per M.
13.	23,360 brick	@ 8.75 per M;	@ 9.90 per M.
14.	14,500 shingles	@ 5.62½ per M;	@ 5.25 per M.
15.	346 ft. lumber	@ 33.00 per M;	@ 31.25 per M.
16.	23,465 ft. lumber	@ 37.50 per M;	@ 33.75 per M.
17.	47,860 brick	@ 11.25 per M;	@ 12.50 per M.
18.	12,375 brick	@ 13.75 per M;	@ 15.00 per M.
19.	7850 shingles	@ 6.75 per M;	@ 7.50 per M.
20.	319 ft. lumber	@ 45.00 per M;	@ 55.00 per M.
21.	2660 lb. hay	@ 18.00 per T.;	@ 22.50 per T.
22.	3474 lb. hay	@ 26.00 per T.;	@ 24.00 per T.
23.	3096 lb. straw	@ 14.50 per T.;	@ 13.75 per T.
24.	7276 lb. straw	@ 13.75 per T.;	@ 12.50 per T.
25.	1680 lb. feed	@ 24.60 per T.;	@ 27.90 per T.
26.	4288 lb. feed	@ 26.00 per T.;	@ 27.00 per T.
27.	13,890 lb. coal	@ 6.25 per T.;	@ 5.50 per T.
28.	47,976 lb. coal	@ 6.75 per T.;	@ 6.60 per T.
29.	1890 lb. coal	@ 4.50 per T.;	@ 4.40 per T.
30.	560 lb. coal	@ 6.40 per T.;	@ 6.25 per T.

ALIQUOT PARTS APPLIED TO SIMPLE INTEREST

56. Simple interest * is treated (briefly) in this part of the text for two reasons: first, because it is required early in the course by all students who are at the same time pursuing the study of bookkeeping; and second, to correlate it with the work in "Aliquot Parts."

57. **Interest** is the price paid for the use of money.

58. The **principal** is the money for the use of which interest is paid.

59. For ordinary interest calculations, a **year** is considered as having 12 mo. of 30 da. each, or 360 da.

At 6 % for one year the interest on $ 1 is $ .06.
2 mo., or 60 da., is ⅙ of a year.
Hence, at 6 % for 2 mo. or 60 da., the interest

on $1 is $.01
on $2 is $.02
on $15 is $.15
on $125 is $1.25
on $750.25 is $7.5025 = $7.50.

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* For further work in interest see pages 202–213.

60. Find the interest on \$450.50 for 60 da. at 6%.

\$4|50.50 = \$4.51, int. at 6% for 60 da.

Since at 6% for 60 da. the interest on \$1 is \$.01, the interest on \$450.50 is \$.01 for each dollar, or \$4.5050, which equals \$4.51. (Answers in interest problems are always given in the nearest whole cent.)

A vertical line is used in place of a decimal point in solutions of this kind.

61. Find the interest at 6% on each of the following for 60 da.:

1. \$630
2. \$750
3. \$365
4. \$975
5. \$846
6. \$932
7. \$1036
8. \$1145
9. \$2465
10. \$3675
11. \$4270
12. \$362.50
13. \$824.75
14. \$968.66
15. \$732.45
16. \$1248.72

62. In the preceding pages on aliquot parts 100 was the basis of comparison. In interest solutions 60 is the basis. Memorize the following aliquot parts of 60. (Compare with the parts of an hour.)

$30 = \frac{1}{2}$ of 60
$20 = \frac{1}{3}$ of 60
$15 = \frac{1}{4}$ of 60
$12 = \frac{1}{5}$ of 60
$10 = \frac{1}{6}$ of 60
$6 = \frac{1}{10}$ of 60
$5 = \frac{1}{12}$ of 60
$4 = \frac{1}{15}$ of 60
$3 = \frac{1}{20}$ of 60

63. What combinations of 60 da. and aliquot parts of 60 da. will best give (*a*) 45 da.? (*b*) 27 da.? (*c*) 85 da.?

(*a*) 60 da.
$\underline{15}$ da. $= \frac{1}{4}$ of 60
45 da.

Subtract 15 da.

(*b*) $\underline{60}$ da.
20 da. $= \frac{1}{3}$ of 60
6 da. $= \frac{1}{10}$ of 60
$\underline{1}$ da. $= \frac{1}{6}$ of 6
27 da.

Add 20 da. 6 da. and 1 da.

(*c*) 60 da.
20 da. $= \frac{1}{3}$ of 60
$\underline{5}$ da. $= \frac{1}{4}$ of 20
85 da.

Add 60 da. 20 da. and 5 da.

64. Find combinations of 60 da. and aliquot parts of 60 da. that will best amount to:

1. 55 da.
2. 40 da.
3. 33 da.
4. 35 da.
5. 25 da.
6. 26 da.
7. 66 da.
8. 75 da.
9. 72 da.
10. 80 da.
11. 84 da.
12. 90 da.
13. 11 da.
14. 14 da.
15. 16 da.
16. 76 da.
17. 96 da.
18. 105 da.
19. 110 da.
20. 135 da.

65. **1.** Find the interest at 6 % on $ 540 for 22 da.

$ 5 | 40 = int. for 60 da.
1 | 80 = int. for 20 da.
| 18 = int. for 2 da.
1 | 98 = int. for 22 da.

22 da. is composed of the aliquot parts 20 da. and 2 da.

$ 5.40 is the interest for 60 da.

20 da. is $\frac{1}{3}$ of 60 da., hence the interest for 20 da. is $\frac{1}{3}$ of $ 5.40, or $ 1.80. The remaining 2 da. is $\frac{1}{10}$ of 20 da., and the interest for 2 da. is $\frac{1}{10}$ of $ 1.80, or $ .18. The interest for 22 da. is $ 1.98.

2. Find the interest at 6 % on $ 1716.75 for 87 da.

$ 17 | 1675 = int. for 60 da.
5 | 7225 = int. for 20 da.
1 | 7167 = int. for 6 da.
| 2861 = int. for 1 da.
$ 24 | 8928 = int. for 87 da.
or $ 24 | 89 = int. for 87 da.

87 da. = 60 da. + 20 da. + 6 da. + 1 da. Find 60 da. interest, which is $ 17.1675 and add to it $\frac{1}{3}$ of itself (for 20 da.), $\frac{1}{10}$ of itself (for 6 da.), and $\frac{1}{6}$ of the interest for 6 da. (for 1 da.) which gives $ 24.89, the interest for 87 da.

Notes. (1) The vertical line takes the place of the decimal points in the solution.

(2) The interest for fractional parts of the time should be kept correct to the 4th decimal place to insure accuracy in the cents column.

66. Find the interest at 6 % on:

1. $ 260 for 30 da.
2. $ 360 for 45 da.
3. $ 480 for 50 da.
4. $ 560 for 36 da.
5. $ 840 for 26 da.
6. $ 720 for 25 da.
7. $ 960 for 40 da.
8. $ 900 for 44 da.
9. $ 1260 for 70 da.
10. $ 1600 for 90 da.
11. $ 1800 for 80 da.
12. $ 2000 for 75 da.
13. $ 2100 for 72 da.
14. $ 2200 for 96 da.
15. $ 2600 for 105 da.
16. $ 3000 for 16 da.
17. $ 4200 for 30 da.
18. $ 4800 for 33 da.
19. $ 5100 for 45 da.
20. $ 8000 for 45 da.
21. $ 9000 for 50 da.
22. $ 9600 for 90 da.
23. $ 1000 for 75 da.
24. $ 1340 for 120 da.
25. $ 1600 for 150 da.
26. $ 1800 for 210 da.
27. $ 3000 for 180 da.
28. $ 4000 for 240 da.
29. $ 4500 for 320 da.
30. $ 4800 for 160 da.
31. $ 5000 for 150 da.
32. $ 6300 for 160 da.
33. $ 6600 for 140 da.
34. $ 7200 for 115 da.
35. $ 7500 for 100 da.
36. $ 8400 for 99 da.

67. **1.** What is the interest on $100 at 6 % for 6 da.?

6 da. is what part of 60 da.?

What is the interest on $100 at 6% for 60 da.?

What is the interest on $100 at 6% for 6 da.?

What is the quickest way of finding the interest on $100 at 6% for 6 da.?

What is the quickest way of finding the interest for 6 da. at 6% on any sum of money?

2. Find the interest on $1460 at 6 % for 6 da.

$1.460 = interest for 6 da.

Move the decimal point 3 places to the left. The result is the interest at 6% for 6 da.

Having the interest for 6 da., how may the interest be found for

1 da.?
2 da.?
3 da.?
4 da.?
5 da.?
7 da.?
8 da.?
9 da.?
10 da.?
18 da.?
24 da.?
36 da.?

68. Find the interest at 6 % on the following:

1. $400 for 6 da.
2. $800 for 6 da.
3. $1000 for 9 da.
4. $1200 for 8 da.
5. $1400 for 18 da.
6. $1500 for 7 da.
7. $1800 for 24 da.
8. $1560 for 36 da.
9. $1280 for 42 da.
10. $2000 for 39 da.
11. $1250 for 48 da.
12. $1125 for 24 da.
13. $1440 for 19 da.
14. $1280 for 21 da.
15. $1480 for 24 da.
16. $1600 for 27 da.
17. $2100 for 7 da.
18. $2400 for 3 da.
19. $3600 for 2 da.
20. $4200 for 1 da.
21. $9000 for 4 da.

69. Find the total interest at 6 % on:

1.	**2.**	**3.**
$420 for 18 da.	$630 for 11 da.	$1300 for 66 da.
$540 for 20 da.	$750 for 9 da.	$1560 for 72 da.
$640 for 21 da.	$780 for 26 da.	$1480 for 78 da.
$350 for 13 da.	$960 for 36 da.	$1260 for 84 da.
$432 for 15 da.	$1200 for 49 da.	$1450 for 90 da.
$128 for 16 da.	$840 for 50 da.	$1660 for 91 da.

4.

$ 1728 for 95 da.
$ 1890 for 98 da.
$ 1820 for 100 da.
$ 1675 for 108 da.
$ 1440 for 110 da.
$ 1360 for 120 da.

5.

$ 800 for 31 da.
$ 900 for 41 da.
$ 1000 for 48 da.
$ 460 for 27 da.
$ 390 for 19 da.
$ 260 for 16 da.

6.

$ 320 for 24 da.
$ 390 for 2 da.
$ 410 for 12 da.
$ 812 for 15 da.
$ 915 for 20 da.
$ 990 for 10 da.

7.

$ 828 for 45 da.
$ 936 for 15 da.
$ 832 for 75 da.
$ 648 for 70 da.
$ 444 for 80 da.
$ 555 for 40 da.

8.

$ 736 for 105 da.
$ 888 for 110 da.
$ 960 for 115 da.
$ 822 for 150 da.
$ 440 for 130 da.
$ 375 for 88 da.

9.

$ 482.75 for 12 da.
$ 321.50 for 18 da.
$ 428.48 for 15 da.
$ 320.10 for 20 da.
$ 516.74 for 24 da.
$ 608.31 for 30 da.

10.

$ 597.30 for 36 da.
$ 312.47 for 40 da.
$ 218.38 for 42 da.
$ 712.80 for 6 da.
$ 811.75 for 10 da.
$ 928.30 for 30 da.

11.

$ 731.45 for 13 da.
$ 806.40 for 14 da.
$ 318.25 for 11 da.
$ 208.14 for 10 da.
$ 327.38 for 16 da.
$ 445.50 for 19 da.

12.

$ 673.87 for 21 da.
$ 832.49 for 22 da.
$ 738.27 for 25 da.
$ 387.39 for 29 da.
$ 873.93 for 5 da.
$ 783.79 for 3 da.

13.

$ 720.30 for 25 da.
$ 628.50 for 44 da.
$ 731.20 for 14 da.
$ 906.75 for 9 da.
$ 887.63 for 12 da.
$ 674.70 for 13 da.

14.

$ 475 for 26 da.
$ 938 for 33 da.
$ 840 for 17 da.
$ 729 for 19 da.
$ 325 for 39 da.
$ 480 for 47 da.

15.

$ 425 for 38 da.
$ 576 for 67 da.
$ 545 for 88 da.
$ 750 for 112 da.
$ 800 for 99 da.
$ 235 for 81 da.

16.

$ 329.60 for 32 da.
$ 572.40 for 38 da.
$ 635.70 for 52 da.
$ 596.75 for 17 da.
$ 831.50 for 12 da.
$ 938.75 for 31 da.

17.

$ 800 for 37 da.
$ 900 for 44 da.
$ 1200 for 39 da.
$ 1500 for 53 da.
$ 1800 for 67 da.
$ 2000 for 78 da.

18.

$ 450 for 28 da.
$ 480 for 21 da.
$ 540 for 19 da.
$ 660 for 23 da.
$ 720 for 32 da.
$ 840 for 40 da.

Find the total interest at 6 % on:

19.

$ 700 for 57 da.
$ 1100 for 83 da.
$ 1300 for 96 da.
$ 1700 for 47 da.
$ 1600 for 81 da.
$ 1400 for 102 da.

20.

$ 521.60 for 23 da.
$ 387.60 for 32 da.
$ 573.20 for 41 da.
$ 753.30 for 55 da.
$ 839.70 for 64 da.
$ 931.75 for 20 da.

21.

$ 220 for 12 da.
$ 330 for 15 da.
$ 560 for 18 da.
$ 640 for 24 da.
$ 720 for 30 da.
$ 960 for 10 da.

22.

$ 830 for 6 da.
$ 870 for 9 da.
$ 910 for 12 da.
$ 530 for 15 da.
$ 780 for 45 da.
$ 880 for 42 da.

23.

$ 380 for 46 da.
$ 780 for 40 da.
$ 190 for 57 da.
$ 350 for 51 da.
$ 870 for 18 da.
$ 880 for 24 da.

24.

$ 540 for 82 da.
$ 840 for 12 da.
$ 670 for 91 da.
$ 900 for 29 da.
$ 1200 for 39 da.
$ 1400 for 90 da.

25.

$ 4500 for 28 da.
$ 4800 for 33 da.
$ 4000 for 27 da.
$ 3300 for 38 da.
$ 3600 for 70 da.
$ 2400 for 100 da.

26.

$ 88.50 for 40 da.
$ 97.50 for 17 da.
$ 31.75 for 90 da.
$ 75.60 for 40 da.
$ 86.40 for 45 da.
$ 68.70 for 75 da.

27.

$ 588 for 71 da.
$ 329 for 49 da.
$ 713 for 58 da.
$ 657 for 23 da.
$ 486 for 31 da.
$ 867 for 33 da.

28.

$ 425 for 36 da.
$ 575 for 48 da.
$ 625 for 45 da.
$ 335 for 40 da.
$ 475 for 20 da.
$ 835 for 24 da.

29.

$ 500 for 6 da.
$ 800 for 3 da.
$ 900 for 4 da.
$ 700 for 2 da.
$ 600 for 7 da.
$ 400 for 12 da.

30.

$ 1235 for 19 da.
$ 1440 for 17 da.
$ 1680 for 18 da.
$ 1840 for 13 da.
$ 1960 for 14 da.
$ 2160 for 16 da.

31.

$ 385 for 126 da.
$ 745 for 114 da.
$ 335 for 132 da.
$ 265 for 140 da.
$ 755 for 144 da.
$ 245 for 150 da.

32.

$ 885 for 160 da.
$ 795 for 180 da.
$ 175 for 168 da.
$ 675 for 174 da.
$ 680 for 105 da.
$ 870 for 110 da.

33.

$ 2315 for 80 da.
$ 4410 for 75 da.
$ 6810 for 72 da.
$ 8410 for 66 da.
$ 9610 for 56 da.
$ 1620 for 48 da.

ADDITION

70. Addition is the process of finding one number or quantity equal in value to two or more other numbers or quantities.

71. The several numbers or quantities to be added are called **addends.**

72. Investigation shows that the most important arithmetical operation is simple addition. It is important that it be done rapidly and accurately.

The following suggestions will be found helpful.

73. Drill on the 45 primary combinations is the first essential. They must be perfectly memorized. They are

$$\begin{array}{ccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\underline{1} & \underline{2} & \underline{3} & \underline{4} & \underline{5} & \underline{6} & \underline{7} & \underline{8} & \underline{9} \\
 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
 & \underline{2} & \underline{3} & \underline{4} & \underline{5} & \underline{6} & \underline{7} & \underline{8} & \underline{9} \\
 & & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\
 & & \underline{3} & \underline{4} & \underline{5} & \underline{6} & \underline{7} & \underline{8} & \underline{9} \\
 & & & 4 & 4 & 4 & 4 & 4 & 4 \\
 & & & \underline{4} & \underline{5} & \underline{6} & \underline{7} & \underline{8} & \underline{9} \\
 & & & & 5 & 5 & 5 & 5 & 5 \\
 & & & & \underline{5} & \underline{6} & \underline{7} & \underline{8} & \underline{9} \\
 & & & & & 6 & 6 & 6 & 6 \\
 & & & & & \underline{6} & \underline{7} & \underline{8} & \underline{9} \\
 & & & & & & 7 & 7 & 7 \\
 & & & & & & \underline{7} & \underline{8} & \underline{9} \\
 & & & & & & & 8 & 8 \\
 & & & & & & & \underline{8} & \underline{9} \\
 & & & & & & & & 9 \\
 & & & & & & & & \underline{9}
\end{array}$$

When the combinations on p. 33 are perfectly memorized, drill on the following and others like them:

11	11	11	12	13	15	15	16	14	16
2	4	6	3	4	5	7	6	8	5

16	17	15	19	17	18	19	18	19	15
7	4	8	6	7	8	8	9	9	4

Note that the units' figures present the same combinations as before.

Continue the drill on such numbers as the following until the results are recognized instantly:

32	26	43	27	45	53	76	47	98	54
9	7	8	8	7	9	8	6	3	7

ORAL DRILL EXERCISES

74. 1. Count by 3's from 1 to 100; from 2 to 101.

2. Count by 4's from 1 to 101; from 2 to 102; from 3 to 103.

3. Count by 5's beginning with 0; 1; 2; 3; 4, until 100 is passed.

4. Count by 6's beginning with 0; 1; 2; 3; 4; 5, until 100 is passed.

5. Count by 7's beginning with 0; 1; 2; 3; 4; 5; 6, until 100 is passed.

6. Count by 8's beginning with 0; 1; 2; 3; 4; 5; 6; 7, until 100 is passed.

7. Count by 9's beginning with 0; 1; 2; 3; 4; 5; 6; 7; 8, until 100 is passed.

8. Count by 10's beginning with 0; 1; 2; 3; 4; 5; 6; 7; 8; 9, until 100 is passed.

9. Count by 11's beginning with 0; 1; 2; 3; 4; 5; 6; 7; 8; 9; 10, until 100 is passed.

10. Count by 12's beginning with 0; 1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11, until 100 is passed.

RAPID DRILL EXERCISES

75. Drill on the following until you can state the results immediately:

9	8	7	6	9	4	5	3	7	9
1	8	4	2	1	3	5	6	1	2
2	1	1	3	3	3	5	2	4	2
7	1	5	5	6	4	4	2	5	6

9	8	7	6	9	9	7	8	9	6
1	2	3	4	5	2	4	4	7	3
9	8	7	6	5	8	6	6	3	7

17	18	24	39	43	67	76	43	61	55
2	3	5	2	8	4	2	2	1	3
3	3	4	6	1	4	7	4	7	5
5	6	1	2	1	2	1	4	2	2

38	47	64	89	79	97	48	53	89	62
9	8	3	4	3	6	5	1	9	8
1	2	7	6	7	4	5	9	1	2

56	67	83	47	82	69	43	67	79	88
2	3	6	3	8	1	4	2	3	4
4	2	3	4	1	4	4	6	6	1
4	5	1	3	1	5	2	2	1	5

81	47	74	82	71	89	74	90	27	44
6	3	4	4	6	2	6	8	3	4
2	4	5	3	1	2	1	1	4	4
2	3	1	3	3	6	3	1	3	2

76. Two important aids to accuracy and speed in addition work are: (1) to make the figures neatly and of uniform size; and (2) to write them exactly in columns.

Add:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**
64	59	94	56	49	59	67	61	67	56
28	63	89	38	84	63	38	49	84	73
39	87	21	27	47	72	79	72	98	87

11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
69	19	64	56	59	53	48	53	43	53
76	66	83	79	32	87	72	89	72	87
87	74	39	96	27	98	28	72	29	91
24	47	77	87	87	82	63	68	38	76

21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
43	74	92	94	69	88	63	48	56	45
82	91	28	76	78	72	79	27	74	97
79	43	74	87	86	38	64	68	83	86
87	87	68	92	97	76	87	93	92	73
64	96	93	47	74	43	99	27	76	49
47	78	72	83	83	89	76	64	57	36
82	47	46	97	72	96	64	56	83	17
96	64	87	48	87	74	87	74	92	85

77. The rapidity of addition may be increased by adding figures by groups. Note the groups of 10 in the following. Add rapidly:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**
4	7	4	6	3	9	7	4	5	6
6	3	3	3	4	1	3	4	3	2
8	9	3	1	3	4	6	2	2	2
2	1	7	4	5	2	2	5	4	3
7	5	2	4	2	3	2	4	2	5
3	5	1	2	3	1	8	1	4	2

11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
37	49	67	56	56	47	31	77	47	49
84	36	84	34	74	39	63	69	83	84
97	83	97	73	64	28	39	83	76	39
26	92	32	91	59	37	87	65	64	73
84	78	43	79	76	73	79	27	96	42
97	46	88	65	85	81	73	39	89	87
43	87	79	82	93	79	47	47	97	65

CHECKING THE WORK

78. Checking is a means of testing the accuracy of the work. Accuracy can be obtained by frequently checking the results.

79. The best method of checking addition is to add the columns in the reverse order. If the column is first added from the bottom up, check by adding from the top down.

80. Many bookkeepers and accountants use the method shown in the following illustrative solution:

Add:

(1st)	3967	(Check)
26	8234	27
31	7987	32
32	4372	31
27	5976	26
30536	30536	30536

By adding the units' column, a sum of 26 is obtained, which is written as shown. Next, by adding the tens' column, the sum is found to be 31, which is written as in the illustration. The sums of the hundreds' and thousands' columns are 32 and 27, respectively, which are written as here shown.

Beginning with the thousands' column and adding in the opposite direction, we find the sum to be 27, which is written as shown in the column marked "Check." The sums of the hundreds', tens', and units' columns, which are 32, 31, and 26, respectively, are written as in the illustration. By adding these several sums the result is seen to agree with the first result obtained. Hence the work is probably correct.

81. Casting out nines.

$$\begin{array}{rrr} 147 = & 12 = & 3 \\ 229 = & 13 = & 4 \\ 331 = & 7 = & 7 \\ \hline 707 & 14 = & 14 \\ & 5 = & 5 \end{array}$$

The sum of the digits 1, 4, and 7 is 12, equal to one 9 and 3 over. The sum of the digits in the second addend is 13, equal to one 9 and 4 over; and the sum in the third addend is 7, equal to no 9's and 7 over. The sum of the several remainders, 3, 4, and 7, is 14, equal to one 9 and 5 over. The sum of the digits in the sum, 707, is 14, equal to one 9 and 5 over. Hence the addition is probably correct.

NOTES. (1) The digits of 147 are the separate numbers 1, 4, and 7.

(2) The remainders 3, 4, 7, and 5 are the sums of the digits in 12, 13, 7, and 14, respectively. 12 is equal to one 9 with a remainder of 3. The division is unnecessary, however, as the excess (remainder) can always be found by adding the digits.

(3) The remainders 3, 4, and 7 are generally called the "excess of nines," and the principle is stated thus: "The excess of nines in a sum is equal to the excess in the sum of the excesses."

(4) Casting out nines is not an absolute test of accuracy. Transposition of figures cannot be detected by this check, nor can the omission or addition of 9's or ciphers be discovered.

82. Casting out elevens.

$$\begin{array}{r} 475 = 2 \\ 382 = 8 \\ \underline{479 = 6} \\ 1336 = 5 \end{array}$$

Find the sum of the digits in the odd places (beginning at units) and from it deduct the sum of the digits in the even places. If the sum of the digits in the even places exceeds the sum of those in the odd places, add 11, or some multiple of 11, to the sum of the digits in the odd places.

Thus,

$$\left.\begin{array}{ll} 5 + 4 - 7 & = 2 \\ 2 + 3 + 11 - 8 & = 8 \\ \underline{9 + 4 - 7} & \underline{\ \ = 6} \\ 6 + 3 - 3 - 1 & = 5 \end{array}\right\} \begin{array}{l} 6 + 8 + 2 = 16; \\ 16 = \text{one } 11 \text{ and } 5 \text{ over.} \end{array}$$

Note. Casting out elevens will detect errors not located by the nine check, as indicated in note (4), p. 37.

83. Add the following, and check the results:

1.	2.	3.	4.	5.	6.
2874	8727	8237	9371	6738	3796
3957	3469	6489	6487	9674	3987
6498	5487	5387	5238	5976	4598
7237	3245	6959	9764	8238	3294
4596	5894	6432	5437	9783	5786
3872	7978	8643	6087	4598	5487
5295	5673	9782	9121	1359	7372
9638	9178	9176	4327	9385	9765

84. Each of the following columns should be added in fifteen seconds. Drill on them until correct results can be obtained in that time.

1.	2.	3.	4.	5.	6.
4789	7968	4894	4893	4389	1531
3847	7324	3762	3469	5767	4574
5963	8759	5437	9276	9247	8276
8769	4365	9864	1498	6389	9832
5271	9859	4767	9763	9127	4976
8728	7312	5943	7987	8784	2872
9592	8764	8976	6477	4728	3193
7352	5231	4382	9816	2854	8715
5897	9827	4767	8125	1645	2810
7684	5876	5723	7784	5196	4567

7.	8.	9.	10.	11.
32478672	42876597	45967248	12345678	24681357
45972861	27638976	53876289	98765432	91352468
56972764	45972489	93856214	87654321	57913546
36824579	72510764	72387276	34567892	94872468
59472876	63897123	10468217	45678923	74692789

12.	13.	14.	15.	16.
76945897	54763247	57689727	47897684	98753214
34872469	98674382	68972543	32076483	76543216
89778976	76948269	74202604	82478978	97531246
69877978	63897476	70876437	68764376	89765382
45987678	48769531	97687248	98765432	74974768

85. Each of the following columns should be added in one minute. Drill on them until it can be done with facility and accuracy.

	1.	2.	3.	4.	5.
1.	34568	93876	69784	38976	94768
2.	72487	47876	72864	72487	48694
3.	95438	83297	14897	92764	38972
4.	64876	45872	32571	12476	66666
5.	45987	76978	89724	53897	89764
6.	98321	32589	89724	69748	92787
7.	87643	13489	97247	48724	77777
8.	97987	87943	83971	93697	48938
9.	34786	76897	72481	54328	55555
10.	49876	37643	92482	97648	48743
11.	24689	28971	76893	63872	33333
12.	39872	59761	64762	59763	98697
13.	97246	97246	59769	97687	88888
14.	72489	38972	43872	53792	46479
15.	93247	89726	74672	47867	99999
16.	59876	97642	94897	83897	72687
17.	72489	57921	58932	46876	22222
18.	72979	38761	72468	93872	48769

Suggestion. The number of problems in the above columns can be increased by requiring pupils to find the sum of numbers 1 to 10, inclusive; 2 to 11; 3 to 12; 4 to 13; etc.

6. The following is a list of the twenty largest cities in the United States in order of size, according to the census of 1910. Find their total population. New York, N.Y., 4,766,883; Chicago, Ill., 2,185,283; Philadelphia, Pa., 1,549,008; St. Louis, Mo., 687,029; Boston, Mass., 670,585; Cleveland, O., 560,663; Baltimore, Md., 558,485; Pittsburgh, Pa., 533,905; Detroit, Mich., 465,766; Buffalo, N. Y., 423,715; San Francisco, Cal., 416,912; Milwaukee, Wis., 373,857; Cincinnati, O., 363,591; Newark, N. J., 347,469; New Orleans, La., 339,075; Washington, D. C., 331,069; Los Angeles, Cal., 319,198; Minneapolis, Minn., 301,408; Jersey City, N. J., 267,779; Kansas City, Mo., 248,381.

86. Problems like the following will be interesting and helpful for rapid addition drills:

32347
46943

The teacher dictates two numbers of five figures each, the student writing them as shown in the margin.

32347
46943
79290√
126233
205523
331756√
537279
869035
1406314√
2275349
5910069

The remaining numbers in the column are found by the student in this manner:

The third number is the sum of the first two numbers; the fourth number is the sum of the second and third numbers; the fifth number is the sum of the third and fourth numbers; etc. When there are ten numbers the whole column is added. As the column is written, every third number is checked, as here shown, so as to tell quickly when the tenth number is written. With a little practice, students should find the final total in one minute and thirty seconds from the time the second number is dictated.

HORIZONTAL ADDITION

87. Addition from left to right is important on invoices, time sheets, sales sheets, statistical reports, etc.

88. Add from left to right. Check by adding from right to left.

1. 4, 8, 7, 6, 3, 2, 7, 9.
2. 7, 9, 6, 4, 5, 8, 9, 7.
3. 6, 4, 7, 9, 8, 7, 9, 5, 6.
4. 15, 18, 29, 73, 96, 47, 85, 93, 88.
5. 57, 93, 84, 96, 82, 78, 49, 56, 97.
6. 72, 89, 37, 43, 87, 92, 46, 34, 73.
7. 67, 83, 96, 38, 72, 47, 93, 89, 92.

Add the following by columns and from left to right. Check the result by adding the horizontal and vertical totals.

8.	9.	10.
54 + 78 + 72 =	38 + 28 + 54 =	74 + 37 + 45 =
36 + 29 + 83 =	27 + 36 + 38 =	56 + 78 + 58 =
85 + 34 + 96 =	54 + 79 + 29 =	89 + 56 + 79 =
92 + 56 + 87 =	93 + 87 + 98 =	79 + 33 + 46 =
+ + =	+ + =	+ + =

11.	12.	13.
382 + 592 + 848 =	568 + 827 + 584 =	382 + 916 + 538 =
961 + 796 + 796 =	729 + 968 + 763 =	976 + 693 + 872 =
729 + 816 + 382 =	384 + 817 + 972 =	438 + 821 + 279 =
384 + 723 + 794 =	754 + 927 + 475 =	728 + 572 + 186 =
+ + =	+ + =	+ + =

14. Find the total sales in all departments for each month, and the total sales in each department for the year. Check the work by adding the two sets of totals.

SUMMARY OF MONTHLY SALES

	CLOTHING	DRESS GOODS	GLOVES	HATS	SHOES	TOTALS
Jan. . .	$16,427.85	$12,427.83	$3,472.50	$5,472.60	$7,592.50	
Feb. . .	15,138.20	11,428.50	2,896.75	4,854.30	6,438.90	
Mar. . .	18,278.26	12,736.25	3,124.60	4,472.60	5,372.50	
Apr. . .	17,493.87	10,964.78	2,148.75	4,134.50	5,796.25	
May . .	15,382.97	9,974.63	2,004.50	3,834.75	5,176.85	
June . .	14,582.76	9,864.59	1,995.70	3,564.75	6,487.50	
July . .	13,996.80	9,645.72	1,875.50	3,472.25	6,193.85	
Aug. . .	14,592.73	8,796.21	1,850.60	3,678.50	5,040.45	
Sept. . .	15,827.64	9,793.13	1,924.30	4,075.65	6,384.90	
Oct. . .	18,973.90	12,897.08	2,394.60	4,763.83	6,876.75	
Nov. . .	22,738.75	14,594.77	3,548.75	5,276.90	7,564.55	
Dec. . .	28,693.72	18,798.99	3,875.50	5,798.75	8,097.55	
Totals .						

SUBTRACTION

89. **Subtraction** is the process of finding a number or quantity equal to the difference between two other numbers or quantities.

90. In expressions involving a parenthesis ( ), as $16-(5+7)$, the operations indicated within the parenthesis must be performed before the operation indicated by the sign before the parenthesis. Thus, in the example given, 5 and 7 must be added first, and their sum, 12, taken from 16.

The vinculum ——— has the same effect as the parenthesis. Thus, $16-\overline{5+7}=4$.

ORAL DRILL EXERCISE

91. 1. Count backwards by 2's beginning with 100; 99.

2. Count backwards by 3's beginning with 100; 99; 98.

3. Count backwards by 4's beginning with 100; 99; 98; 97.

4. Count backwards by 5's beginning with 100; 99; 98; 97; 96.

5. Count backwards by 6's beginning with 100; 99; 98; 97; 96; 95.

6. Count backwards by 7's beginning with 100; 99; 98; 97; 96; 95; 94.

7. Count backwards by 8's beginning with 100; 99; 98; 97; 96; 95; 94; 93.

8. Count backwards by 9's beginning with 100; 99; 98; 97; 96; 95; 94; 93; 92.

9. Count backwards by 10's beginning with 100; 99; 98; 97; 96; 95; 94; 93; 92; 91.

10. Count backwards by 11's beginning with 100; 99; 98; 97; 96; 95; 94; 93; 92; 91; 90.

11. Count backwards by 12's beginning with 100; 99; 98; 97; 96; 95; 94; 93; 92; 91; 90; 89.

92. Subtract quickly:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**
27	23	24	27	22	23	25	27	28	26
6	8	12	9	4	7	9	8	9	7

11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
32	35	38	32	39	37	36	34	33	31
19	17	19	17	18	18	19	17	16	15

21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
54	59	58	54	56	52	51	57	53	55
36	32	39	37	38	39	34	38	37	38

RAPID DRILL EXERCISE

93. Subtract:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
890467	740231	947204	943876	740023	630704	845700
387589	674678	678327	427628	287204	470568	687354

8.	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**
472907	907240	548724	324876	597860	938701	300470
293879	689382	279876	128987	407976	764902	298784

15.	**16.**	**17.**	**18.**	**19.**	**20.**	**21.**
487246	697287	320765	596872	729684	864200	975310
398472	472349	162782	372489	537315	795301	682405

22.	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**
108724	398764	587639	438076	987654	438762	487296
106897	297876	395301	278649	321076	287964	245498

29.	**30.**	**31.**	**32.**	**33.**	**34.**	**35**
568397	497872	495287	863829	397286	327693	978531
298729	387692	278469	438297	276397	278798	299729

CHECKING THE WORK

94. The best method of checking the work in subtraction is to add the remainder to the subtrahend. If the sum is equal to the minuend, the work is correct.

From 1476 take 938.

$$\begin{array}{r} 1476 \\ \underline{938} \\ 538 \end{array} \qquad \text{CHECK. } 938 + 538 = 1476.$$

Check the work on the preceding page.

THE "MAKING CHANGE" OR AUSTRIAN METHOD

95. This method of subtraction consists in adding to the amount of the purchase enough change to make the sum equal to the amount paid.

1. A man buys groceries to the value of $1.38, and gives a two-dollar bill in payment. How much change should he receive?

The cashier in making change will return to the customer 2 cents, a ten-cent piece, and a half dollar (saying, "$1.38, 40, 50, $2.00," which means $1.38 + $.02 = $1.40; $1.40 + $.10 = $1.50; and $1.50 + $.50 = $2.00).

Of course, other coins than those mentioned might be returned by the cashier, but it is customary to make change in the largest coins possible.

NOTE. This method is used by many in ordinary subtraction problems.

2. From 1231 take 573.

1231 3 + **8** = 11; write 8.
573 7 + 1 (carried) + **5** = 13; write 5.
658 5 + 1 (carried) + **6** = 12; write 6.

96. What change should be given for a two-dollar bill if the following purchases were made? for a five-dollar bill?

Name the coins or bills given as change. Use the largest denominations possible.

1. $1.56
2. 1.27
3. .87
4. .58
5. 1.33
6. 1.64
7. 1.29
8. $1.17
9. .96
10. .88
11. 1.49
12. 1.68
13. 1.21
14. 1.11
15. $.72
16. .49
17. .27
18. 1.67
19. 1.76
20. 1.87
21. 1.19
22. $1.82
23. 1.54
24. 1.43
25. .40
26. 1.32
27. 1.41
28. .99

This method of subtraction has an important application in all problems in which the sum of several numbers is to be taken from another number.

29. A man deposits $1625 in a bank and draws checks against it for $122, $245, and $347. Find his balance.

$1625
122
245
347
911

7 + 5 + 2 = 14; 14 + 1 = 15. The units' figure of 14 is 1 less than the units' figure of the minuend; hence 1 is added to the 14 to make its units' figure equal to the units' figure of the minuend. The 1 is written as the first or units' figure of the difference. Carry 1 (the tens' figure in 15) and add the tens' column. 1 + 4 + 4 + 2 = 11; 11 + 1 = 12. As before, write 1 in the remainder, and carry 1. Add the hundreds' column. 1 + 3 + 2 + 1 = 7; 7 + 9 = 16. Write 9 in the remainder. The bank balance, then, is $911.

Find the balance of the following bank accounts:

	Total Deposits		Checks Drawn			
30.	$ 785.00	. . . .	$ 76.00,	$45.00.		
31.	$ 427.50	. . . .	$ 37.50,	$25.00,	$100.00.	
32.	$1287.70	. . . .	$128.50,	$97.60,	$ 52.70.	
33.	$2576.28	. . . .	$528.75,	$ 7.50,	$923.65.	
34.	$3789.10	. . . .	$227.50,	$ 1.75,	$ 85.00,	$931.80.

35. Find the balance of the following account:

Withdrawal J. McLOUGHLIN Investment Gain

1911						1911					
Jan.	10		3	75		Jan.	3		1	4250	
							31	Loss and Gain	2	365	50

36. Find the loss or gain on merchandise:

Cost MERCHANDISE Returns

1911						1911					
Jan.	4		1	1764	50	Jan.	5		2	950	75
							20		8	574	50
							31	Inventory		874	75

37. Find the balance of cash:

RECEIVED CASH PAID OUT

1911						1911					
Jan.	3		1	4250		Jan.	3		1	250	
	5		2	950	75		4		1	1764	50
	20		8	574	50		10		8	75	
							12		4	500	

38. Find the loss:

COST EXPENSE RETURNS

1911						1911					
Jan.	3		1	250		Jan.	31	Inventory		480	
	12		4	500							

THE COMPLEMENT METHOD

97. The **complement** of a number is the difference between that number and the unit of a next higher order.

8 is the complement of 7, because 3 is the difference between 7 and 10; 31 is the complement of 69, because 31 is the difference between 69 and 100; etc.

98. The **complement method of subtraction** is based on the principle illustrated in the following examples:

1. From 23 take 8.

$$23 - 8 = (23 + 2) - (8 + 2) = 25 - 10 = 15.$$

The complement of 8 is 2.

If 2 is added to the minuend and is then deducted by adding it to the subtrahend also, it is evident that the value of the remainder will be unchanged. Hence, in the above example we think 23, 25, 15.

2. From 173 take 96.

$$173 - 96 = (173 + 4) - (96 + 4) = 177 - 100 = 77.$$

The complement of 96 is 4.

As stated in the preceding example, if 4 is added to both minuend and subtrahend, the remainder is unchanged. Think 173, 177, 77.

This method of subtraction is specially convenient when one number is to be deducted from the sum of several other numbers.

3. From the sum of 827, 136, and 472, take 658.

827
136
472
658
777

Arrange the numbers as shown. The complement of 658 is 342. (The complement should be carried in the mind, not written on the paper.) Add the complement to the several numbers of the minuend, and deduct 1000 (that is, 1 unit of the next higher order than the subtrahend). The result is 777.

99. By the method just explained solve the following:

1. $48 + 27 + 63 - 85$
2. $39 + 48 + 97 - 64$
3. $149 + 164 + 387 - 456$
4. $827 + 639 + 564 - 387$
5. $964 + 763 + 816 - 978$
6. $839 + 572 + 634 - 857$
7. $768 + 173 + 824 - 643$
8. $246 + 369 + 741 - 927$
9. $329 + 417 + 726 - 843$
10. $274 + 976 + 834 - 748$
11. $1238 + 1376 + 2473 - 3764$
12. $3872 + 4391 + 7684 - 8727$

13. The following statement shows the earnings and the expenses of a certain railroad company for two months. Find the total earnings and expenses; the increase or decrease of each class of earnings and expenses for November as compared with October; and the net increase or decrease.

	NOVEMBER	OCTOBER	INCREASE	DECREASE
Earnings				
Coal traffic	$850,456.25	$550,321.30		
Merchandise	625,382.40	582,498.22		
Passengers	227,421.36	231,375.60		
Express	16,172.44	17,275.85		
Mails	11,328.38	10,531.28		
Miscellaneous	24,275.60	25,386.24		
Total earnings				
Expenses				
Maintenance of road	$150,381.35	$146,327.88		
Maintenance of equipment	192,571.90	207,364.76		
Transportation	602,892.70	570,192.60		
General	60,428.75	63,175.80		
Total expenses				
Net earnings				

MULTIPLICATION

100. **Multiplication** is the process of taking one number as many times as there are units in another.

101. The **multiplier** and the **multiplicand** are called **factors** of the product.

SHORT METHODS

102. To multiply by 10 or a multiple of 10.

1. Multiply 4586 by 10; by 100; by 1000.

$$4586 \times 10 = 45860$$
$$4586 \times 100 = 458600$$
$$4586 \times 1000 = 4586000$$

Hence, to multiply by 10, 100, 1000, etc., annex as many ciphers to the multiplicand as there are ciphers in the multiplier.

2. Multiply 345 by 30; by 400; by 5000.

$$345 \times 30 = 10350$$
$$345 \times 400 = 138000$$
$$345 \times 5000 = 1725000$$

Hence, to multiply by 30, 400, 5000, etc., multiply the multiplicand by the 3, 4, or 5, etc., and annex as many ciphers to the product as there are ciphers in the multiplier.

3. Multiply 4300 by 6000.

$$43 \times 6 = 258; \text{ hence } 25800000.$$

Omit all ciphers on the right of the factors, and multiply the remaining numbers. To the result annex as many ciphers as were omitted from both factors.

103. Make the following multiplications mentally where possible:

1. 472 by 10
2. 563 by 100
3. 596 by 1000
4. 738 by 10
5. 920 by 100
6. 6472 by 1000

7. 5345 by 10
8. 824 by 300
9. 9300 by 500
10. 7250 by 6000
11. 82,400 by 5000
12. 93,875 by 100
13. 92,700 by 2000
14. 4300 by 700
15. 89,600 by 4000
16. 97,620 by 100
17. 83,472 by 10
18. 94,800 by 800
19. 7000 by 5000
20. 8900 by 900
21. 9800 by 3000

104. To multiply by 25, 50, 75, etc.

1. Multiply 413 by 25; by 50; by 75.

$4\overline{)41300}$

10325

25 is ¼ of 100. Multiplying by 25 gives a product ¼ as great as the product obtained by multiplying by 100.

Hence, to multiply by 25, annex two ciphers to the multiplicand (that is, multiply by 100) and divide by 4.

$2\overline{)41300}$

20650

50 is ½ of 100.

Hence, to multiply by 50, annex two ciphers to the multiplicand and divide by 2.

$4\overline{)41300}$

10325

3

30975

75 is ¾ of 100.

Hence, to multiply by 75, annex two ciphers to the multiplicand, divide by 4, and multiply the result by 3.

2. Multiply 475 by 125; by 250.

$8\overline{)475000}$

59375

$4\overline{)475000}$

118750

125 is ⅛ of 1000. To multiply by 1000, annex three ciphers. The product is now 8 times as large as it should be. Therefore, divide by 8 to obtain the correct result.

In like manner, 250 is ¼ of 1000. To multiply by 250, annex three ciphers to the multiplicand and divide by 4.

105. Find the products mentally where possible:

1. 24 × 25
2. 64 × 25
3. 88 × 25
4. 94 × 50
5. 53 × 50
6. 87 × 50
7. 99 × 75
8. 103 × 25
9. 137 × 50
10. 246 × 75
11. 824 × 125
12. 938 × 250
13. 728 × 500
14. 1498 × 125
15. 2387 × 25
16. 7296 × 250
17. 8974 × 750
18. 4976 × 75
19. 8397 × 250
20. 17986 × 125

106. To find the product of two factors when each ends in 5.

1. Multiply 45 by 45. 45 × 45 = 2025.

To find the square of numbers ending in 5, write 25 for the right-hand two figures of the product. Next add 1 to the tens' figure (4 + 1 = 5 in this example), multiply by the other tens' figure (4 × 5 = 20), and write the product next to the 25 already written. The result is 2025.

2. Multiply 35 by 55. $35 \times 55 = 1925$.

If the sum of the tens' figures is even, write 25 as the first part of the product. The sum of the tens' figures $(5 + 3 = 8)$ is even. Take $\frac{1}{2}$ of the sum of the tens' figures ($\frac{1}{2}$ of $8 = 4$) and add it to the product of the tens' figures multiplied together. $5 \times 3 = 15$; $15 + 4 = 19$. Write 19 in the product, making 1925.

3. Multiply 45 by 115. $45 \times 115 = 5175$.

The sum of the tens here $(4+11=15)$ is odd; hence write 75 as the first part of the product. Take $\frac{1}{2}$ the sum of the tens' figures ($\frac{1}{2}$ of $15 = 7\frac{1}{2}$), disregard the fraction, and add the half sum to the product of the tens' figures multiplied together. $(4 \times 11)+7=51$. Write 51 in the product, making 5175.

107. Practice on the following until you can read the products at sight:

1. 15 × 15
2. 25 × 25
3. 35 × 35
4. 45 × 45
5. 55 × 55
6. 65 × 65
7. 75 × 75
8. 85 × 85
9. 95 × 95
10. 85 × 65
11. 85 × 55
12. 45 × 65
13. 55 × 75
14. 85 × 35
15. 95 × 85
16. 65 × 55
17. 45 × 35
18. 75 × 85
19. 95 × 15
20. 85 × 45
21. 65 × 75
22. 125 × 45
23. 125 × 125
24. 105 × 35
25. 95 × 105
26. 105 × 105
27. 115 × 115
28. 115 × 55

108. To square any number of two figures.

1. Multiply 53 by 53. $53 \times 53 = (50 \times 56) + 3^2 = 2809$.

53 is 3 *more* than 50; 53 *plus* 3 is 56. Hence, $53 \times 53 = (50 \times 56) + (3 \times 3) = 2809$. $[(53)^2 = (50+3)^2 = (50 \times 50) + (\overline{2 \times 3} \times 50) + (3 \times 3) = (50 \times 50) + (6 \times 50) + (3 \times 3) = 50 \times 56 + 9 = 2809.]$

2. Multiply 69 by 69. $69 \times 69 = (70 \times 68) + 1^2 = 4761$.

69 is 1 *less* than 70; 69 *minus* 1 is 68. Hence, $69 \times 69 = (70 \times 68) + (1 \times 1) = 4761$.

109. Write the squares of all the numbers from 40 to 60.

To find the product of two odd or two even numbers of two figures each.

Multiply 23 by 19.

$23 = 21 + 2$; $19 = 21 - 2$. Hence, $23 \times 19 = (21 + 2) \times (21 - 2) = 21^2 - 2^2 = 441 - 4 = 437$. This is the principle known as finding the product of the sum and difference of two numbers, and is stated thus: "The product of the sum and difference of two numbers is equal to the square of the first number minus the square of the second number." The square of 21 is 441, and the square of 2 is 4. Hence the result is 441 minus $4 = 437$.

110. Multiply mentally:

1. 27 by 23
2. 29 by 19
3. 31 by 23
4. 32 by 24
5. 43 by 27
6. 38 by 26
7. 33 by 29
8. 37 by 23
9. 44 by 38
10. 17 by 29
11. 18 by 26
12. 18 by 14

111. To find the product of any two numbers of two figures each when the units or the tens are alike.

1. Multiply 42 by 72. $42 \times 72 = 3024$.

$2 \times 2 = 4$; write 4. Take the sum of the tens' figures, and multiply by one of the units' figures. $(7 + 4) \times 2 = 22$; write 2, and carry 2. Take the product of the tens, and add 2. $7 \times 4 + 2 = 30$; write 30, making 3024.

2. Multiply 37 by 34. $37 \times 34 = 1258$.

$4 \times 7 = 28$; write 8, carry 2. Take the sum of the units' figures, and multiply by one of the tens' figures. $(7 + 4) \times 3 = 33$; $33 + 2 = 35$; write 5, carry 3. Take the product of the tens' figures, and carry the 3; $3 \times 3 + 3 = 12$. Therefore, the product is 1258.

NOTE. The student should take careful notice of the fact that when the tens are alike, the units' figures are added, and *vice versa*.

112. Multiply:

1. 43 by 33
2. 24 by 44
3. 52 by 62
4. 67 by 27
5. 83 by 53
6. 69 by 29
7. 74 by 84
8. 56 by 86
9. 37 by 32
10. 48 by 45
11. 58 by 54
12. 88 by 87
13. 93 by 98
14. 69 by 63
15. 67 by 69
16. 78 by 76
17. 39 by 49
18. 58 by 57
19. 67 by 63
20. 72 by 92
21. 86 by 46
22. 55 by 57
23. 39 by 38
24. 46 by 36

113. To multiply by 11 or by a multiple of 11.

1. Multiply 287 by 11.

287 × 11 = 3157.

$$\begin{array}{r} 287 \\ \underline{11} \\ 287 \\ \underline{287} \\ 3157 \end{array}$$

By referring to the solution in the margin, it is seen that the first figure in the product is 7, the units' figure of the multiplicand; the second figure of the product is the sum of the units and the tens; the third is the sum of the tens and the hundreds, including the carrying figure, and the fourth is the hundreds' figure of the multiplicand plus the carrying figure.

The principle involved in the preceding example applies to problems in which it is desired to multiply by a multiple of 11.

2. Multiply 329 by 44. 329 × 44 = 14476.

44 is 4 times 11. Make the multiplication mentally by 11, and multiply the result by 4 at the same time. In multiplying by 11 the first figure of the product would be 9, which multiplied by 4 gives 36. Write 6, and carry 3.

Continue the multiplication by 11: $9 + 2 = 11$; $4 \times 11 = 44$, which, with 3 to carry, makes 47; write 7, carry 4. $2 + 3 = 5$; $4 \times 5 = 20$, which, with 4 to carry, makes 24; write 4, carry 2. $4 \times 3 + 2 = 14$; write 14, making 14476.

114. Multiply, mentally, each of the following numbers by 11:

1. 54
2. 35
3. 43
4. 78
5. 96
6. 58
7. 39
8. 76
9. 244
10. 372
11. 628
12. 579
13. 396
14. 927
15. 489
16. 767
17. 4872
18. 3976
19. 8498
20. 2976
21. 4507
22. 5240
23. 3007
24. 9080

115. Multiply each of the above numbers by 22; 33; 55; 77; 88.

116. To multiply by 21, 31, etc., and by 101, 201, etc.

1. Multiply 287 by 41.

287 × 41 = 11767.

$$\begin{array}{r} 287 \\ \underline{41} \\ 287 \\ \underline{1148} \\ 11767 \end{array}$$

The solution in the margin will help to make the explanation clear. The first figure of the product is plainly 7. Now, begin to multiply by 4: $4 \times 7 = 28$. $28 + 8$ (the tens' figure of the multiplicand) $= 36$; write 6, carry 3. $4 \times 8 = 32$; $32 + 2$ (the hundreds' figure of the multiplicand) $+ 3$ (carried) $= 37$; write 7, carry 3. $4 \times 2 + 3 = 11$; write 11, making 11767.

2. Multiply 458 by 601.

$458 \times 601 = 275258.$

$$\begin{array}{r} 458 \\ \underline{601} \\ 458 \\ \underline{2748} \\ 275258 \end{array}$$

The solution in the margin makes it clear that the first two figures of the product are 58. Now multiply by 6. $6 \times 8 = 48$. Add 4, the hundreds' figure of the multiplicand, which gives 52; write 2 and carry 5. Next, $6 \times 45 = 270$. To this, add the 5 to be carried, which gives 275. The entire product, then, is 275258.

117. Multiply, as directed by the teacher, the following numbers by 21, 41, 51, 81, 101, 301, 601, 901:

1. 234
2. 456
3. 728
4. 342
5. 371
6. 542
7. 295
8. 463
9. 756
10. 637

118. To multiply by a number composed of factors.

1. Multiply 369 by 497.

$$\begin{array}{r} 369 \\ \underline{497} \\ 2583 \\ \underline{18081} \\ 183393 \end{array}$$

First multiply by 7 in the usual way. The other part of the multiplier, 49, is 7 times 7; hence instead of multiplying 369 by 49, multiply 2583 by 7, placing the result as shown in the illustration, since the multiplier is 49 *tens*. Then add the partial products.

2. Multiply 657 by 321.

$$\begin{array}{r} 657 \\ \underline{321} \\ 1971 \\ \underline{13797} \\ 210897 \end{array}$$

First multiply by 3, placing the result as shown. 21 is 7 times 3. Multiply the result already found by 7, writing the product two places to the right of the first partial product, since the 3 represents *hundreds* and the 21, *units*.

CHECKING THE WORK

119. The following checks will be helpful in proving multiplication.

1. Divide the product by one of the factors: the quotient will be the other factor. If there are more than two factors, the product divided by one of them will give a quotient equal to the product of all the other factors.

(*a*) $18 \times 25 = 450$. Proof. $450 \div 18 = 25$.

(*b*) $4 \times 8 \times 9 \times 5 = 1440$. Proof. $1440 \div 4 = 360 = 8 \times 9 \times 5$.

2. **Casting out the nines.**

$$\begin{array}{rl} 41 = 5 & \\ \underline{22 = 4} & \\ 82 & 20 = 2 \\ \underline{82} & \\ 902 = 2 & \end{array}$$

The excess of nines in the multiplicand is 5, and in the multiplier is 4. The product of the excesses 4 and 5 is 20, and the excess of nines in 20 is 2. The excess of nines in the product, 902, is 2. Hence the multiplication is probably correct.

The excess of nines in the product is equal to the excess of nines in the product of the excesses.

3. **Casting out the elevens.**

$$\begin{array}{rl} 248 = 6 & \\ \underline{37 = 4} & \\ 1736 & 24 = 2 \\ \underline{744} & \\ 9176 = 2 & \end{array}$$

The excess of elevens in 248 is 6, (8 − 4 + 2); the excess of elevens in 37 is 4, (7 − 3). The product of the excesses is 4 × 6, or 24, and the excess of elevens in 24 is 2. The excess of elevens in 9176 is 2, (6 + 1 + 11 − 7 − 9). Since the results agree, the multiplication is probably correct.

120. Multiply, and check the results. Use the check assigned by the teacher.

1. 376 by 287
2. 428 by 327
3. 543 by 357
4. 896 by 416
5. 732 by 824
6. 932 by 427
7. 967 by 488
8. 853 by 366
9. 976 by 459
10. 875 by 756
11. 1249 by 648
12. 3147 by 639
13. 2489 by 763
14. 3271 by 654
15. 5298 by 324
16. 6476 by 549
17. 8749 by 624
18. 9483 by 424
19. 8965 by 545
20. 6797 by 864
21. 6859 by 218

THE SUPPLEMENT AND COMPLEMENT METHOD OF MULTIPLICATION

121. 1. Multiply 107 by 104.

$$\begin{array}{r} 107 \\ \underline{104} \\ 428 \\ \underline{107} \\ 11128 \end{array}$$

The supplement of 107 is 7, and of 104 is 4. Take the product of the supplements for the first two figures of the result. Next add the supplement of either number to the other number, (107 + 4 = 111), and write the sum as the remaining part of the result.

2. Multiply 97 by 96.

$\overset{3}{97} \times \overset{4}{96} = 9312.$

First write the complements of 97 and 96, respectively, as here shown. Write the product of the complements, (4 × 3 = 12). Next deduct the complement of either number from the other number, (97 − 4 = 93), and write the result, 93, in the product, making 9312.

122. Multiply:

1. 93 by 98
2. 97 by 95
3. 96 by 94
4. 92 by 99
5. 93 by 97
6. 93 by 94
7. 91 by 96
8. 98 by 99
9. 93 by 95
10. 92 by 93
11. 94 by 99
12. 104 by 102
13. 106 by 107
14. 109 by 108
15. 101 by 107
16. 109 by 102
17. 107 by 103
18. 108 by 105
19. 106 by 108
20. 104 by 109

123. To multiply any two numbers of two figures.

Multiply 46 by 92. $46 \times 92 = 4232.$

First, 2 × 6 = 12; write 2, carry 1. Next, 2 × 4 + 1 (carried) = 9; also 9 × 6 = 54. Add 54 and 9, giving 63; write 3, carry 6. Next, 9 × 4 + 6 (carried) = 42, which makes a completed product of 4232.

124. Multiply and check the results:

1. 43 by 29
2. 72 by 37
3. 48 by 27
4. 37 by 84
5. 43 by 56
6. 91 by 28
7. 96 by 47
8. 56 by 46
9. 83 by 34
10. 87 by 65
11. 67 by 48
12. 64 by 36
13. 93 by 67
14. 94 by 69
15. 81 by 74
16. 82 by 29
17. 34 by 85
18. 35 by 63
19. 73 by 57
20. 79 by 67

DIVISION

125. Division is the process of finding how many times one number is contained in another.

SHORT METHODS

126. To divide by 10, 100, 1000, etc.

Divide 4370 by 10.

4370 ÷ 10 = 437.

$$\begin{array}{r} 437.0 \\ 10\overline{)4370.} \end{array}$$

The solution at the left shows that to divide by 10 it is necessary only to move the decimal point one place to the left.

In like manner, to divide by 100, 1000, etc., move the point 2 places, 3 places, etc., to the left.

127. Divide each of the following numbers by 10; 100; 1000; 10,000.

1. 130000
2. 1250000
3. 40000
4. 750000
5. 1160000
6. 24750000
7. 400000
8. 1800000
9. 2000000
10. 14500000
11. 3489
12. 7248
13. 5673
14. 8470
15. 19872

128. To divide by 25, 50, 75, etc.

In many cases the work of division can be lessened by making the operation one of multiplication.

Divide 2800 by 25. 2800 ÷ 25 = 28 × 4 = 112.

25 is ¼ of 100. Divide 2800 by 100 by dropping the two ciphers. But, in dividing by 100, you have divided by a number 4 times too large, consequently the quotient is only ¼ as large as it should be (Principle 17, page 101). To correct the error, multiply the 28 by 4.

NOTE. Further application of the principle illustrated in the last example is made in the chapter on Aliquot Parts, page 21.

129. Divide:

1. 1400 by 25
2. 1600 by 25
3. 1800 by 50
4. 2400 by 50
5. 142 by 25
6. 153000 by 250
7. 72000 by 500
8. 16000 by 500
9. 27000 by 75
10. 36000 by 250
11. 48000 by 500
12. 54000 by 750

130. To divide by a number composed of factors.

1. Divide 12726 by 63.

$9 \times 7 = 63.$

9)12726
7) 1414
      202

The factors of 63 are 9 and 7. Instead of making a long division, divide by the factors, 9 and 7, using short division.

2. Divide 38971 by 64.

$8 \times 8 = 64.$

8)38971
8) 4871   3 rem.
      608   7 rem.
$7 \times 8 + 3$ 59 rem.

The factors of the divisor are 8 and 8. Divide by the factors as before, reserving the remainders as shown. To find the remainder, multiply the first divisor, 8, by the second remainder, 7, and add the first remainder, 3. Thus, $7 \times 8 + 3 = 59$, remainder. Hence the result is 608, with a remainder of 59.

3. Divide 174684 by 504.

$7 \times 8 \times 9 = 504.$

7)174684
8) 24954   6 rem.
9)   3119   2 rem.
        346   5 rem.

$$\begin{aligned} 5 \times 8 \times 7 &= 280 \\ 2 \times 7 &= 14 \\ 6 &= 6 \\ \hline && 300 \end{aligned}$$

300, remainder.

The factors of 504 are 7, 8, and 9. Divide by the factors as before. The quotient is 346. To find the remainder, multiply the third remainder by the first and second divisors, 7, and 8, ($5 \times 8 \times 7 = 280$). Next multiply the second remainder by the first divisor, 7, ($2 \times 7 = 14$). To the sum of these two products, add the first remainder, 6, giving 300 as the remainder.

4. Divide 58750 by 2400.

$4 \times 6 \times 100 = 2400.$

4)587 . 50
6)146   3 rem.
    24   2 rem.

$2 \times 4 + 3 = 11,$
1150, remainder.

The factors are 4, 6, and 100. Divide by 100 by "cutting off" the right-hand two figures of the dividend. Then divide by the factors 4 and 6. The quotient is 24. To find the remainder, first treat the divisors and remainders as though the divisor, 100, and the remainder, 50, were not present. $2 \times 4 + 3 = 11$. To complete the remainder, annex the remainder, 50, to the 11, giving a remainder of 1150.

CHECKING THE WORK

131. The following methods will be found helpful in checking the work.

1. Multiply the divisor by the quotient and add the remainder (if any); the result should equal the dividend.

2. Cast out the nines.

$8879 \div 247 = 35$, with a remainder of 234.
Therefore, $8879 = 247 \times 35 + 234$.
The excess of 9's in 8879 is 5.
The excess of 9's in 247 is 4.
The excess of 9's in 35 is 8.
The excess of 9's in 234 is 0.
$4 \times 8 + 0 = 32.$
The excess of 9's in 32 is 5.

Since the excess of 9's in the dividend is equal to the excess of 9's in the product of the excesses in the divisor and quotient plus the excess in the remainder, the division is probably correct.

3. Cast out the elevens.

$3504 \div 54 = 64$, with a remainder of 48.
Therefore, $3504 = 54 \times 64 + 48$
The excesses of 11's in 3504 is 6.
The excess of 11's in 54 is 10.
The excess of 11's in 64 is 9.
The excess of 11's in 48 is 4.
$10 \times 9 + 4 = 94.$
The excess of 11's in 94 is 6.

Since the excesses agree, the division is probably correct.

132. Divide and check the work. Use the check assigned by the teacher.

1. 4872 by 36
2. 7349 by 42
3. 5987 by 56
4. 7879 by 72
5. 9478 by 84
6. 9764 by 96
7. 11492 by 99
8. 24763 by 108
9. 45972 by 54
10. 79489 by 88
11. 48900 by 3500
12. 147260 by 3600
13. 248750 by 4200
14. 329764 by 4800
15. 247689 by 4900
16. 324798 by 5600
17. 498749 by 54000
18. 897200 by 63000
19. 948720 by 48000
20. 672000 by 24000
21. 486000 by 18000

DIVISION BY CONTINUED SUBTRACTION

133. This method of division is often required in civil service examinations.

134. By the method of continued subtraction divide 784 by 149, and prove the work.

$$\begin{array}{rr} & 784 \\ 1. & \underline{149} \\ & 635 \\ 2. & \underline{149} \\ & 486 \\ 3. & \underline{149} \\ & 337 \\ 4. & \underline{149} \\ & 188 \\ 5. & \underline{149} \\ & 39 \end{array}$$

Subtract the divisor from the dividend, and from the successive remainders, until a remainder less than the divisor is obtained. The number of subtractions is the quotient.

PROOF.

$5 \times 149 + 39 = 784$; or, $784 \div 149 = 5$, with a remainder of 39.

135. Divide by continued subtraction and prove:

1. 489764 by 42875
2. 447859 by 48727
3. 128976 by 54738
4. 249768 by 55987
5. 473964 by 75428
6. 963874 by 85925
7. 389726 by 98762
8. 598370 by 87820
9. 397645 by 98435
10. 947680 by 94768
11. 247682 by 31463
12. 437263 by 43721

FACTORING

136. 6 is the product of what numbers?

Because $2 \times 3 = 6$, 2 and 3 are called the **factors** of 6.

137. What are the factors of 10? 15? 18? 21?

138. A **prime number** is one not exactly divisible by any number except itself and 1.

139. Prime factors are prime numbers.

140. **Factoring** is the process of separating a number or quantity into its factors.

REMARK. Factoring is important, not merely for the purpose of resolving a given number into its factors, but also for its assistance in the solution of problems. In many problems in fractions, practical measurements, and percentage, in fact, in all problems in which cancellation is used, the work involved in the solution is very materially lessened by the ability to see factors quickly.

The ability quickly to factor a given number depends upon a knowledge of the tests of divisibility, with which the student should become thoroughly familiar.

141. Tests of Divisibility.

1. All even numbers are divisible by 2.

Thus, 4, 8, 18, 38, 96, 144, are divisible by 2.

2. If the sum of the digits of any number is divisible by 3, the number is divisible by 3.

Thus, 144957 is divisible by 3 because $1 + 4 + 4 + 9 + 5 + 7 = 30$, and 30 is divisible by 3.

3. All numbers whose right-hand two figures are ciphers, or express a number divisible by 4, are divisible by 4.

Thus, 100, 144, and 3288, are divisible by 4.

4. All numbers whose units' figure is either a cipher or 5 are divisible by 5.

Thus, 60 and 135 are divisible by 5.

5. All even numbers, the sum of whose digits is divisible by 3, are divisible by 6.

Thus, 126, 918, and 45252 are divisible by 6.

In practice, notice first whether the number is even. If so, proceed as in test for 3.

6. 7, 11, and 13 will divide 1001 and any of its multiples; as, 3003, 8008, 12012, etc.

7. All numbers whose right-hand three digits are ciphers, or express a number divisible by 8, are divisible by 8.

Thus, 3000, 6624, and 18232 are divisible by 8.

8. All numbers, the sum of whose digits is divisible by 9, are divisible by 9.

Thus, 45621 and 234819 are divisible by 9.

9. All numbers whose right-hand figure is 0 are divisible by 10.

Thus, 60, 140, 1190, are divisible by 10.

10. All numbers whose right-hand two figures are ciphers, or express a number divisible by 25, are divisible by 25.

Thus, 9800, 4125, and 875 are divisible by 25.

11. All numbers whose right-hand three figures are ciphers, or express a number divisible by 125, are divisible by 125.

Thus, 3000, 4625, and 3375 are divisible by 125.

142. Applying the tests given above, find the prime factors of 12,464.

$$\begin{array}{r} 8\underline{)12464} \\ 2\underline{)\ \ 1558} \\ 19\underline{)\ \ \ \ 779} \\ 41 \end{array}$$

Therefore, $2 \times 2 \times 2 \times 2 \times 19 \times 41 = 12{,}464$.

The "test" for 8 shows that 8 is a factor. The quotient 1558 is even; hence, 2 is a factor. By trial 19 is found to be a factor, resulting in a quotient of 41, a prime number.

8 was used as a divisor first to save time. Its prime factors are known to be $2 \times 2 \times 2$. Therefore, the prime factors of 12,464 are $2 \times 2 \times 2 \times 2 \times 19 \times 41$.

143. There is no regular method of determining large factors. Unless one has access to factor tables, nothing remains but to try the prime numbers until the right one is found. There are, however, two fundamental principles of factoring that should be borne in mind:

1. **A factor of a number is also a factor of all the multiples of that number.**

Thus, 7 is a factor of 21. 7 is also a factor of 42, 63, 105, 189, 420, 1071, all of which are multiples of 21.

2. **A common factor of two numbers is a factor also of the sum or of the difference of those two numbers.**

Thus 4 is a common factor of 20 and 36. It is a factor also of 56 (20 + 36), and of 16 (36 − 20).

144. Find the prime factors of the following numbers:

1. 48
2. 64
3. 80
4. 105
5. 144
6. 160
7. 729
8. 343
9. 1125
10. 3375
11. 3654
12. 3248
13. 2664
14. 4064
15. 4512
16. 3824
17. 9664
18. 9750
19. 16296
20. 25728
21. 6875
22. 5375
23. 4004
24. 3224
25. 5872
26. 9536
27. 3856
28. 7328
29. 15309
30. 27783

GREATEST COMMON DIVISOR

145. The **greatest common divisor** of two or more numbers is the largest number that will exactly divide each of them.

146. The only practical application (so far as this work is concerned) of the principles involved in finding the G. C. D. is in reducing common fractions to their lowest terms.

147. 1. Find the G. C. D. of 177 and 295.

177—295
177—118
59—118
59 is the G. C. D.

Divide the greater number by the smaller, writing the remainder, 118, and the divisor, 177, as shown. Divide again by the smaller number (118 this time) and write results as before. Dividing again, it is seen that 59 is exactly contained in 118, hence 59 is the G. C. D.

2. Find the G. C. D. of 188, 470, 705, and 940.

188—470—705—940
188— 94—141— 0
0— 94— 47— 0
47 is the G. C. D

Divide all the other numbers by the smallest number, writing the remainders as shown in the solution. The "divisor" must also be "brought down." Divide again by the smallest number. By inspection, it is now seen that 47 is the G. C. D

148. Find the G. C. D. of:

1. 912 and 1121.
2. 319 and 551.
3. 629 and 1147.
4. 2813 and 3589.
5. 791 and 1016.
6. 2167, 2561, and 985.
7. 3749, 8313, and 1141.
8. 6859, 11,191, and 3610.
9. 961, 1271, and 372.
10. 1681, 2501, and 410.

GREATEST COMMON DIVISOR BY INSPECTION

149. 1. Find the G. C. D. of 115 and 161.

$$161 - 115 = 46. \qquad 46 = 2 \times 23.$$

The G. C. D. of two numbers is their difference, or a factor of their difference. The factors of the difference, 46, are 2 and 23. By inspection, it is seen that 2 is not a common divisor, and that 23 *is* a common divisor. Hence 23 is the G. C. D.

2. Find the G. C. D. of 57, 95, 152, and 228.

The G. C. D. of several numbers is their *smallest* difference, or a factor of their smallest difference. The smallest difference of the numbers given is 38 (95 − 57). The factors of 38 are 2 and 19. By inspection, it is seen that 2 is not a common factor, and that 19 *is* a common factor. Hence 19 is the G. C. D.

150. Find, by inspection, the G. C. D. of:

1. 145 and 203.
2. 296 and 407.
3. 387 and 559.
4. 85 and 119.
5. 52, 91, and 143.
6. 68, 102, and 187.
7. 48, 72, 120, and 288.
8. 90, 108, 144, and 111.

LEAST COMMON MULTIPLE

151. A **multiple** of any given number is a number that will contain the given number without a remainder. Thus, 24 is a multiple of 4.

152. 24 is a **common multiple** of 4 and 6, because it contains 4 and 6 each without a remainder.

153. 24 is the **least common multiple** of 4, 6, 8, and 12, because it is the smallest number that will contain them without remainders.

Finding the least common multiple has its practical application in reducing several fractions to equivalent fractions having the least common denominator.

154. Find the L. C. M. of 6, 8, 12, 15, 20, 24, 30, and 36.

$$\begin{array}{r|cccccccc} 30 & \cancel{6} & \cancel{8} & \cancel{12} & \cancel{15} & 20 & 24 & 30 & 36 \\ \hline & & & & 4) & \cancel{2} & 4 & 1 & 6 \\ \cline{6-9} & & & & & & 1 & 1 & 3 \end{array}$$

$$30 \times 4 \times 3 = 360, \text{ the L. C. M.}$$

Write all the numbers in one line. Cancel all the numbers that are contained in any of the other numbers. Divide the remaining numbers by any number that will exactly divide one of them. 30 is taken as the divisor. It is contained in 30, once. Divide the other numbers (which do not exactly contain 30) by the largest factor that is common to the divisor, 30, and to any of the numbers. 10 is a common factor of both 30 and 20. 20 is divided by 10, which gives 2. 6 is a factor common to 30, and to both 24 and 36. Dividing 24 and 36 by 6 gives 4 and 6, respectively. A new line of numbers is now obtained. Proceed as at first, and continue the operation until the remaining numbers have no common factor except unity. The L. C. M. is the product of all the divisors and the numbers remaining in the last line.

NOTE. In case any of the given numbers do not contain the divisor, and do not have any factor of the divisor, "bring them down" into the next line.

155. Find the L. C. M. of:

1. 2, 4, 6, 12, 15.
2. 3, 6, 9, 18, 30.
3. 5, 12, 15, 30, 40.
4. 8, 12, 24, 60, 84.
5. 15, 25, 36, 75, 100.
6. 13, 15, 25, 65, 75.
7. 24, 36, 48, 42, 120.
8. 6, 20, 48, 100, 144.
9. 9, 16, 64, 80, 128.
10. 36, 45, 72, 105, 150.

CANCELLATION

156. Problems frequently arise in which the solution is best made like the following:

Divide the product of 36 × 18 × 21 by the product of 9 × 28 × 12.

$$\frac{\overset{3}{\cancel{36}} \times \overset{\cancel{2}}{\cancel{18}} \times \overset{3}{\cancel{21}}}{\cancel{9} \times \underset{\underset{2}{\cancel{4}}}{\cancel{28}} \times \cancel{12}} = \frac{9}{2} = 4\tfrac{1}{2}.$$

Write the factors of the dividend above a horizontal line, and the factors of the divisor below the line, as shown in the solution. 9, below the line, is contained in 18, above the line. Both numbers are canceled, and the quotient, 2, is written near the 18. In the same

way 12 is contained in 36, 3 times. Since 28 is not divisible by 21, both numbers are divided by their common factor, 7. The numbers 21 and 28 are both canceled, and the quotients, 3 and 4, are written near the numbers, respectively. 2 of the dividend now divides the 4 in the divisor; that is, the common factor, 2, is canceled in both dividend and divisor. (See Principles of Division, No. 21, page 101.) The product of the remaining factors above the line is now divided by the product of the factors left below the line, thus $\frac{3 \times 3}{2} = \frac{9}{2} = 4\frac{1}{2}$, the desired quotient.

PROBLEMS

157. Using cancellation, divide:

1. 8 × 6 × 12 × 15 by 3 × 4 × 5 × 18.
2. 15 × 12 × 24 × 39 by 10 × 13 × 4 × 16.
3. 125 × 80 × 96 × 216 by 50 × 60 × 32 × 540.
4. 48 × 56 × 63 × 72 by 36 × 78 × 42 × 108.
5. 30 × 45 × 9 × 12 by 5 × 50 × 18 × 60.
6. 6 × 12 × 80 × 20 by 4 × 9 × 75 × 120.
7. 8 × 24 × 36 × 160 by 12 × 18 × 30 × 60.
8. 36 × 45 × 54 × 75 × 68 by 144 × 15 × 36 × 51 × 65.
9. 57 × 39 × 76 × 91 × 96 by 114 × 65 × 95 × 160.
10. 38 × 64 × 68 × 69 × 217 by 133 × 12 × 17 × 460.

11. How many tubs of butter averaging 30 lb. each, at 28¢ per pound, will pay for 12 pieces of muslin, each piece containing 49 yd. at 10¢ per yard?

The cost of the muslin is represented by the product of 12 × 49 × $.10; the value of 1 tub of butter, by the product of 30 × $.28. The number of tubs of butter required is found by dividing the cost of the muslin by the value of 1 tub of butter.

Thus, $\frac{\overset{4}{\cancel{12}} \times \overset{7}{\cancel{49}} \times .\cancel{10}}{\underset{3}{\cancel{30}} \times .\underset{7}{\cancel{28}}} = 7.$ *Ans.* 7 tubs of butter.

NOTE. It should be observed that in case of a 2-place decimal in both dividend and divisor, the decimal points may be ignored and the decimals treated as whole numbers. Ignoring the points, as in this case, is equivalent to multiplying both dividend and divisor by 100. (See Principles of Division, No. 21, page 101.)

12. How many pounds of sugar at 5 ¢ per pound can be had in exchange for 15 doz. eggs at 21 ¢ per dozen?

13. At 60 ¢ a bushel, how many bushels of potatoes will pay for 3 chests of tea, weighing 36 lb. each, at 40 ¢ per pound?

14. At 45 ¢ a yard, how many pieces of cloth, averaging 48 yd. each, must be given in exchange for 6 loads of wheat at $ 1.08 per bushel, if each load contains 60 bu.?

15. How many days of 8 hr. each will it require a mechanic to work at 55 ¢ an hour to earn enough to pay for 11 A of land at $ 40 per acre?

16. How many boys earning 10 ¢ an hour, 8 hr. a day in 15 da., will earn as much as 4 men at 25 ¢ an hour, in 18 da., 10 hr. a day?

17. At $ 1.60 per barrel, how many loads of apples, 24 bbl. to the load, will be required to pay for 20 bbl. of sugar, each barrel weighing 288 lb., at 6 ¢ per pound?

18. If 17 T. of coal cost $ 106.25, how much will 51 T. cost at the same rate?

19. Find the cost of 40 tubs of butter, each weighing 56 lb., if 28 tubs of butter of the same quality, weighing 48 lb. each, cost $ 336.

20. How many yards of cloth worth 35 ¢ a yard would be given in exchange for 4 loads of oats, each load containing 40 bags of 2 bu. each, if oats are worth 42 ¢ a bushel?

21. How many cases of eggs, each containing 36 doz., at 22 ¢ a dozen will pay for 4 bbl. of sugar weighing 216 lb. each, at 5½ ¢ a pound?

22. If $ 560 earns $ 84 interest in 2 yr. 6 mo., how much interest will $ 640 earn in 3 yr. 6 mo. at the same rate?

FRACTIONS

158. What part of a dollar is 50¢? 25¢? 75¢? $12\frac{1}{2}$¢? 10¢? 5¢? 1¢?

Each of these amounts is less than one dollar. They are fractional parts of a dollar.

159. A **fraction** is a part of a unit.

50¢ = $\frac{1}{2}$ of a dollar = .50 of a dollar.
25¢ = $\frac{1}{4}$ of a dollar = .25 of a dollar.
75¢ = $\frac{3}{4}$ of a dollar = .75 of a dollar.
5¢ = $\frac{1}{20}$ of a dollar = .05 of a dollar.
1¢ = $\frac{1}{100}$ of a dollar = .01 of a dollar.

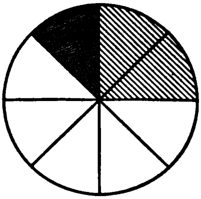
How many ways are there of writing a fraction?

Are $\frac{1}{2}$, $\frac{5}{10}$, and .5 equal in value?
Are $\frac{3}{4}$, $\frac{75}{100}$, and .75 equal in value?
Are $\frac{1}{20}$, $\frac{5}{100}$, and .05 equal in value?
Are $\frac{1}{100}$ and .01 equal in value?

160. There are, then, two ways of writing a fraction. In the common fraction form any number may be used as a denominator of the fraction. In the fractions $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{20}$, and $\frac{1}{100}$, 2, 4, 20, and 100, respectively, are the denominators.

161. In the decimal fraction form, the denominator must be 10 or some power* of 10. In the fractions .5, .75, .05, and .125, the denominators are 10, 100, 100, and 1000, respectively.

162. Into how many parts is this circle divided?



What part of the circle is heavily shaded?

How much of it is lightly shaded?

How does the lightly shaded part compare in size with the heavily shaded part? $\frac{1}{4} = 2 \times \frac{1}{8}$.

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* By "power of 10" is meant the product resulting from using 10 a given number of times as a factor. Thus, 10 × 10, or 100, is the 2d power of 10; 10 × 10 × 10, or 1000, is the 3d power of 10, etc.

This illustration emphasizes an important principle:

The larger the denominator, the smaller the fraction. Compare with the Principles of Division (No. 17, page 101).

What part of the circle is not shaded?

Which part of the fraction, $\frac{5}{8}$, shows the number of parts into which the circle is divided?

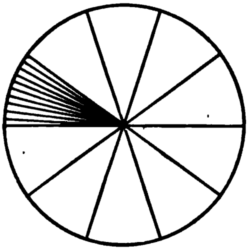
Which part shows the number of eighths in the fraction?

In the fraction, $\frac{5}{8}$, 8 is the **denominator**, and shows that the unit (the unit, in this case, is the circle) has been divided into 8 equal parts. 5 is the **numerator**, and shows that 5 of the 8 parts have been taken to form the fraction, $\frac{5}{8}$.

Important. For all practical purposes, a common fraction represents a division. $5 \div 8 = \frac{5}{8}$. These expressions are the same except in form. Each may be read "5 divided by 8." It is clear, therefore, that 5, the numerator of the fraction, is a dividend, and that 8, the denominator, is a divisor.

163. Decimal and common fractions are so closely associated in business problems, that it has been deemed wise to treat them together.

There are three things to learn about decimals: (1) The use of decimal point. (2) Notation and numeration. (3) The common fraction equivalent, and *vice versa.*



Into how many equal parts has this circle been divided?

One of these equal parts is called one tenth, written $\frac{1}{10}$ or .1.

Into how many equal parts has one of the tenths been divided?

If each of the tenths were divided into ten equal parts, into how many smaller parts would the entire circle be divided?

One of these small parts is what part of the circle? $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$; or .1 of .1 = .01. (Principles 2 and 4, page 100.)

One tenth equals how many hundredths?

164. .1 and .01 are called **decimal fractions**, or, more often, **decimals**. (From the Latin word *decem*, meaning ten.) They are decimals because their denominators are 10 or some power of 10.

165. The **denominator** of a decimal is not written, but is indicated by the number of figures used to write the decimal.

166. In reading decimals, the denominator is named the same as though it were written. Thus, the decimal .015 is read "fifteen thousandths," the same as if written $\frac{15}{1000}$. The denominator is determined by the fact that the decimal contains 3 figures ("places"). For each place, one cipher is written to the right of the figure 1, making, in this case, 1000.

ORAL EXERCISE

167. Read the following decimals:

1. .4
2. .14
3. .05
4. .125
5. .001
6. .101
7. .427
8. .821
9. .025
10. .0025
11. 15.0125*
12. 28.02345
13. 75.00001
14. .00125
15. .00025
16. 112.01875
17. .125484
18. 236.324562
19. .000125
20. 10000.0001
21. .10101
22. .4755
23. 125.0125
24. .0000001

REDUCTION OF FRACTIONS

168. How many eighths equal $\frac{1}{4}$? How many quarters equal $\frac{1}{2}$? How many eighths equal $\frac{1}{2}$?

$$\frac{4}{8} = \frac{2}{4} = \frac{1}{2}, \text{ and } \frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

These equations illustrate what is meant by reduction of fractions. The form of the fraction, $\frac{4}{8}$, has been changed so as to read "$\frac{1}{2}$," but its value is not changed.

In the first equation, the fraction, $\frac{4}{8}$, has been reduced to its lowest terms.

In the second equation, the fraction, $\frac{1}{2}$, has been reduced to higher terms.

169. The **terms** of a fraction are its numerator and denominator.

170. A fraction is reduced to its **lowest terms** when its numerator and denominator cannot both be exactly divided by any whole number except 1.

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* Read "fifteen, *and* one hundred twenty-five ten-thousandths."

171. 1. Reduce $\frac{126}{189}$ to its lowest terms.

$$3\,\Big|\,\frac{126}{189}=3\,\Big|\,\frac{42}{63}=7\,\Big|\,\frac{14}{21}=\frac{2}{3}$$

Divide both terms of the fraction by any common divisor, say 3. The resulting fraction is $\frac{42}{63}$. Divide again by any common divisor, as 3, obtaining $\frac{14}{21}$. Now divide both terms by 7, and the result is $\frac{2}{3}$, the lowest terms, because the numbers 2 and 3 cannot be divided by the same number.

2. Reduce $\frac{493}{899}$ to its lowest terms.

$$29\,\Big|\,\frac{493}{899}=\frac{17}{31}$$

In this fraction the common divisor cannot be readily determined by inspection. Find the greatest common divisor by the method explained on page 63. 29 is the G. C. D. Dividing both terms of the fraction by 29 gives $\frac{17}{31}$, the lowest terms.

Compare these solutions with the principles of division (No. 21, page 101).

State the principle involved in the last two examples.

Fractions in the decimal form generally cannot be reduced to lower terms, unless they are changed to the common fraction form. Thus, .143 and .0025 as decimals cannot be reduced to lower terms, but .40 and .0030 may be reduced by omitting the ciphers at the right. .40 = .4 and .0030 = .003.

Practically the only instances in which a cipher is written at the right of a decimal is in writing 10¢, 20¢, 30¢, etc., decimally, as $.10, $.20, $.30, etc.

3. Reduce $\frac{7}{8}$ to fortieths.

$$40 \div 8 = 5$$
$$\frac{7 \times 5 = 35}{8 \times 5 = 40}$$

The denominator of the fraction desired is 40, which is 5 times as large as the denominator of the given fraction. If $\frac{7}{8}$ is to be changed to a fraction having a denominator 5 times as large, the numerator also must be 5 times as large. Hence, multiplying both numerator and denominator by 5 gives the desired result, $\frac{35}{40}$.

Compare with the Principles of Division (No. 21, page 101).

4. Reduce .13 to ten-thousandths.

$$10{,}000 \div 100 = 100$$
$$13 \times 100 = 1300. \text{ Therefore, } .1300$$

The required denominator is 100 times as large as the denominator of the given fraction. Therefore the numerator should be 100 times as large. Hence, the required decimal is .1300.

Observe that the process of reduction consists in annexing ciphers. Show the application of the principles of division.

EXERCISE

172. Reduce the following fractions to their lowest terms (mentally, when possible):

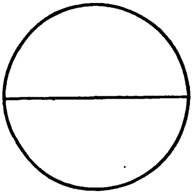
1. $\frac{72}{80}, \frac{84}{120}, \frac{96}{116}$
2. $\frac{125}{150}, \frac{48}{144}, \frac{56}{168}$
3. $\frac{111}{117}, \frac{156}{207}, \frac{165}{220}$
4. $\frac{45}{60}, \frac{45}{180}, \frac{55}{495}$
5. $\frac{1240}{1648}, \frac{1640}{2050}, \frac{3088}{4632}$
6. $\frac{1998}{2997}, \frac{5922}{6732}, \frac{2448}{7272}$
7. $\frac{437}{551}, \frac{609}{783}, \frac{341}{465}$
8. $\frac{648}{756}, \frac{244}{427}, \frac{201}{335}$
9. $\frac{355}{568}, \frac{207}{552}, \frac{152}{171}$
10. $\frac{153}{187}, \frac{91}{143}, \frac{209}{231}$
11. $\frac{253}{299}, \frac{435}{464}, \frac{555}{740}$
12. $\frac{970}{1455}, \frac{410}{615}, \frac{684}{1026}$

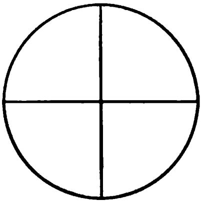
Change to higher terms:

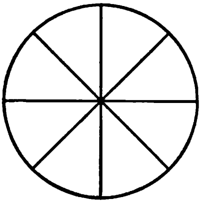
13. $\frac{3}{4}$ to 24ths.
14. $\frac{5}{8}$ to 48ths.
15. $\frac{7}{9}$ to 63ds.
16. .05 to thousandths.
17. .04 to hundred-thousandths.
18. $\frac{5}{6}$ to 36ths.
19. $\frac{8}{11}$ to 55ths.
20. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$ to 12ths.
21. $\frac{3}{5}, \frac{5}{8}, \frac{3}{4}, \frac{9}{10}$ to 40ths.
22. $\frac{3}{4}, \frac{13}{20}$, .5, .8 to 100ths.
23. $\frac{5}{8}, \frac{5}{6}$, .75, .125 to 24ths.
24. $\frac{3}{16}, \frac{2}{3}$, $.33\frac{1}{3}$, $.16\frac{2}{3}$ to 48ths.

173. To reduce an improper fraction to a whole or a mixed number.

How many halves are there in 1 circle? in 2 circles? in 3 circles?
How many quarters are there in 1 circle? in 2 circles? in 3 circles?
How many eighths are there in 1 circle? in 2 circles? in 3 circles?







1 circle $= \frac{?}{2} = \frac{?}{4} = \frac{?}{8}$. 2 circles $= \frac{?}{2} = \frac{?}{4} = \frac{?}{8}$. 3 circles $= \frac{?}{2} = \frac{?}{4} = \frac{?}{8}$.

$\frac{3}{2}$ circles = how many circles?
$\frac{5}{2}$ circles = how many circles?
$\frac{4}{2}$ circles = how many circles?
$\frac{6}{4}$ circles = how many circles?
$\frac{9}{4}$ circles = how many circles?
$\frac{12}{8}$ circles = how many circles?
$\frac{16}{8}$ circles = how many circles?
$\frac{20}{8}$ circles = how many circles?

All the fractions in this paragraph are **improper fractions,** because their numerators are equal to, or greater than, their denominators.

174. A fraction having a numerator smaller than its denominator is a **proper fraction.**

175. Improper fractions may be reduced to whole or mixed numbers.

176. A **mixed number** is the sum of a whole number and a fraction. Thus, $2\frac{1}{4}$ is a mixed number; it is the sum of 2 and $\frac{1}{4}$.

177. Reduce $\frac{37}{5}$ to a mixed number.

$$37 \div 5 = 7\frac{2}{5}.$$

There are $\frac{5}{5}$ in one unit. In $\frac{37}{5}$ there are as many units as 5 is contained times in 37, or 7 times and $\frac{2}{5}$ over.

Therefore, $\frac{37}{5} = 7\frac{2}{5}$.

EXERCISE

178. Reduce the following improper fractions to whole or mixed numbers:

1. $\frac{43}{6}$
2. $\frac{72}{10}$
3. $\frac{81}{4}$
4. $\frac{48}{8}$
5. $\frac{58}{7}$
6. $\frac{91}{8}$
7. $\frac{88}{9}$
8. $\frac{124}{7}$
9. $\frac{268}{15}$
10. $\frac{328}{12}$
11. $\frac{264}{12}$
12. $\frac{866}{15}$
13. $\frac{472}{4}$
14. $\frac{268}{16}$
15. $\frac{224}{128}$
16. $\frac{168}{10}$
17. $\frac{248}{100}$
18. $\frac{387}{100}$
19. $\frac{4897}{100}$
20. $\frac{2481}{1000}$
21. $\frac{6987}{100}$
22. $\frac{3480}{100}$
23. $\frac{5625}{1000}$
24. $\frac{4375}{1000}$
25. $\frac{675}{200}$
26. $\frac{990}{300}$
27. $\frac{4897}{25}$
28. $\frac{8000}{400}$
29. $\frac{4725}{25}$
30. $\frac{3125}{125}$

179. To reduce a mixed number to an improper fraction.

Reduce $4\frac{5}{8}$ to an improper fraction.

$$\begin{array}{r} 4\frac{5}{8} \\ \hline 32 \\ 5 \\ \hline \frac{37}{8} \end{array}$$

There are $\frac{8}{8}$ in one unit. In 4 units, or 4, there are 4 times $\frac{8}{8}$, or $\frac{32}{8}$; $\frac{32}{8} + \frac{5}{8} = \frac{37}{8}$. Therefore, $4\frac{5}{8} = \frac{37}{8}$.

In practice, the solution is as given in the margin, viz., multiply the whole number by the denominator of the fraction, and to the product add the numerator. Under the sum write the denominator of the fraction.

EXERCISE

180. Reduce the following mixed numbers to improper fractions (mentally, when possible):

1. $3\frac{1}{2}$
2. $4\frac{2}{5}$
3. $8\frac{3}{4}$
4. $9\frac{3}{8}$
5. $6\frac{3}{7}$
6. $9\frac{8}{9}$
7. $10\frac{3}{4}$
8. $5\frac{9}{10}$
9. $6\frac{7}{8}$
10. $9\frac{3}{7}$
11. $17\frac{3}{4}$
12. $19\frac{3}{8}$
13. $72\frac{3}{4}$
14. $96\frac{5}{9}$
15. $82\frac{3}{4}$

16. $85\frac{5}{7}$
17. $45\frac{5}{8}$
18. $19\frac{9}{11}$
19. $38\frac{8}{11}$
20. $87\frac{1}{2}$
21. $128\frac{3}{4}$
22. $236\frac{3}{7}$
23. $328\frac{5}{6}$
24. $482\frac{5}{9}$
25. $437\frac{3}{5}$
26. $927\frac{3}{8}$
27. $726\frac{6}{7}$
28. $322\frac{5}{8}$
29. $972\frac{3}{16}$
30. $1846\frac{3}{7}$

In the following pages both ways of writing fractions will be used. The student must be able to handle either form alone or both together.

As illustrated in the chapter on Aliquot Parts, the common fraction is the more convenient form to use in many instances. In other cases, the decimal form is better.

INTERCHANGE OF FRACTIONAL FORMS

181. The student already knows that $\frac{1}{2}$ of a dollar = 50 cents = .50 of a dollar. If this were not known, it could be determined thus: One dollar is written $1.00. To find $\frac{1}{2}$ of any number, divide by 2.

$$\begin{array}{r} \$.50 \\ 2)\overline{\$1.00} \end{array}$$

This simple example illustrates the method of changing any common fraction to its equivalent decimal form, viz.,

Annex ciphers to the numerator and divide by the denominator.

EXERCISE

182. Change the following fractions to their equivalent decimal form. (Find results mentally, when possible.)

1. $\frac{3}{20}$
2. $\frac{5}{8}$
3. $\frac{9}{32}$
4. $\frac{7}{25}$
5. $\frac{4}{15}$ *
6. $\frac{5}{18}$
7. $\frac{9}{27}$
8. $\frac{4}{9}$
9. $\frac{4}{11}$
10. $\frac{3}{14}$
11. $\frac{1}{80}$
12. $\frac{3}{40}$
13. $\frac{1}{200}$
14. $\frac{13}{14}$
15. $\frac{15}{44}$
16. $\frac{4}{125}$
17. $\frac{18}{25}$
18. $\frac{21}{500}$
19. $\frac{4}{3125}$
20. $\frac{89}{75}$

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* A common fraction in its lowest terms having a denominator that contains any factor other than 2 or 5 cannot be changed to a pure decimal. In such cases the division should be carried out two places and the fraction retained.

Thus,

$$\begin{array}{r} .26\frac{2}{3} \\ 15)\overline{4.00} \\ \underline{30} \\ 100 \\ \underline{90} \\ \frac{10}{15}=\frac{2}{3} \end{array}$$

Hence, $\frac{4}{15} = .26\frac{2}{3}$.

If it were not known that 25¢ = .25 of a dollar = $\frac{1}{4}$ of a dollar, that fact could be determined by comparing 25¢ with 100¢ (100¢ being equal to one dollar). 25¢ is $\frac{25}{100}$ of 100¢. $5 \mid \frac{25}{100} = 5 \mid \frac{5}{20} = \frac{1}{4}$. **Hence, .25 =** $\frac{1}{4}$.

Observe that the decimal is written in the common fraction form by supplying its denominator, and reducing the resulting fraction to its lowest terms.

183. The denominator of the decimal is 1, with as many ciphers after it as there are places in the decimal.

Change $.46\frac{2}{3}$ to its common fraction form in its lowest terms.

$$.46\frac{2}{3} = \frac{46\frac{2}{3}}{100}$$

$$\frac{46\frac{2}{3} \times 3}{100 \times 3} = 10 \left| \frac{140}{300} = 2 \right| \frac{14}{30} = \frac{7}{15}, \text{ or } \frac{\overset{.46\frac{2}{3}}{\overline{140}}}{300} = \frac{7}{15}.$$

$.46\frac{2}{3}$ is a complex decimal. Its denominator is 100, giving a complex common fraction of $\frac{46\frac{2}{3}}{100}$. By the Principles of Division (No. 21, page 101), multiplying both dividend and divisor (numerator and denominator) by the same number does not change the value of the quotient. Hence, multiply both numerator and denominator by 3 (the denominator of the fraction $\frac{2}{3}$). The result is $\frac{140}{300}$, which equals $\frac{7}{15}$, the desired result.

EXERCISE

184. Change the following decimals to their equivalent common fraction form in their lowest terms. Determine results mentally, when possible.

1. .8
2. .24
3. .56
4. .625
5. $.71\frac{3}{7}$
6. .888
7. .032
8. .00125
9. .0005
10. $.012\frac{1}{2}$
11. $.006\frac{1}{4}$
12. $.002\frac{1}{2}$
13. $.003\frac{1}{3}$
14. .1875
15. $.09\frac{3}{8}$
16. $.09\frac{1}{11}$
17. $.07\frac{1}{7}$
18. .0015
19. .056
20. .104
21. $.0001\frac{1}{4}$
22. $.0003\frac{1}{3}$
23. $.0142\frac{6}{7}$
24. $.006\frac{2}{3}$

LEAST COMMON DENOMINATOR

185. What is the denominator of each of the following fractions: $\frac{5}{16}$, $\frac{11}{16}$, $\frac{9}{16}$, $\frac{13}{16}$, $\frac{1}{16}$, $\frac{3}{16}$, $\frac{7}{16}$, $\frac{15}{16}$?

Because their denominators are all alike, the fractions are said to have a **common denominator.** They are also called **similar fractions.**

186. Compare these denominators with each other: $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{4}{5}$, $\frac{9}{10}$, $\frac{11}{16}$, $\frac{8}{9}$, .18, .232.

They are unlike, or **dissimilar.**

What are dissimilar fractions?

187. Fractions must be made similar before they can be added. (See Principles of Addition, No. 5, page 100.)

188. What is the least common multiple of 2, 4, 8, and 16?

189. 1. Change $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{9}{16}$ to similar fractions, having the smallest denominator possible.

$\frac{1}{2} = \frac{8}{16}$
$\frac{3}{4} = \frac{12}{16}$
$\frac{5}{8} = \frac{10}{16}$
$\frac{9}{16} = \frac{9}{16}$

If the fractions are to be similar, the denominators must be alike. The least common denominator is the same as the least common multiple of the denominators. The least common denominator can be determined mentally. (See page 64 for finding L. C. M.) It is 16.

As explained on page 70, reduce each of the fractions to 16ths. (State the principle of division that applies here.)

2. Reduce $\frac{5}{8}$, $\frac{11}{24}$, $.12\frac{1}{2}$, $.33\frac{1}{3}$, and .25 to similar fractions, having their least common denominator.

$\frac{5}{8} = \frac{15}{24}$
$\frac{11}{24} = \frac{11}{24}$
$.12\frac{1}{2} = \frac{1}{8} = \frac{3}{24}$
$.33\frac{1}{3} = \frac{1}{3} = \frac{8}{24}$
$.25 = \frac{1}{4} = \frac{6}{24}$

Change the fractions in decimal form to their equivalent common fraction form, in their lowest terms. By inspection, it is seen that 24 is the least common denominator. Each fraction is now changed to a fraction having 24 for its denominator.

EXERCISE

190. Reduce the following fractions to similar fractions having their least common denominator. (Do the work mentally when possible, using pencil only to tabulate results.)

1. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ 3. $\frac{3}{4}$, $\frac{1}{8}$, .25   5. $\frac{3}{4}$, $.06\frac{1}{4}$, $.12\frac{1}{2}$ 7. $\frac{5}{8}$, $\frac{2}{3}$, $\frac{1}{12}$
2. $\frac{4}{5}$, $\frac{9}{10}$, $\frac{1}{2}$ 4. $\frac{5}{6}$, $\frac{2}{3}$, .1   6. $\frac{7}{8}$, .9, .375   8. $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$
9. $\frac{5}{16}$, $\frac{7}{24}$, $.62\frac{1}{2}$ 15. $\frac{5}{12}$, $\frac{4}{9}$, $.08\frac{1}{3}$, $.33\frac{1}{3}$
10. $\frac{8}{9}$, $\frac{2}{3}$, $\frac{5}{18}$ 16. $\frac{7}{8}$, $\frac{9}{16}$, $\frac{3}{4}$, $\frac{1}{6}$, $\frac{5}{12}$
11. $\frac{5}{9}$, $\frac{3}{4}$, $\frac{7}{18}$, $\frac{2}{3}$, $\frac{5}{36}$ 17. .25, .625, $.06\frac{1}{4}$, .5
12. $\frac{5}{16}$, .125, $\frac{3}{4}$, $.18\frac{3}{4}$ 18. $\frac{3}{7}$, $\frac{5}{21}$, $\frac{9}{28}$, $\frac{3}{4}$, $\frac{1}{2}$
13. $\frac{3}{4}$, $.87\frac{1}{2}$, $\frac{1}{2}$, .5, .25   19. $\frac{3}{8}$, $.31\frac{1}{4}$, $\frac{2}{3}$, $.16\frac{2}{3}$, $\frac{5}{12}$
14. $\frac{17}{24}$, $\frac{5}{6}$, $.33\frac{1}{3}$, $.16\frac{2}{3}$ 20. $\frac{3}{11}$, $\frac{5}{9}$, $\frac{7}{33}$, $\frac{18}{99}$, $\frac{2}{3}$

ADDITION OF FRACTIONS

191. 1 qt. + 3 qt. + 2 qt. + 3 qt. = how many quarts?

What kind of numbers can be added?

1 fourth + 3 fourths + 2 fourths + 3 fourths = how many fourths?

$\frac{1}{4}+\frac{3}{4}+\frac{2}{4}+\frac{3}{4}=?$

What kind of fractions can be added?

192. 1. Add $\frac{5}{8}$, $\frac{7}{8}$, $\frac{3}{8}$, $\frac{1}{8}$, $\frac{2}{8}$.

$\frac{5}{8}+\frac{7}{8}+\frac{3}{8}+\frac{1}{8}+\frac{2}{8}=\frac{18}{8}=2\frac{1}{4}$.

Since these are similar fractions, they may be added by taking the sum of their numerators, which is 18, and writing it over the common denominator, 8. $\frac{18}{8}$, reduced to a mixed number, $=2\frac{1}{4}$.

2. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{11}{12}$.

$$\begin{array}{ll} \frac{2}{3} = & 16 \\ \frac{3}{4} = & 18 \\ \frac{5}{8} = & 15 \\ \frac{11}{12} = & \underline{22} \\ & \frac{71}{24} = 2\frac{23}{24}. \end{array}$$

Before these fractions can be added they must be changed to similar fractions. By inspection, the least common denominator is seen to be 24. Instead of writing the 24 for each fraction, write it where it is needed, under the sum of the new numerators, as shown in the solution. Now find the numerators, writing each one after the fraction it represents. This arrangement makes the addition of the numerators very easy and quick, their sum being written, as shown, over the common denominator, 24; $\frac{71}{24}=2\frac{23}{24}$, the desired result.

3. Find the sum of $18\frac{2}{3}$, $33\frac{4}{5}$, and $68\frac{1}{2}$.

$$\begin{array}{lll} 18 & \frac{2}{3} = & 20 \\ 33 & \frac{4}{5} = & 24 \\ \underline{68} & \frac{1}{2} = & \underline{15} \\ 120\frac{29}{30} & & \frac{59}{30} = 1\frac{29}{30} \end{array}$$

Write the integers and fractions, slightly separated, as shown in the solution. Add the fractions first by the method already explained. Their sum is $1\frac{29}{30}$. Add the integers, carrying the 1 from the sum of the fractions. The total result is $120\frac{29}{30}$.

4. Find the sum of $13\frac{1}{3}$, 18.25, 64.345, and $56\frac{1}{7}$.

$$\begin{array}{ll} 13.333\frac{1}{3} = & 7 \\ 18.25 & \\ 64.345 & \\ \underline{56.142\frac{6}{7}} = & \underline{18} \\ 152.071\frac{4}{21} & \frac{25}{21} = 1\frac{4}{21} \end{array}$$

These fractions are best added in their decimal form. The first thing to do is to change each common fraction to its decimal form. Next write the numbers as shown, with the decimal points under one another. In changing $\frac{1}{3}$ and $\frac{1}{7}$ to their decimal form, the student should observe that the decimals are carried out as many places as there are places in the longest decimal in any of the other numbers. Thus, the longest decimal given is .345, having 3 places. Therefore, $\frac{1}{3}$ and $\frac{1}{7}$ are changed to 3-place decimals. This being done, the remaining common fractions, $\frac{1}{3}$ and $\frac{6}{7}$, are parts of like orders and can be added. Add the common fractions as before, and carry the integral part as in the preceding example.

5. How many yards are there in 5 pieces of silk as follows: 41^2, 43^1, 44, 42^3, 47^1?

41^2
43^1
44
42^3
47^1
218^3

The small numbers written a little above and to the right of the numbers 41, 43, etc., indicate quarter yards. That is, 41^2 yd. is $41\frac{1}{2}$ yd.; 41^3 yd. is $41\frac{3}{4}$ yd., etc. This method of writing quarters is much used in the dry-goods business. These being similar fractions, the addition is made as already explained.

EXERCISE

193. Add:

1. $\frac{3}{4}, \frac{1}{2}, \frac{2}{3}, \frac{5}{8}$
2. $\frac{5}{6}, \frac{1}{3}, \frac{1}{2}, \frac{3}{4}$
3. $\frac{5}{7}, \frac{1}{2}, \frac{3}{14}, \frac{2}{3}$
4. $\frac{3}{11}, \frac{1}{2}, \frac{5}{22}, \frac{1}{3}$
5. $18\frac{1}{2}, 16\frac{2}{3}, 17\frac{3}{8}$
6. $24\frac{5}{6}, 31\frac{3}{4}, 47\frac{1}{3}$
7. $14\frac{1}{4}$, 18.125, 27.5
8. $126\frac{1}{8}$, 162.5, 204.475
9. $\frac{5}{8}, 2\frac{1}{2}$, 13.324
10. $\frac{3}{5}$, .4, $\frac{5}{8}$, .875
11. $\frac{4}{5}, \frac{7}{8}$
12. $\frac{5}{7}, \frac{9}{10}$
13. $18^1, 17^2, 16^3, 19^2, 22^1$
14. $48^1, 72^3, 64^2, 89^2, 94^1$
15. $75\frac{2}{3}, 81\frac{1}{2}$, 63.427
16. $41\frac{5}{8}$, 72.325, $47.18\frac{2}{3}$
17. $32\frac{7}{8}, 91\frac{3}{4}$, 52.135
18. $46^1, 72^2, 84^1, 92, 87^3$
19. $19\frac{1}{2}, 18\frac{1}{4}, 82\frac{3}{8}, 76\frac{3}{4}$
20. $21\frac{2}{3}, 76\frac{4}{5}, 91\frac{2}{3}$, 4.847

MENTAL ADDITION OF FRACTIONS

194. 1. Find the sum of $\frac{1}{3}$ and $\frac{1}{4}$.

$\frac{1}{3}=\frac{4}{12}$
$\frac{1}{4}=\frac{3}{12}$
$\frac{7}{12}$

Observe that, in changing the fractions to similar fractions, the numerators of the similar fractions are the same numbers as the denominators of the given fractions, and that the common denominator is the product of the same numbers, 3 and 4. Therefore, to find the sum of two fractions having 1 for a numerator, take the sum of the denominators for the numerator, and the product of the denominators for the denominator.

2. Add $\frac{1}{3}, \frac{1}{8}, \frac{5}{24}$.

$\frac{1}{3}+\frac{1}{8}=\frac{11}{24}$
$\frac{11}{24}+\frac{5}{24}=\frac{16}{24}=\frac{2}{3}$

Notice that the sum of the first two fractions, $\frac{1}{3}$ and $\frac{1}{8}$, is a fraction ($\frac{11}{24}$) similar to the third fraction, $\frac{5}{24}$, which may then be added mentally.

The student should be always on the alert, looking for short methods.

195. Add the following:

1. $\frac{1}{3}+\frac{1}{5}$
2. $\frac{1}{3}+\frac{1}{7}$
3. $\frac{1}{2}+\frac{1}{9}$
4. $\frac{1}{3}+\frac{1}{8}$
5. $\frac{1}{5}+\frac{1}{4}$
6. $\frac{1}{6}+\frac{1}{7}$
7. $\frac{1}{8}+\frac{1}{9}$
8. $\frac{1}{7}+\frac{1}{8}$
9. $\frac{1}{12}+\frac{1}{7}$
10. $\frac{1}{11}+\frac{1}{2}$
11. $\frac{1}{10}+\frac{1}{11}$
12. $\frac{1}{9}+\frac{1}{5}$
13. $\frac{1}{8}+\frac{1}{5}$
14. $\frac{1}{9}+\frac{1}{4}$
15. $\frac{1}{12}+\frac{1}{5}$
16. $\frac{1}{11}+\frac{1}{12}$
17. $\frac{1}{8}+\frac{1}{11}$
18. $\frac{1}{7}+\frac{1}{11}$
19. $\frac{1}{6}+\frac{1}{11}$
20. $\frac{1}{7}+\frac{1}{5}$
21. $\frac{1}{2}+\frac{1}{3}+\frac{5}{6}$
22. $\frac{1}{4}+\frac{1}{5}+\frac{9}{20}$
23. $\frac{1}{3}+\frac{1}{4}+\frac{5}{6}$
24. $\frac{1}{7}+\frac{1}{4}+\frac{3}{14}$
25. $\frac{1}{9}+\frac{1}{2}+\frac{2}{6}$
26. $\frac{1}{8}+\frac{1}{3}+\frac{11}{12}$
27. $\frac{1}{9}+\frac{1}{3}+\frac{5}{27}$
28. $\frac{1}{10}+\frac{1}{3}+\frac{4}{15}$
29. $\frac{1}{9}+\frac{1}{5}+\frac{8}{45}$
30. $\frac{1}{7}+\frac{1}{9}+\frac{5}{21}$

196. Find the sum of $\frac{2}{5}$ and $\frac{2}{7}$.

$$\frac{2}{5}+\frac{2}{7}=\frac{24}{35}$$
$$(7+5)\times 2=24$$

$\frac{2}{5}$ is 2 times $\frac{1}{5}$, and $\frac{2}{7}$ is 2 times $\frac{1}{7}$. The sum of $\frac{1}{5}$ and $\frac{1}{7}$ is $\frac{12}{35}$. Therefore, the sum of $\frac{2}{5}$ and $\frac{2}{7}$ is 2 times $\frac{12}{35}$, or $\frac{24}{35}$. That is, take 2 times the sum of the denominators for the numerator of the sum. The denominator is the product of the denominators, the same as before.

197. Add the following:

1. $\frac{2}{5}+\frac{2}{6}$
2. $\frac{2}{5}+\frac{2}{8}$
3. $\frac{2}{3}+\frac{2}{7}$
4. $\frac{2}{8}+\frac{2}{9}$
5. $\frac{2}{7}+\frac{2}{9}$
6. $\frac{2}{5}+\frac{2}{9}$
7. $\frac{2}{3}+\frac{2}{8}$
8. $\frac{2}{7}+\frac{2}{11}$
9. $\frac{2}{5}+\frac{2}{11}$
10. $\frac{3}{4}+\frac{3}{5}$
11. $\frac{3}{5}+\frac{3}{6}$
12. $\frac{3}{7}+\frac{3}{8}$
13. $\frac{4}{5}+\frac{4}{7}$
14. $\frac{4}{9}+\frac{4}{11}$
15. $\frac{5}{8}+\frac{5}{9}$
16. $\frac{5}{6}+\frac{5}{7}$
17. $\frac{6}{7}+\frac{6}{8}$
18. $\frac{6}{7}+\frac{6}{10}$
19. $\frac{2}{3}+\frac{2}{7}+\frac{8}{21}$
20. $\frac{2}{5}+\frac{2}{9}+\frac{4}{15}$
21. $\frac{2}{7}+\frac{2}{9}+\frac{1}{21}$
22. $\frac{2}{3}+\frac{2}{5}+\frac{11}{15}$
23. $\frac{3}{4}+\frac{3}{5}+\frac{7}{20}$
24. $\frac{4}{5}+\frac{4}{7}+\frac{8}{35}$

198. The student should drill carefully and thoroughly on additions like the following, till he can add simple fractions as readily as whole numbers.

Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{9}{16}$.

To add these fractions, the student should "*think*" all operations, naming only the results. ("Think" the part in italics). *The least common denominator is* 16; *the first numerator is* 8; *the second is* 12, 20; *the third is* 10; 30, 39; $\frac{39}{16}=2\frac{7}{16}$.

199. Add the following:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**	**11.**	**12.**
$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{8}$	$\frac{7}{9}$	$\frac{9}{12}$	$\frac{7}{12}$	$\frac{1}{2}$
$\frac{3}{4}$	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{7}{10}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{7}{16}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{5}{16}$
$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{7}{12}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{7}{8}$	$\frac{5}{6}$	$\frac{7}{8}$
$\frac{5}{12}$	$\frac{2}{3}$	$\frac{9}{16}$	$\frac{1}{10}$	$\frac{7}{20}$	$\frac{7}{12}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{5}{18}$	$\frac{3}{8}$	$\frac{2}{3}$	$\frac{3}{4}$
						$\frac{1}{2}$	$\frac{5}{8}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{4}$

13.	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**	**21.**	**22.**	**23.**	**24.**
$\frac{1}{3}$	$\frac{5}{9}$	$\frac{5}{8}$	$\frac{5}{16}$	$\frac{9}{10}$	$\frac{15}{32}$	$\frac{5}{18}$	$\frac{9}{16}$	$\frac{11}{15}$	$\frac{11}{24}$	$\frac{2}{3}$	$\frac{5}{9}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{16}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{3}$
$\frac{5}{6}$	$\frac{5}{6}$	$\frac{3}{4}$	$\frac{9}{16}$	$\frac{2}{3}$	$\frac{7}{8}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{7}{8}$	$\frac{5}{6}$	$\frac{3}{4}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{9}{32}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{11}{12}$	$\frac{11}{12}$
$\frac{1}{6}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{10}$	$\frac{3}{4}$	$\frac{5}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{5}{24}$	$\frac{1}{2}$	$\frac{1}{2}$

200. Add (mentally, when possible):

1. $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}+\frac{7}{8}$
2. $5\frac{4}{5}+\frac{1}{3}+\frac{1}{2}+\frac{7}{10}$
3. $\frac{5}{8}+\frac{9}{16}+.25+.37\frac{1}{2}$
4. $16\frac{3}{4}+17\frac{2}{3}+24\frac{5}{8}+56.5$
5. $47\frac{3}{5}+96.2+143\frac{9}{10}+54.25$
6. $122.125\frac{1}{3}+63\frac{1}{2}+88.0625+15\frac{1}{6}$
7. $49\frac{3}{4}+72\frac{5}{8}+99\frac{11}{16}+\frac{1}{2}$
8. $51^1+64^2+48^3+54+50^3$
9. $32\frac{3}{16}+29\frac{3}{8}+19\frac{3}{4}+54$
10. $78.125+31.5+83.333\frac{1}{3}+.66\frac{2}{3}$

11–30. For further drill add the fractions on page 75.

SUBTRACTION OF FRACTIONS

201. 1. 5 yd. minus 3 yd. = ?

2. 5 eighths minus 3 eighths = ?
$\frac{5}{8}-\frac{3}{8}=$?

3. What kind of fractions can be subtracted?

4. From $\frac{9}{11}$ take $\frac{3}{11}$.
$\frac{9}{11}-\frac{3}{11}=\frac{6}{11}$.

Since the fractions are similar, it is necessary only to take the difference between the numerators, 9 and 3, which equals 6, and under the 6 to write the common denominator. Thus, $\frac{6}{11}$.

202. Find the difference between:

1. $\frac{5}{8}$ and $\frac{3}{8}$
2. $\frac{9}{11}$ and $\frac{7}{11}$
3. $\frac{5}{6}$ and $\frac{1}{6}$
4. $\frac{3}{7}$ and $\frac{2}{7}$
5. $\frac{9}{14}$ and $\frac{7}{14}$
6. $\frac{7}{12}$ and $\frac{5}{12}$
7. $\frac{7}{8}$ and $\frac{5}{8}$
8. $\frac{9}{16}$ and $\frac{7}{16}$
9. $\frac{4}{5}$ and $\frac{2}{5}$
10. .8 and .5
11. .11 and .09
12. .50 and .25

203. 1. From $\frac{5}{8}$ take $\frac{1}{3}$.

$$\begin{array}{rl} \frac{5}{8} = & 15 \\ \frac{1}{3} = & \underline{8} \\ & \frac{7}{24} \end{array}$$

Reduce the fractions to similar fractions, as in addition. Instead of adding the new numerators, subtract the one from the other, which gives the result, $\frac{7}{24}$.

2. From $15\frac{1}{3}$ take $9\frac{3}{4}$.

$$\begin{array}{lrl} 14 & , \frac{4}{3} = & 16 \\ \underline{9} & , \frac{3}{4} = & \underline{9} \\ 5\frac{7}{12} & & \frac{7}{12} \end{array}$$

$\frac{3}{4}$ is larger than $\frac{1}{3}$. First, therefore, "borrow" 1 from the 15, and add it to the $\frac{1}{3}$, expressing the result as an improper fraction, $\frac{4}{3}$. Now proceed as before: Change to similar fractions, and subtract, obtaining $5\frac{7}{12}$ as the result.

204. Find the value of:

1. $\frac{7}{8} - \frac{3}{4}$
2. $\frac{9}{10} - \frac{5}{6}$
3. $\frac{3}{4} - \frac{2}{3}$
4. $\frac{9}{16} - \frac{1}{4}$
5. $1\frac{1}{2} - \frac{3}{4}$
6. $5\frac{2}{3} - 3\frac{5}{6}$
7. $5\frac{1}{2} - 2.75$
8. $18\frac{2}{3} - 12.16\frac{2}{3}$
9. $24.83\frac{1}{3} - 18\frac{1}{3}$
10. $63\frac{3}{8} - 12.06\frac{1}{4}$
11. $14\frac{5}{9} - 7.33\frac{1}{3}$
12. $38\frac{7}{8} - 21\frac{11}{12}$
13. $56\frac{3}{7} - 29.5$
14. $76\frac{3}{16} - 67.18\frac{3}{4}$
15. $98.16\frac{2}{3} - 21.135$
16. $88.124 - 57\frac{1}{3}$
17. $74\frac{7}{16} - 28\frac{11}{12}$
18. $58\frac{8}{9} - 27.9$
19. $73\frac{3}{7} - 24\frac{9}{14}$
20. $84\frac{1}{4} - 72.13$
21. $18.127 - 12.33\frac{1}{3}$
22. $26.872\frac{1}{3} - 5\frac{1}{6}$
23. $39.458\frac{1}{3} - 18.12\frac{1}{2}$
24. $79\frac{3}{11} - 18\frac{3}{4}$
25. $71\frac{4}{5} - 28\frac{7}{8}$
26. $82\frac{9}{10} - 43\frac{1}{4}$
27. $67\frac{3}{16} - 51\frac{3}{4}$
28. $54.128 - 27.063$
29. $57.45\frac{1}{3} - 33.128$
30. $99\frac{5}{8} - 74.25$

MENTAL SUBTRACTION OF FRACTIONS

205. 1. From $\frac{1}{2}$ take $\frac{1}{3}$.

$$\begin{array}{rl} \frac{1}{2} = & \frac{3}{6} \\ \frac{1}{3} = & \underline{\frac{2}{6}} \\ & \frac{1}{6} \end{array}$$

In changing the fractions to similar fractions, notice that the numerators of the similar fractions are the same numbers as the denominators of the given fractions. The difference is $\frac{1}{6}$. Therefore, take the difference between the denominators for the required numerator, and the product of the denominators for the required denominator.

2. From $\frac{2}{3}$ take $\frac{2}{5}$.

$\frac{2}{3} = \frac{10}{15}$
$\frac{2}{5} = \frac{6}{15}$
$\frac{4}{15}$

The common denominator is 15. Observe that the new numerators are just twice the denominators of the given fractions, and that their difference, 4, is just twice the difference between the denominators 5 and 3. Therefore, take twice the difference between the denominators of the given fractions and write it over the common denominator.

206. Find the value of:

1. $\frac{1}{3} - \frac{1}{4}$
2. $\frac{1}{3} - \frac{1}{5}$
3. $\frac{1}{3} - \frac{1}{7}$
4. $\frac{1}{3} - \frac{1}{8}$
5. $\frac{1}{3} - \frac{1}{9}$
6. $\frac{1}{3} - \frac{1}{10}$
7. $\frac{1}{4} - \frac{1}{5}$
8. $\frac{1}{4} - \frac{1}{6}$
9. $\frac{1}{4} - \frac{1}{7}$
10. $\frac{1}{4} - \frac{1}{8}$
11. $\frac{1}{4} - \frac{1}{9}$
12. $\frac{1}{4} - \frac{1}{10}$
13. $\frac{1}{4} - \frac{1}{11}$
14. $\frac{1}{4} - \frac{1}{12}$
15. $\frac{1}{5} - \frac{1}{6}$
16. $\frac{1}{5} - \frac{1}{7}$
17. $\frac{1}{5} - \frac{1}{8}$
18. $\frac{1}{5} - \frac{1}{9}$
19. $\frac{1}{5} - \frac{1}{10}$
20. $\frac{1}{5} - \frac{1}{11}$
21. $\frac{1}{5} - \frac{1}{12}$
22. $\frac{2}{5} - \frac{2}{7}$
23. $\frac{2}{3} - \frac{2}{5}$
24. $\frac{2}{3} - \frac{2}{7}$
25. $\frac{2}{7} - \frac{2}{9}$
26. $\frac{3}{5} - \frac{3}{7}$
27. $\frac{3}{4} - \frac{3}{8}$
28. $\frac{3}{7} - \frac{3}{11}$

207. Find (mentally, when possible) the value of:

1. $\frac{1}{2} + \frac{1}{3} - \frac{1}{6}$
2. $\frac{1}{3} + \frac{1}{4} - \frac{5}{12}$
3. $\frac{1}{3} + \frac{1}{5} - \frac{7}{15}$
4. $\frac{2}{3} + \frac{2}{7} - \frac{5}{21}$
5. $\frac{3}{4} + \frac{3}{5} - \frac{7}{20}$
6. $\frac{2}{7} + \frac{2}{9} - \frac{1}{3}$
7. $\frac{1}{3} + \frac{3}{8} - \frac{1}{12}$
8. $\frac{1}{4} + \frac{1}{5} - \frac{3}{10}$
9. $\frac{1}{8} + \frac{1}{6} - \frac{5}{24}$
10. $\frac{1}{4} + \frac{1}{7} - \frac{5}{14}$
11. $\frac{1}{2} + \frac{2}{3} - \frac{5}{6}$
12. $\frac{2}{3} + \frac{1}{4} - \frac{5}{12}$
13. $\frac{3}{4} + \frac{1}{3} - \frac{7}{12}$
14. $\frac{3}{5} + \frac{1}{2} - \frac{9}{10}$
15. $\frac{3}{4} + \frac{3}{8} - \frac{1}{2}$
16. $\frac{5}{6} + \frac{1}{2} - \frac{2}{3}$
17. $\frac{3}{8} + \frac{2}{3} - \frac{5}{12}$
18. $\frac{4}{5} + \frac{1}{3} - \frac{11}{15}$
19. $\frac{5}{8} + \frac{3}{4} - \frac{7}{8}$
20. $\frac{3}{5} + \frac{3}{7} - \frac{9}{35}$
21. $\frac{7}{8} - \frac{1}{2} + \frac{3}{4} - \frac{3}{8} + \frac{5}{16}$
22. $\frac{9}{16} - \frac{1}{2} + \frac{7}{8} - \frac{3}{16} + \frac{1}{4}$
23. $\frac{5}{6} + \frac{1}{3} - \frac{1}{2} - \frac{1}{6} + \frac{2}{3}$
24. $\frac{4}{9} + \frac{2}{3} - \frac{5}{6} + \frac{1}{2} - \frac{1}{3}$
25. $\frac{5}{8} + \frac{1}{4} + \frac{1}{2} - \frac{3}{8} - \frac{1}{2}$
26. $\frac{15}{16} - \frac{1}{2} - \frac{1}{4} + \frac{3}{8} + \frac{3}{4}$
27. $\frac{9}{10} - \frac{4}{5} + \frac{1}{2} + \frac{3}{4} - \frac{17}{20}$
28. $\frac{8}{9} - \frac{1}{6} - \frac{1}{3} + \frac{2}{18} - \frac{1}{4}$
29. $\frac{11}{12} - \frac{3}{4} + \frac{1}{6} + \frac{1}{3} - \frac{1}{2}$
30. $\frac{11}{16} + \frac{1}{4} - \frac{5}{8} + \frac{1}{2} - \frac{3}{4}$

MULTIPLICATION OF FRACTIONS

208. To multiply a whole number by a fraction or a fraction by a whole number.

1. 4 times 3 gallons = how many gallons?
2. 4 times 3 fifths = how many fifths? $4 \times \frac{3}{5} = ?$
3. Does $4 \times \frac{3}{5} = \frac{3}{5} \times 4$? (Principle 13, page 101.)

4. Compare these two solutions.

$$5 \times \tfrac{3}{7} = \tfrac{15}{7} = 2\tfrac{1}{7}. \qquad \tfrac{3}{7} \times 5 = \tfrac{15}{7} = 2\tfrac{1}{7}.$$

(× may be read "times," "multiplied by," or "of." The first of these two equations may be read "5 times $\frac{3}{7} = \frac{15}{7}$," or "5 multiplied by $\frac{3}{7} = \frac{15}{7}$"; the second may be read "$\frac{3}{7}$ of $5 = \frac{15}{7}$," or "$\frac{3}{7}$ multiplied by $5 = \frac{15}{7}$." It is not proper to say "5 of $\frac{3}{7}$," or "$\frac{3}{7}$ times 5.")

State the principle of division that applies in these equations. See page 101.

5. Compare these two solutions:

$$7 \times \tfrac{5}{14} = \tfrac{35}{14} = \tfrac{5}{2} = 2\tfrac{1}{2}.$$

$$\cancel{7} \times \frac{5}{\underset{2}{\cancel{14}}} = \frac{5}{2} = 2\tfrac{1}{2}.$$

Observe that, in the first solution, the numerator of the fraction is multiplied by 7, and that the resulting fraction is reduced to its lowest terms by *dividing both* terms of the fraction by 7.

In the second solution, notice that by dividing the denominator of the fraction, 14, by the multiplier, 7, the result, $\frac{5}{2}$, is obtained at once, without the intermediate step of multiplying by 7 (Principles 18, 19, page 101).

TO THE STUDENT. Study each problem carefully and determine the shortest solution. As far as possible, make all computations mentally, using pen or pencil to tabulate results.

209. Find the value of:

1. $5 \times \frac{7}{8}$
2. $8 \times \frac{3}{4}$
3. $9 \times \frac{7}{9}$
4. $6 \times \frac{3}{5}$
5. $5 \times \frac{4}{7}$
6. $9 \times \frac{8}{11}$
7. $3 \times \frac{2}{7}$
8. $4 \times \frac{8}{15}$
9. $7 \times \frac{6}{11}$
10. $5 \times \frac{9}{10}$
11. $6 \times \frac{5}{6}$
12. $8 \times \frac{9}{16}$
13. $\frac{5}{8} \times 16$
14. $\frac{7}{10} \times 15$
15. $\frac{7}{9} \times 27$
16. $\frac{15}{16} \times 48$
17. $\frac{3}{4} \times 64$
18. $\frac{8}{11} \times 55$
19. $\frac{11}{12} \times 11$
20. $\frac{5}{9} \times 15$
21. $\frac{8}{9} \times 24$
22. $\frac{9}{8} \times 24$
23. $\frac{7}{15} \times 60$
24. $\frac{8}{15} \times 40$
25. $\frac{5}{9}$ of 148
26. $\frac{3}{4}$ of 123
27. $\frac{8}{9}$ of 72
28. $\frac{5}{16}$ of 38
29. $\frac{11}{12}$ of 66
30. $\frac{7}{8}$ of 84
31. $\frac{15}{16}$ of 5
32. $\frac{19}{20}$ of 50
33. $\frac{21}{22}$ of 66
34. $\frac{12}{25}$ of 75
35. $\frac{13}{20}$ of 48
36. $\frac{15}{19}$ of 57

210. In all cases in which the fraction is in its decimal form, proceed in one of two ways: (1) If the decimal is one of the familiar aliquot or aliquant parts, use it in its common fraction form as illustrated in the chapter on Aliquot Parts. (2) In all other cases solve as follows:

Multiply 425 by .217.

$$\begin{array}{r} 425 \\ \underline{.217} \\ 2975 \\ \underline{8925} \\ 92.225 \end{array}$$

Perform the multiplication in the same manner as for simple numbers, and point off as many places in the product as there are places in either or both the multiplier and multiplicand.

211. Multiply:

1. 48 by .16⅔
2. 93 by .123
3. 133 by .416
4. 216 by .75
5. 874 by .613
6. 838 by .456
7. 971 by .375
8. 1240 by .625
9. 368 by .649
10. 487 by .1245
11. 696 by .875
12. 389 by .0625
13. 496 by .083
14. 979 by .005
15. 845 by .001
16. 859 by .0101
17. 59 by .101
18. 84 by .0125
19. 96 by .125
20. 88 by .09
21. 138 by .0001
22. 4585 by .01
23. 7345 by .003
24. 250 by .004

The student should observe carefully that on multiplying by such decimals as .1, .01, .001, .0001, and so on, the product contains the same figures as the multiplicand, with as many places pointed off as there are places in the decimal. Thus, multiply 4789 by .001. The product is 4.789, the same order of digits, with 3 places pointed off.

Write the answers to the following:

25. 8432 × .0001
26. 9678 × .1
27. 8439 × .001
28. 9637 × .00001
29. 45000 × .01
30. 64873 × .00001
31. 69478 × .000001
32. 83902 × .01
33. 84800 × .001
34. 12489 × .01
35. 34876 × .0001
36. 48749 × .01
37. 87634 × .001
38. 85634 × .1
39. 7964 × .0001

212. To find the product of an integer and a mixed number.

1. At \$½ a yard, how much will 4 yd. of cloth cost?
2. At \$1 a yard, how much will 4 yd. of cloth cost?

Compare the cost of 4 yd. at \$½ a yard and 4 yd. at \$1 a yard, with the cost of 4 yd. at \$1½ a yard.

3. At $3 a yard, how much will $\frac{1}{2}$ a yard of silk cost? How much will 6 yd. cost? How much will $6\frac{1}{2}$ yd. cost?

4. How many are $6 \times 4\frac{1}{2}$? $8 \times 3\frac{1}{4}$? $9 \times 5\frac{1}{3}$? $3\frac{1}{2} \times 8$? $7\frac{1}{5} \times 10$? $9\frac{1}{4} \times 4$? $6\frac{1}{3} \times 6$?

213. 1. Multiply 864 by $13\frac{3}{8}$.

$$\begin{array}{r} 864 \\ \underline{13\frac{3}{8}} \\ 324 \\ 2592 \\ \underline{864} \\ 11556 \end{array}$$

First multiply mentally by the fraction $\frac{3}{8}$; thus, $\frac{1}{8}$ of 864 is 108; $3 \times 108 = 324$. Next, multiply by the whole number, 13, and add the results.

2. Multiply $231\frac{2}{3}$ by 43.

$$\begin{array}{r} 231\frac{2}{3} \\ \underline{43} \\ 3\overline{)86} \\ 28\frac{2}{3} \\ 693 \\ \underline{924} \\ 9961\frac{2}{3} \end{array}$$

First, multiply the fraction $\frac{2}{3}$ by 43. If the integer, 43, is not exactly divisible by the denominator of the fraction, multiply the numerator of the fraction by 43; $43 \times 2 = 86$. Divide 86 by 3, the denominator of the fraction, obtaining $28\frac{2}{3}$. Then multiply 231 by 43, and add results.

3. Multiply 583 by $31.64\frac{3}{4}$.

$$\begin{array}{r} 31.64\frac{3}{4} \\ \underline{583} \\ 4\overline{)1749} \\ 437\frac{1}{4} \\ 9492 \\ 25312 \\ \underline{15820} \\ 18450.49\frac{1}{4} \end{array}$$

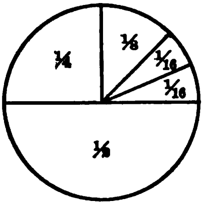
Note that the multiplicand, 583, is used as the multiplier for the purpose of shortening the work (Prin. 13, page 101). First multiply 583 by the numerator of the fraction, and divide the product by the denominator, as in the preceding example. Multiply 31.64 by 583 the same as though 31.64 were a whole number, and add the results. In the product point off 2 places, for the 2 places in the number $31.64\frac{3}{4}$.

214. Find the value of:

1. $10 \times 8\frac{1}{2}$
2. $12 \times 4\frac{2}{3}$
3. $16 \times 8\frac{3}{4}$
4. $24 \times 5\frac{3}{8}$
5. $33 \times 11\frac{5}{6}$
6. $76 \times 51\frac{3}{5}$
7. $21 \times 7\frac{3}{7}$
8. $67 \times 18\frac{4}{9}$
9. $49 \times 88\frac{3}{8}$
10. $83 \times 31\frac{2}{3}$
11. $59 \times 83\frac{3}{5}$
12. $38 \times 17\frac{5}{7}$

13. $16\frac{3}{5} \times 28$
14. $18\frac{3}{4} \times 115$
15. $14\frac{5}{12} \times 93$
16. $15\frac{6}{7} \times 99$
17. $24\frac{3}{14} \times 50$
18. $89\frac{11}{15} \times 160$
19. $39\frac{3}{11} \times 138$
20. $58\frac{4}{7} \times 44$
21. $91\frac{15}{16} \times 110$
22. $38\frac{3}{16} \times 92$
23. $47\frac{7}{9} \times 86$
24. $69\frac{3}{17} \times 85$
25. $67 \times 4.13\frac{3}{8}$
26. $87 \times .312\frac{1}{8}$
27. $34 \times 4.37\frac{5}{6}$
28. $39 \times 3.82\frac{3}{4}$
29. $98 \times 74.66\frac{2}{3}$
30. $38.13\frac{3}{7} \times 45$
31. $56.42\frac{3}{11} \times 66$
32. 3.123×86
33. $71 \times 16.10\frac{2}{3}$
34. $5.91\frac{2}{3} \times 84$
35. $8.31\frac{1}{4} \times 34$
36. $19.36\frac{3}{16} \times 59$

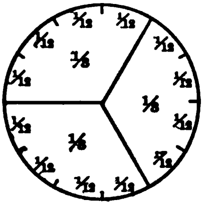
215. To multiply a fraction by a fraction.



1. How much is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{8}$?

2. If a yard of cloth costs $\frac{2}{3}$ of a dollar, how much will $\frac{1}{2}$ of a yard cost?

216. 1. What is $\frac{3}{4}$ of $\frac{2}{3}$?



Let the circle represent a unit divided into thirds. If each third is divided into four equal parts, the circle will be divided into 12 equal parts, or twelfths. $\frac{2}{3} = \frac{8}{12}$. $\frac{3}{4}$ of $\frac{2}{3}$, then, is the same as $\frac{3}{4}$ of $\frac{8}{12}$, which is equivalent to $\frac{3}{4}$ of 8 parts. $\frac{1}{4}$ of 8 parts is 2 parts; 3 × 2 parts is 6 parts, or $\frac{6}{12} = \frac{1}{2}$. Therefore, $\frac{3}{4}$ of $\frac{2}{3} = \frac{1}{2}$.

Instead of all the work indicated in this explanation, which has been given to make clear the principles involved, the solution may be made very short. Observe the following:

$$\frac{3}{4} \text{ of } \frac{2}{3} = \frac{\cancel{3} \times \cancel{2}}{\underset{2}{\cancel{4}} \times \cancel{3}} = \frac{1}{2}.$$

By applying cancellation, the work of the solution is reduced to a minimum.

Some problems do not admit of cancellation. For instance, in finding the value of $\frac{5}{6}$ of $\frac{7}{8}$, cancellation cannot be applied. The solution consists in multiplying together the numerators for the numerator of the result, and multiplying together the denominators for the denominator of the result.

The expression is equivalent to $\frac{5 \times 7}{6 \times 8} = \frac{35}{48}$.

2. Multiply .43 by .032.

$$\begin{array}{r} .43 \\ \underline{.032} \\ 86 \\ \underline{129} \\ .01376 \end{array}$$

Multiply as in whole numbers, pointing off as many places in the product as there are places in both multiplicand and multiplier.

217. Find the products:

1. $\frac{2}{3}$ of $\frac{9}{10}$
2. $\frac{5}{8}$ of $\frac{4}{5}$
3. $\frac{7}{9}$ of $\frac{9}{14}$
4. $\frac{5}{6}$ of $\frac{11}{15}$
5. $\frac{3}{4}$ of $\frac{7}{8}$
6. $\frac{11}{12}$ of $\frac{3}{4}$
7. $\frac{7}{15}$ of $\frac{9}{11}$
8. $\frac{8}{9}$ of $\frac{11}{12}$
9. $\frac{7}{10}$ of $\frac{15}{28}$
10. $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{2}{3}$
11. $\frac{9}{10}$ of $\frac{5}{6}$ of $\frac{6}{7}$
12. $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{14}$
13. $\frac{15}{16}$ of $\frac{32}{45}$ of $\frac{90}{128}$
14. $\frac{11}{12}$ of $\frac{6}{7}$ of $\frac{1}{2}$
15. $\frac{3}{5}$ of $\frac{10}{21}$ of $\frac{11}{15}$
16. $\frac{5}{6}$ of $\frac{3}{8}$ of $\frac{9}{16}$
17. $\frac{9}{10}$ of $\frac{8}{15}$ of $\frac{45}{56}$
18. $\frac{7}{12}$ of $\frac{24}{35}$ of $\frac{15}{16}$
19. $.4123 \times .814$
20. $.963 \times .0001$
21. $8.24 \times .001$
22. $.4237 \times 1.001$
23. $.954 \times 10.11$
24. 76.98×11.11
25. 8.329×8.96
26. 4.73×100.10
27. 89.48×3.824

218. To multiply a mixed number by a fraction.

1. What is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{2}$ of 4? Add the two answers.
2. What is $\frac{1}{2}$ of $4\frac{1}{2}$? $\frac{1}{3}$ of $6\frac{1}{2}$? $\frac{1}{4}$ of $8\frac{1}{4}$?
3. What is $\frac{2}{3}$ of $6\frac{1}{2}$? $\frac{3}{4}$ of $12\frac{1}{2}$? $\frac{4}{5}$ of $20\frac{1}{2}$?

219. Find the value of $\frac{7}{8}$ of $313\frac{4}{5}$.

$$\frac{7}{\underset{2}{\cancel{8}}} \text{ of } \frac{\cancel{4}}{5} = \frac{7}{10}$$

$$\frac{7}{8} \text{ of } 313 = \frac{2191}{8} = 273\frac{7}{8}$$

$$\begin{array}{rl} 273\ \frac{7}{8} = & \frac{35}{} \\ \underline{\frac{7}{10}} = & \frac{28}{} \\ 274\frac{23}{40} & \frac{63}{40} = 1\frac{23}{40} \end{array}$$

First, multiply the fractions together, obtaining $\frac{7}{10}$.

Next, take $\frac{7}{8}$ of 313, which gives $273\frac{7}{8}$. (For explanation, see page 84.)

Now add $\frac{7}{10}$ and $273\frac{7}{8}$, and the desired result is obtained.

220. Find the products:

1. $\frac{3}{5} \times 18\frac{8}{9}$
2. $\frac{4}{7} \times 33\frac{3}{8}$
3. $\frac{9}{11} \times 16\frac{4}{7}$
4. $\frac{8}{9} \times 39\frac{4}{5}$
5. $\frac{5}{8} \times 91\frac{3}{7}$
6. $\frac{7}{9} \times 88\frac{4}{7}$
7. $\frac{9}{14} \times 84\frac{7}{10}$
8. $\frac{7}{12} \times 127\frac{5}{8}$
9. $\frac{6}{7} \times 148\frac{7}{8}$
10. $\frac{3}{4} \times 131\frac{3}{5}$
11. $48\frac{7}{8} \times \frac{9}{10}$
12. $68\frac{7}{9} \times \frac{3}{4}$

13. $137\frac{3}{4} \times \frac{6}{7}$
14. $246\frac{7}{8} \times \frac{5}{6}$
15. $337\frac{4}{5} \times \frac{3}{4}$
16. $498\frac{3}{7} \times \frac{2}{3}$
17. $468\frac{15}{16} \times \frac{3}{4}$
18. $483\frac{3}{11} \times \frac{4}{5}$
19. $591\frac{4}{5} \times \frac{4}{5}$
20. $563\frac{2}{3} \times \frac{3}{4}$
21. $\frac{3}{16} \times 416\frac{3}{8}$
22. $\frac{5}{9} \times 382\frac{3}{4}$
23. $\frac{7}{11} \times 438\frac{2}{3}$
24. $\frac{5}{6} \times 938\frac{3}{4}$
25. $\frac{3}{4} \times 414\frac{5}{8}$
26. $\frac{7}{8} \times 241\frac{2}{3}$
27. $\frac{3}{8} \times 328\frac{2}{5}$
28. $\frac{4}{5} \times 564\frac{5}{8}$
29. $\frac{7}{12} \times 366\frac{6}{7}$
30. $\frac{9}{10} \times 455\frac{5}{9}$

221. To multiply a mixed number by a mixed number.

Find the value of $31\frac{3}{4} \times 17\frac{5}{8}$.

$$\begin{array}{l} 31\frac{3}{4} \\ \underline{17\frac{5}{8}} \\ \quad \frac{15}{32} = {}^{15} \\ 19\ \frac{3}{8} = {}^{12} \\ 12\ \frac{3}{4} = {}^{24} \\ 217 \\ \underline{31\qquad\qquad\quad} \\ 559\frac{19}{32} \quad \frac{51}{32} = 1\frac{19}{32} \end{array}$$

1. $\frac{5}{8}$ of $\frac{3}{4} = \frac{15}{32}$.
2. $\frac{5}{8}$ of $31 = \frac{155}{8} = 19\frac{3}{8}$.
3. $17 \times \frac{3}{4} = \frac{51}{4} = 12\frac{3}{4}$.
4. $17 \times 31 = 217$

$$\begin{array}{r} 217 \\ \underline{31} \\ 527 \end{array}$$

This solution is what is known as the "four-step process" of multiplying mixed numbers. To make the work clearer, the four steps are written out in full. The student should observe these four steps very carefully: (1) $\frac{5}{8}$ of $\frac{3}{4} = \frac{15}{32}$, which is written as a part of the product. (2) $\frac{5}{8}$ of $31 = 19\frac{3}{8}$, which is written as in the solution. (3) $17 \times \frac{3}{4} = 12\frac{3}{4}$, which is written under the $19\frac{3}{8}$. (4) $31 \times 17 = 217 + 310$. Add the several results. The final result is $559\frac{19}{32}$.

In adding the fractions of the several partial products, the student will find abundant opportunity to make use of the "short cuts" in addition of fractions. He should use them wherever possible.

222. Multiply:

1. $15\frac{3}{8} \times 16\frac{1}{4}$
2. $18\frac{3}{4} \times 26\frac{1}{2}$
3. $27\frac{5}{6} \times 34\frac{3}{5}$
4. $37\frac{3}{7} \times 29\frac{4}{7}$
5. $38\frac{3}{4} \times 34\frac{1}{4}$
6. $56\frac{4}{9} \times 9\frac{3}{4}$
7. $4\frac{1}{2} \times 9\frac{3}{8}$
8. $76\frac{3}{4} \times 12\frac{1}{2}$
9. $89\frac{4}{5} \times 17\frac{1}{2}$
10. $93\frac{3}{5} \times 27\frac{3}{4}$
11. $87\frac{3}{8} \times 29\frac{7}{9}$
12. $91\frac{1}{2} \times 41\frac{1}{3}$
13. $47\frac{2}{3} \times 38\frac{5}{6}$
14. $93\frac{3}{4} \times 86\frac{2}{3}$
15. $74\frac{3}{4} \times 19\frac{1}{5}$
16. $67\frac{7}{8} \times 18\frac{1}{4}$
17. $47\frac{3}{5} \times 21\frac{4}{5}$
18. $78\frac{2}{3} \times 37\frac{3}{4}$
19. $98\frac{3}{7} \times 42\frac{5}{9}$
20. $87\frac{4}{9} \times 56\frac{3}{8}$
21. $91\frac{3}{4} \times 36\frac{4}{13}$

22. $88\frac{4}{5} \times 55\frac{4}{5}$
23. $1.34\frac{3}{8} \times .19\frac{4}{7}$
24. $2.28\frac{4}{5} \times .67\frac{3}{4}$
25. $2.14\frac{5}{16} \times .48\frac{1}{4}$
26. $.246\frac{4}{5} \times .96\frac{3}{4}$
27. $3.24\frac{5}{8} \times .56\frac{4}{5}$
28. $6.32\frac{3}{4} \times .16\frac{3}{8}$
29. $4.56\frac{3}{8} \times .59\frac{3}{8}$
30. $5.98\frac{3}{4} \times 1.22\frac{3}{4}$
31. $8.46\frac{5}{8} \times 1.38\frac{5}{8}$
32. $9.66\frac{2}{3} \times 1.44\frac{4}{7}$
33. $8.62\frac{1}{2} \times 2.33\frac{1}{3}$

Find the total value of each of the following:

34.	35.	36.
27^{3} yd. at $\$.07^{2}$.	35^{1} yd. at $\$.07^{1}$.	64^{1} yd. at $\$.08^{2}$.
31^{1} yd. at $.08^{3}$.	43^{3} yd. at $.08^{1}$.	76^{2} yd. at $.07^{3}$.
45^{2} yd. at $.09^{1}$.	72^{1} yd. at $.05^{2}$.	48^{3} yd. at $.08^{2}$.
56^{3} yd. at $.07^{3}$.	53^{3} yd. at $.06^{2}$.	39^{2} yd. at $.04^{3}$.
38^{2} yd. at $.06^{1}$.	58^{2} yd. at $.04^{3}$.	17^{2} yd. at $.12^{3}$.
51^{2} yd. at $.07^{3}$.	63^{1} yd. at $.07^{1}$.	19^{3} yd. at $.13^{3}$.

37.	38.	39.
72^{3} yd. at $\$.10^{1}$.	21^{3} yd. at $\$.07^{2}$.	44^{1} yd. at $\$.08^{1}$.
42^{2} yd. at $.09^{2}$.	34^{3} yd. at $.06^{3}$.	29^{3} yd. at $.07^{1}$.
63^{1} yd. at $.07^{2}$.	56^{2} yd. at $.08^{2}$.	64^{2} yd. at $.07^{3}$.
16^{3} yd. at $.15^{2}$.	47^{1} yd. at $.09^{1}$.	29^{3} yd. at $.05^{2}$.
22^{2} yd. at $.18^{3}$.	53^{3} yd. at $.07^{3}$.	12^{2} yd. at $.26^{1}$.
31^{3} yd. at $.23^{2}$.	29^{1} yd. at $.07^{1}$.	14^{3} yd. at $.35^{2}$.

40.	41.	42.
59^{2} yd. at $\$.05^{2}$.	26^{1} yd. at $\$.07^{2}$.	32^{2} yd. at $\$.06^{2}$.
39^{3} yd. at $.09^{3}$.	32^{3} yd. at $.07^{1}$.	54^{1} yd. at $.07^{1}$.
45^{1} yd. at $.06^{2}$.	49^{2} yd. at $.08^{3}$.	69^{3} yd. at $.09^{3}$.
73^{2} yd. at $.08^{2}$.	38^{2} yd. at $.22^{2}$.	57^{1} yd. at $.07^{1}$.
67^{3} yd. at $.07^{2}$.	59^{3} yd. at $.33^{2}$.	48^{2} yd. at $.06^{2}$.
83^{1} yd. at $.09^{1}$.	47^{3} yd. at $.17^{2}$.	27^{3} yd. at $.05^{3}$.

MENTAL MULTIPLICATION OF MIXED NUMBERS

223. When each fraction is $\frac{1}{2}$.

1. Multiply $6\frac{1}{2}$ by $6\frac{1}{2}$.

$$(a)\ \begin{array}{r} 6\frac{1}{2} \\ \underline{6\frac{1}{2}} \\ 3\frac{1}{4} \\ 3 \\ \underline{36} \\ 42\frac{1}{4} \end{array} \qquad (b)\ \begin{array}{r} 6\frac{1}{2} \\ \underline{6\frac{1}{2}} \\ 42\frac{1}{4} \end{array}$$

In (*a*) the complete "four-step" solution (page 87) is given. Note that the sum of the two cross products (3 and 3) is 6. Hence, instead of taking 6 × 6, and adding 6, add 1 to the multiplicand, and take 6 × 7, as in (*b*).

2. Multiply 9.5 by 9.5.

$$\begin{array}{r} 9.5 \\ \underline{9.5} \\ 90.25 \end{array}$$

As in (*b*), problem 1, $.5 \times .5 = .25$. Add 1 to one of the 9's and multiply by the other 9, and obtain 90. Hence, $9.5 \times 9.5 = 90.25$.

224. Multiply (mentally):

1. $8\frac{1}{2}$ by $8\frac{1}{2}$
2. $7\frac{1}{2}$ by $7\frac{1}{2}$
3. 3.5 by 3.5
4. 4.5 by 4.5
5. $2\frac{1}{2}$ by $2\frac{1}{2}$
6. $1\frac{1}{2}$ by $1\frac{1}{2}$
7. $10\frac{1}{2}$ by $10\frac{1}{2}$
8. 11.5 by 11.5
9. $13\frac{1}{2}$ by $13\frac{1}{2}$
10. 12.5 by 12.5
11. $5\frac{1}{2}$ by $5\frac{1}{2}$
12. 6.5 by 6.5
13. $9\frac{1}{2}$ by $9\frac{1}{2}$
14. $15\frac{1}{2}$ by $15\frac{1}{2}$
15. $20\frac{1}{2}$ by $20\frac{1}{2}$

225. 1. Multiply $7\frac{1}{2}$ by $5\frac{1}{2}$.

$$\begin{array}{r} 7\frac{1}{2} \\ \underline{5\frac{1}{2}} \\ 41\frac{1}{4} \end{array}$$

$\frac{1}{2}$ of $7 + \frac{1}{2}$ of $5 = \frac{1}{2}$ of $(7 + 5) = 6$. Take 5 × 7, and add the 6, which gives $41\frac{1}{4}$ as the product.

2. Multiply $9\frac{1}{2}$ by $4\frac{1}{2}$.

$$\begin{array}{r} 9\frac{1}{2} \\ \underline{4\frac{1}{2}} \\ 42\frac{3}{4} \end{array}$$

The sum of the integers $(4 + 9 = 13)$ being an odd number, $\frac{1}{2}$ of the sum gives a mixed number, $6\frac{1}{2}$. $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$, to which is added the $\frac{1}{2}$ of the mixed number $(6\frac{1}{2})$, giving $\frac{3}{4}$, the fractional part of the product. Now take 4 times 9, and add the 6, obtaining $42\frac{3}{4}$, the desired result.

In problems like the last two examples, observe first whether the sum of the integers is even or odd. If even, write as the first part of the product $\frac{1}{4}$, .25, or 25, as the case may require; if odd, write $\frac{3}{4}$, .75, or 75. For the remainder of the product multiply the integers together, and add the integral part of half their sum. (Compare with Art. 106.)

When the fractions are alike.

3. Multiply $8\frac{2}{3}$ by $5\frac{2}{3}$.

$$\begin{array}{r} 8\frac{2}{3} \\ \underline{5\frac{2}{3}} \\ \frac{4}{9} \\ 8\frac{2}{3} \\ \underline{40} \\ 49\frac{1}{9} \end{array}$$

First multiply $\frac{2}{3}$ by $\frac{2}{3}$. Next find $\frac{2}{3}$ of 13, $(8 + 5)$, which is $8\frac{2}{3}$. Then find 5×8, which is 40. It should be observed that although $\frac{2}{3}$ of 13 gives a mixed number, $8\frac{2}{3}$, the solution is shorter than by the other method. Now there are only two fractions to be added, instead of three, as there would be if 8 and 5 were multiplied separately.

The student will be surprised at the rapidity with which he will be able to make multiplications like the above examples, after a little practice.

226. Multiply (mentally):

1. $6\frac{1}{2} \times 10\frac{1}{2}$
2. $8\frac{1}{3} \times 7\frac{1}{3}$
3. $5\frac{3}{4} \times 11\frac{3}{4}$
4. 8.25×4.25
5. $6\frac{1}{3} \times 9\frac{1}{3}$
6. $7\frac{1}{2} \times 9\frac{1}{2}$
7. 11.25×9.25
8. $7\frac{2}{3} \times 8\frac{2}{3}$
9. $10\frac{1}{5} \times 15\frac{1}{5}$
10. 11.5×8.5
11. $14\frac{1}{4} \times 8\frac{1}{4}$
12. $5\frac{3}{5} \times 7\frac{3}{5}$
13. $12\frac{3}{4} \times 9\frac{3}{4}$
14. $14\frac{1}{3} \times 7\frac{1}{3}$
15. $15\frac{3}{7} \times 6\frac{3}{7}$
16. $18\frac{2}{5} \times 7\frac{2}{5}$
17. $16\frac{2}{3} \times 8\frac{2}{3}$
18. $11\frac{3}{4} \times 7\frac{3}{4}$
19. $13\frac{2}{3} \times 2\frac{2}{3}$
20. 15.75×1.75
21. $17\frac{4}{7} \times 4\frac{4}{7}$

227. Find the total value of each of the following groups:

1.

65 crates at	$6.50.
85 crates at	7.50.
$4\frac{1}{2}$ bu. at	.75.
$9\frac{1}{2}$ bu. at	.85.

2.

$10\frac{1}{2}$ lb. at	$.75.
$5\frac{1}{2}$ lb. at	.65.
$7\frac{1}{2}$ lb. at	$.07\frac{1}{2}$.
$3\frac{1}{2}$ lb. at	$.15\frac{1}{2}$.

3.

$12\frac{1}{2}$ A. at	$125.
$15\frac{1}{2}$ A. at	155.
$16\frac{1}{2}$ A. at	85.
$20\frac{1}{2}$ A. at	75.

4.

$5\frac{1}{4}$ yd. at	$\$.07\frac{1}{4}$.
$9\frac{1}{4}$ yd. at	$.18\frac{1}{4}$.
$7\frac{1}{4}$ yd. at	$.09\frac{1}{4}$.
$13\frac{1}{4}$ yd. at	$.07\frac{1}{4}$.

5.

$8\frac{3}{4}$ lb. at	$\$.08\frac{3}{4}$.
$9\frac{3}{4}$ lb. at	$.11\frac{3}{4}$.
$5\frac{3}{4}$ lb. at	3.75.
$12\frac{3}{4}$ lb. at	4.75.

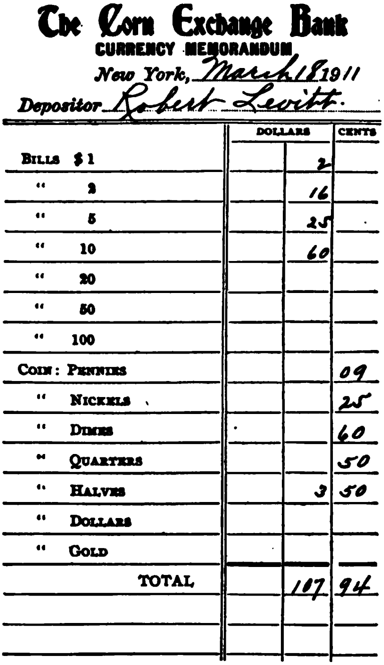
6.

$15\frac{1}{2}$ bu. at	$1.50.
350 lb. at	1.50 per cwt.
2500 ft. at	18.50 per M.
4500 ft. at	6.50 per M.

228. The following is a form of pay roll generally used by time clerks. In addition to the time, rate, and amount due for each clerk, it also shows the number of both bills and minor coins necessary to pay each and all of the clerks. The total of all the different denominations must agree with the total of the amount column.

No.*	M.	T.	W.	T.	F.	S.	Total Time	Rate	Am't	$ 10	$ 5	$ 2	$ 1	¢ 50	¢ 25	¢ 10	¢ 5	¢ 1
1.	8	7	7½	6	9	8	45½	.30	$13.65	1		1	1	1		1	1	
2.	7	8½	6	9	9	7½	47	.33⅓	15.67	1	1			1		1	1	2
3.	9	10	7	6	7½	6½	46	.27½	12.65	1		1		1		1	1	
4.	8	8	7	8	8	6	45	.22	9.90		1	2		1	1	1	1	
5.	9	7	6	4½	8½	9½	44½	.17½	7.79		1	1		1	1			4
6.	10	8	7	9	8	8½	50½	.35	17.68	1	1	1		1		1	1	3
7.	9	7	8	6	4½	9½	44	.37½	16.50	1	1		1	1				
8.	8½	7	6	9½	8½	7½	47	.30	14.10	1		2				1		
									$107.94	6	5	8	2	7	2	6	5	9
										60	25	16	2	3.50	.50	.60	.25	.09

At the right is a currency memorandum and on p. 92 will be found a pay roll check. The memorandum accompanies the check drawn for the total amount of the pay roll to show the paying teller at the bank just what denominations and how many of each are required.



The Corn Exchange Bank
CURRENCY MEMORANDUM
New York, March 18 1911
Depositor Robert Levitt.

	Dollars	Cents
Bills $1	2	
" 2	16	
" 5	25	
" 10	60	
" 20		
" 50		
" 100		
Coin: Pennies		09
" Nickels		25
" Dimes		60
" Quarters		50
" Halves	3	50
" Dollars		
" Gold		
TOTAL	107	94

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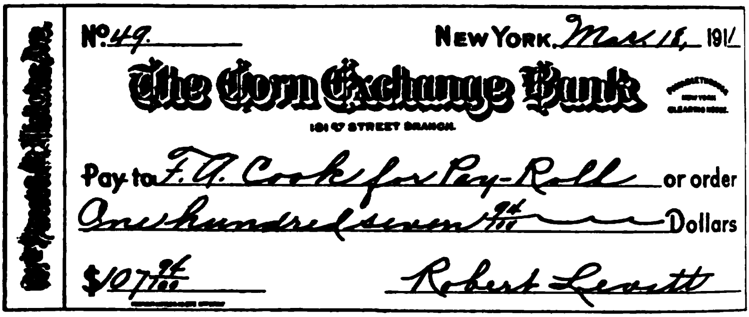
* Frequently the names as well as the numbers of the clerks are given, as shown in the form on page 92.

PAY ROLL CHECK



No. 49.
NEW YORK. Mar 18, 1911
The Corn Exchange Bank
181 ST STREET BRANCH
Pay to F. A. Cook for Pay-Roll or order
One hundred seven 94/100 Dollars
$107 94/100
Robert Leavitt

229. 1. In the following time sheet the rate is for a day of 8 hr., with ½ more for overtime.

Prepare a pay roll, a currency memorandum, and a check like the above.

TIME SHEET

WEEK ENDING, JAN. 21, 1911

CLERK NOS.	NAMES OF EMPLOYEES	M.	T.	W.	T.	F.	S.	TOTAL TIME	RATE	
1.	Addison, James	8	10	9	11	8	9		4.50	
2.	Buell, Henry	9	8	9	8½	9	8		4.00	
3.	Crane, John	10	9	8	9½	8	8		4.00	
4.	Crawford, Wm.	9	9½	8	9	10½	12		3.50	
5.	Jones, Albert	10	8	9	9	10	11		3.25	
6.	Landis, Frank	8	8	9	8	10	9		3.00	
7.	Money, Theo.	9	9	8	8½	9½	10		3.00	
8.	Newton, James	8	10½	9½	8	9	8		3.00	
9.	Orton, George	9	8	9	9	10	8		2.75	
10.	Rooney, Thos.	9	8	8	10	10	11		2.50	

2. The following time card is used by a large drug corporation for recording the time of beginning and ending work each day. The first two columns are for the forenoon; the second two for the afternoon, and the third two for the evening.

Find the total time and the total wages.

Form No. 3

No. ....*18*........

Name......*William MacCormack*..............................

Total Time..............................**Hr.** **Rate, 27½¢.**

Total Wages...**$**.......

Week Ending........*January 7*..............................**19**....

In	Out	In	Out	In	Out
7.50	11.30	12.20	5.30		
		12.30	5.30	6.45	11.30
7.45	11.30	12.45	5.45		
7.55	12.15			6.10	11.15
		12.00	5.40	6.30	11.20
8.00	12.20	12.50	6.10		
8.30	12.15	1.30	6.10		

3. A gang of carpenters working under the 8-hour law, with a maximum day of 10 hours, and one half extra pay for overtime, worked one week as follows.

Time Sheet. **Week Ending March 14, 1914.**

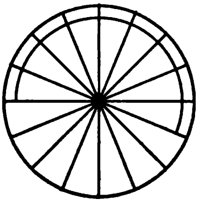
Names	M.	T.	W.	Th.	F.	S.	Total Time	Rate per Day
Bigelow, A. E. (foreman)	8	8½	8	9	8	10		$4.50
Arnold, A. W.	8	8	8	8	8	9		4.00
Babcock, C. D.	8	8	8	9	8	8		4.00
Comstock, L. O. . . .	8	8	8	8½	8	9		4.00
Evans, M. R.	8	8	8	8½	8	8		4.00
Geraghty, P. O.	8	8½	8	8½	8	9		4.00
Morrow, G. W.	8	8¼	8	8	8	8		4.00
Peters, A. B.	8	8½	8	8¼	8	8		4.00
Rogers, H. F.	8	8¼	8	8	8	9		4.00
Simler, H.	8	8	8	8	8	8		4.00
Watson, F. G.	8	8½	8	9	8	10		4.00
Whiteley, I.	8	8	8	8	8	10		4.00

Prepare a pay roll for the gang.

DIVISION OF FRACTIONS

230. To divide a fraction by an integer.

1. Into how many parts is this circle divided? What part of the circle is marked off by the curved line? If these $\frac{9}{16}$ are divided into 3 equal parts, how many sixteenths will there be in each part?



$$\tfrac{9}{16} \div 3 = \tfrac{3}{16}.$$

2. How does the fraction $\frac{3}{16}$ compare in value with the fraction $\frac{9}{16}$? Formulate a second way of dividing a fraction by an integer. (Prin. 20, page 101.)

231. 1. Divide $\frac{5}{8}$ by 3. $\frac{5}{8} \div 3 = \frac{5}{24}$.

Multiply the denominator of the fraction by the integer, 3, and write the numerator, 5, over the product. This divides the fraction, because multiplying the divisor (denominator) divides the quotient (which, in this case is the fraction). (Prin. 17, page 101.)

2. Divide $\frac{6}{11}$ by 2. $\frac{6}{11} \div 2 = \frac{3}{11}$.

Divide the numerator of the fraction by the integer, 2, and write the result over the denominator, 11. This divides the fraction, because dividing the dividend (numerator) divides the quotient (the fraction). (Prin. 20, page 101.)

This solution should always be used when the integer is an exact divisor of the numerator of the fraction.

3. Find the value of .0344 ÷ 8.

$$\begin{array}{r} .0043 \\ 8\overline{).0344} \end{array}$$

The first thing to do when the divisor is a whole number, as in this case, is to place the point for the quotient. Place it directly above the point in the dividend, as shown in the example. Divide thus: "8 is contained in 0, 0 times; 8 in 3, 0 times; 8 in 34, 4 times, etc." When the figures of the quotient are placed above the figures of the dividend, as here shown, the position of the decimal point in the quotient is right.

232. Find the value of:

1. $\frac{6}{7} \div 3$
2. $\frac{8}{9} \div 4$
3. $\frac{5}{8} \div 3$
4. $\frac{9}{10} \div 6$
5. .0125 ÷ 5
6. .464 ÷ 8
7. $\frac{7}{8} \div 3$
8. $\frac{9}{16} \div 3$
9. $\frac{11}{15} \div 3$
10. .968 ÷ 4
11. .8464 ÷ 16
12. .0035 ÷ 5
13. $\frac{7}{20} \div 5$
14. $\frac{10}{17} \div 8$
15. $\frac{11}{18} \div 9$
16. $\frac{24}{81} \div 6$

17. $.3125 \div 25$
18. $.738 \div 9$
19. $.001 \div 100$
20. $.00128 \div 16$
21. $\frac{19}{21} \div 4$
22. $\frac{15}{26} \div 5$
23. $\frac{11}{4} \div 5$
24. $\frac{25}{6} \div 5$
25. $\frac{19}{8} \div 3$
26. $\frac{81}{82} \div 6$
27. $\frac{43}{8} \div 7$
28. $.125 \div 50$
29. $.0125 \div 500$
30. $.8448 \div 8$
31. $\frac{159}{160} \div 3$
32. $\frac{249}{328} \div 83$
33. $\frac{896}{1024} \div 128$
34. $\frac{1089}{10} \div 121$
35. $\frac{1144}{12} \div 104$
36. $.00126 \div 63$
37. $.216 \div 24$
38. $.4888 \div 8$
39. $.03036 \div 11$
40. $.3069 \div 11$

233. To divide a mixed number by an integer.

If \$$1\frac{1}{2}$ is divided equally among 3 boys, how much will each receive? If 9 lb. of coffee cost \$$2\frac{1}{4}$, what part of a dollar does 1 lb. cost?

$1\frac{1}{2} \div 3 = ?$ $\quad 2\frac{1}{4} \div 9 = ?$ $\quad 2\frac{2}{3} \div 8 = ?$ $\quad 1\frac{3}{5} \div 4 = ?$

234. **1.** Divide $5\frac{5}{8}$ by 7.

$5\frac{5}{8} = \frac{45}{8}$

$\frac{45}{8} \div 7 = \frac{45}{56}$

When the mixed number is small, change it to an improper fraction, and divide as in dividing a fraction by an integer.

2. Divide $3478\frac{2}{9}$ by 8.

$$\begin{array}{r} 434\frac{7}{9} \\ 8\overline{)3478\frac{2}{9}} \end{array}$$

$6\frac{2}{9} = \frac{56}{9}$

$\frac{56}{9} \div 8 = \frac{7}{9}$

In problems like this, in which the mixed number is much larger than the divisor, *do not* change the mixed number to an improper fraction. Divide the integral part of the dividend by the divisor as if there were no fraction in the dividend. 8 is contained in 3478, 434 times, with a remainder of 6, which, with the fraction $\frac{2}{9}$, makes a total remainder of $6\frac{2}{9}$. Dividing $6\frac{2}{9}$ by 8 as explained in the preceding example gives the completed quotient, $434\frac{7}{9}$.

The student should divide $6\frac{2}{9}$ by 8, and all similar expressions, mentally. It is written out in full here for the sake of clearness.

3. Divide 42.3 by 25.

```
     1.692
 25)42.300
    25
    17 3
    15 0
     2 30
     2 25
        50
        50
```

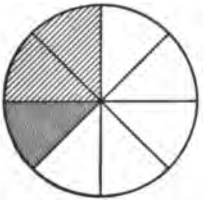
Place the point for the quotient directly above the point in the dividend. The important part, now, is to write the first figure of the quotient in the right place. Observe that 1, the first figure of the quotient, is written over the 2, the figure farthest to the right in the dividend, that is used to contain the divisor, 25. Continue the division, annexing ciphers in the dividend as needed, and placing the figures of the quotient over each succeeding figure in the dividend, respectively.

NOTE. If the figures of the quotient are placed correctly, the last, or right-hand figure, of the quotient will stand directly above the last figure of the dividend.

235. Find the value of:

1. $3\frac{3}{4} \div 5$
2. $6\frac{7}{8} \div 10$
3. $8\frac{3}{4} \div 7$
4. $3\frac{3}{8} \div 7$
5. $9\frac{4}{5} \div 14$
6. $18\frac{3}{4} \div 8$
7. $26\frac{7}{8} \div 4$
8. $47\frac{5}{9} \div 2$
9. $167\frac{8}{9} \div 21$
10. $568\frac{3}{7} \div 43$
11. $828\frac{3}{11} \div 69$
12. $939\frac{4}{15} \div 22$
13. $896\frac{4}{5} \div 15$
14. $948\frac{5}{8} \div 12$
15. $776\frac{7}{9} \div 9$
16. $3924\frac{5}{8} \div 12$
17. $4964\frac{3}{4} \div 11$
18. $6847\frac{5}{9} \div 96$
19. $3478\frac{3}{8} \div 84$
20. $9674\frac{5}{9} \div 99$
21. $8496\frac{7}{11} \div 64$
22. $4589\frac{7}{10} \div 5$
23. $896\frac{3}{11} \div 17$
24. $974\frac{3}{16} \div 11$
25. $897\frac{4}{5} \div 15$
26. $969\frac{3}{14} \div 7$
27. $896\frac{3}{20} \div 8$
28. $487\frac{5}{7} \div 10$
29. $389\frac{7}{12} \div 6$
30. $967\frac{3}{4} \div 5$
31. $478.336 \div 16$
32. $394.128 \div 8$
33. $48.3744 \div 24$
34. $9674.88 \div 48$
35. $364.64 \div 160$
36. $511.375 \div 125$
37. $895.4 \div 110$
38. $6.4872 \div 4$
39. $9.674\frac{3}{8} \div 8$
40. $4.592 \div 16$
41. $9.34\frac{1}{2} \div 12$
42. $6.43\frac{2}{3} \div 5$
43. $6.38\frac{7}{8} \div 9$
44. $9.72\frac{5}{9} \div 11$
45. $106.88\frac{8}{9} \div 15$

236. To divide an integer, a fraction, or a mixed number by a fraction.



How many times does the whole circle contain $\frac{1}{2}$ of the circle?

How many times does it contain $\frac{1}{4}$ of the circle? $\frac{1}{8}$ of the circle?

$$1 \div \tfrac{1}{2} = ? \quad 1 \div \tfrac{1}{4} = ? \quad 1 \div \tfrac{1}{8} = ?$$

If $\frac{1}{2}$ is contained in 1, 2 times, how many times is $\frac{1}{2}$ contained in 2? in 3? in 4? in 6? in 10?

$$1 \div \tfrac{1}{2} = 2 = \tfrac{2}{1}.$$
$$3 \div \tfrac{1}{2} = 6 = \tfrac{6}{1}.$$

Observe, in the first equation, that the answer, 2, or its equivalent, $\frac{2}{1}$, is the fraction $\frac{1}{2}$ inverted; that is, the denominator is written for the numerator, and the numerator for the denominator.

In the second equation, notice that the answer 6, or $\frac{6}{1}$, is 3 times as large as the answer in the first equation. That is, to divide by $\frac{1}{2}$ is to multiply by its reciprocal, 2.

In the same circle count off 3 eighths. How many times will these $\frac{3}{8}$ be contained in the circle?

Call the "eighths" "parts," and divide 8 parts (that is, the circle) by 3 parts. 8 parts ÷ 3 parts = $2\frac{2}{3}$ times. That is, $1 \div \frac{3}{8} = 2\frac{2}{3}$.

But $2\frac{2}{3} = \frac{8}{3}$, and $\frac{8}{3}$ is the fraction $\frac{3}{8}$ inverted.

The result of 1 divided by any fraction is that fraction inverted.

237. The **reciprocal** of a fraction is the fraction inverted.

238. The reciprocal of any number is 1 divided by that number. The reciprocal of 4 is $\frac{1}{4}$; of 8 is $\frac{1}{8}$; etc.

What is the reciprocal of $\frac{3}{4}$? of $\frac{5}{8}$? of $\frac{2}{3}$? of 10? of 12?

How may an integer be divided by a fraction?

To divide by a fraction, multiply by its reciprocal.

239. 1. Divide $6\frac{2}{3}$ by $\frac{4}{5}$.

$$6\frac{2}{3} \div \frac{4}{5} = 6\frac{2}{3} \times \frac{5}{4} = \frac{\overset{5}{\cancel{20}}}{3} \times \frac{5}{\cancel{4}} = \frac{25}{3} = 8\frac{1}{3}.$$

To divide by $\frac{4}{5}$ is to multiply by its reciprocal, $\frac{5}{4}$. $6\frac{2}{3}$ multiplied by $\frac{5}{4}$ gives $\frac{25}{3} = 8\frac{1}{3}$.

2. Divide $\frac{4}{5}$ of $\frac{5}{8}$ of $\frac{2}{3}$ by $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{9}{10}$.

$\frac{4}{5}$ of $\frac{5}{8}$ of $\frac{2}{3} \div \frac{3}{4}$ of $\frac{2}{3}$ of $\frac{9}{10}$ =

$$\frac{\cancel{4}}{\cancel{5}} \times \frac{\cancel{5}}{\underset{2}{\cancel{8}}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\overset{2}{\cancel{4}}}{3} \times \frac{\cancel{3}}{\cancel{2}} \times \frac{10}{9} = \frac{20}{27}.$$

In dividing by a compound fraction, invert all the separate fractions composing the divisor, and treat the problem as an exercise in cancellation.

NOTE. A compound fraction is a fraction of a fraction: thus, $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{4}{5}$ of $\frac{5}{8}$ of $\frac{7}{8}$, etc.

3. Divide 4.8325 by .025.

```
          193.3
  025‸)4.832‸5
        2 5
        2 33
        2 25
          82
          75
           7 5
           7 5
```

When the divisor is a decimal, change it to an integer by moving its point to the right-hand side of the decimal. In this example the point (as indicated by the caret) is moved three places, which is equivalent to multiplying by 1000. Since the divisor is multiplied by 1000, the dividend must likewise be multiplied by 1000 (Prin. 21, page 101). Therefore, the point in the dividend is moved 3 places to the right. Before dividing, place the point for the quotient directly above the point in the dividend after it is moved.

The division is now performed in the same manner as in dividing by integers.

4. Divide 5000 by .0005.

$$\begin{array}{r} 1000\,0000. \\ .0005_{\wedge})\overline{5000.0000_{\wedge}} \end{array}$$

As in the preceding example, change the divisor to an integer by moving the point to the right-hand side of the decimal, that is, by multiplying by 10,000. The dividend also must be multiplied by 10,000; that is, the point must be moved 4 places to the right. In this example it is necessary to make the places by annexing ciphers.

Remember, ***the point in the dividend*** **MUST** ***be moved in the same direction as many places as the point in the divisor is moved to make the divisor a whole number. Annex ciphers when necessary.***

Remember, also, to place the point for the quotient ***before*** **dividing.**

The process of division is then the same as in simple numbers.

240. Find the value of:

1. $\frac{2}{3} \div \frac{3}{4}$
2. $\frac{4}{5} \div \frac{9}{10}$
3. $\frac{5}{11} \div \frac{7}{8}$
4. $\frac{6}{7} \div \frac{5}{9}$
5. $1\frac{1}{2} \div \frac{2}{3}$
6. $2\frac{1}{4} \div \frac{4}{5}$
7. $9\frac{1}{3} \div \frac{7}{8}$
8. $15\frac{1}{4} \div \frac{3}{4}$
9. $\frac{2}{3}$ of $\frac{4}{5} \div \frac{5}{8}$ of $\frac{6}{7}$
10. $\frac{5}{8}$ of $\frac{3}{4} \div \frac{9}{10}$ of $\frac{5}{4}$
11. $\frac{8}{9}$ of $\frac{3}{4} \div \frac{11}{15}$ of $\frac{15}{22}$
12. $28\frac{1}{2} \times \frac{8}{19} \div \frac{5}{16}$ of $\frac{4}{5}$
13. $\frac{7}{15}$ of $22\frac{1}{2} \div \frac{3}{13}$ of $\frac{26}{33}$
14. $28\frac{2}{3} \div \frac{4}{15}$
15. $18\frac{3}{4} \div \frac{5}{8}$
16. $.2862 \div .009$
17. $34.74 \div .018$
18. $49.52 \div .16$
19. $.004 \div .0002$
20. $.400 \div .004$
21. $.800 \div .0001$
22. $.008 \div .0001$
23. $.12\frac{1}{2} \div .62\frac{1}{2}$
24. $4.875 \div .0625$
25. $8.5 \div .3125$
26. $40{,}000 \div .002$
27. $404.04 \div .20$
28. $96.848 \div 1.60$
29. $74.64 \div 1.25$
30. $395.20 \div .0125$
31. $16 \div \frac{5}{8}$
32. $28 \div \frac{4}{7}$
33. $36 \div \frac{12}{17}$
34. $89 \div \frac{2}{3}$
35. $56 \div \frac{6}{7}$
36. $38 \div \frac{19}{20}$
37. $42 \div \frac{6}{11}$
38. $32 \div \frac{16}{17}$
39. $48 \div \frac{9}{10}$

241. To divide an integer or a mixed number by a mixed number.

1. Divide 24 by $7\frac{1}{2}$.

$$\begin{array}{r} 7\frac{1}{2})24 \\ 2\quad 2 \\ \hline 3\frac{1}{5} \\ \hline 15)48 \\ 45 \\ \hline 3 \\ \hline 15 \end{array} = \frac{1}{5}$$

Write the dividend and divisor as for division of simple numbers. The divisor being a mixed number, it is first necessary to "clear of fractions." This is done by multiplying the divisor by 2, the denominator of the fraction in the divisor. Multiplying the divisor by 2 gives 15. If the divisor is multiplied by 2, the dividend must also be multiplied by 2 (why ?), which gives 48. The divisor and the dividend are now both integral, and the divison is easily completed.

2. Divide $19.44\frac{4}{9}$ by $4.16\frac{2}{3}$.

$$\begin{array}{r} 6 \\ 4.16\frac{2}{3})\ 19.44\frac{4}{9} \\ \underline{9 \qquad\quad 9} \end{array}$$

$$\begin{array}{r} 4.66\frac{2}{3} \\ 37\,5.0\,\overline{)175\,0.00} \\ \underline{150\,0} \\ 25\,0\,0 \\ \underline{22\,5\,0} \\ 2\,5\,00 \\ \underline{2\,2\,50} \\ \frac{2\,50}{3\,75} = \frac{2}{3} \end{array}$$

Multiply both dividend and divisor by 9, the least common multiple of the denominators of the fractions $\frac{2}{3}$ and $\frac{4}{9}$, retaining the decimal points in the products. Move the points each one place to the right, and divide as already explained. The quotient is $4.66\frac{2}{3}$.

242. Find the value of:

1. $472\frac{3}{4} \div 24\frac{2}{3}$
2. $867 \div 15\frac{1}{6}$
3. $978\frac{5}{8} \div 19\frac{1}{2}$
4. $5.26\frac{2}{3} \div .46\frac{2}{3}$
5. $8.49\frac{1}{2} \div 2.12\frac{3}{8}$
6. $349\frac{3}{5} \div 16\frac{1}{4}$
7. $8749 \div 22\frac{1}{4}$
8. $8396 \div 78\frac{2}{3}$
9. $7469\frac{3}{5} \div 152\frac{1}{2}$
10. $9119\frac{2}{3} \div 428\frac{1}{2}$
11. $16.31\frac{1}{2} \div 1.21\frac{1}{4}$
12. $329\frac{3}{5} \div 26\frac{1}{4}$
13. $897\frac{2}{3} \div 31\frac{1}{4}$
14. $598\frac{4}{7} \div 24\frac{1}{2}$
15. $939\frac{3}{8} \div 42\frac{2}{3}$
16. $978 \div 85\frac{1}{8}$
17. $125 \div 1\frac{2}{3}$
18. $250 \div 1.66\frac{2}{3}$
19. $375 \div 3.33\frac{1}{3}$
20. $964\frac{5}{8} \div 16\frac{1}{4}$
21. $493\frac{2}{3} \div 42\frac{3}{4}$
22. $967\frac{5}{6} \div 91\frac{1}{3}$
23. $83.92 \div 5.125$
24. $49.87\frac{1}{2} \div 2.49\frac{3}{8}$
25. $39.37\frac{1}{2} \div 1.31\frac{1}{4}$
26. $478\frac{3}{7} \div 21\frac{2}{3}$
27. $674\frac{5}{6} \div 91\frac{1}{2}$
28. $845\frac{3}{8} \div 14\frac{1}{4}$
29. $7689 \div 18\frac{5}{6}$
30. $598\frac{3}{5} \div 14\frac{1}{2}$
31. $963\frac{8}{9} \div 8\frac{2}{3}$
32. $458\frac{3}{4} \div 19\frac{1}{2}$
33. $732\frac{3}{8} \div 19\frac{3}{4}$
34. $864\frac{5}{8} \div 21\frac{1}{2}$
35. $438\frac{6}{7} \div 219\frac{3}{7}$
36. $644\frac{3}{8} \div 16\frac{1}{2}$
37. $849\frac{5}{9} \div 13\frac{2}{3}$
38. $674\frac{5}{9} \div 11\frac{1}{3}$
39. $84.96\frac{1}{2} \div 15\frac{1}{4}$
40. $60.83\frac{1}{3} \div 12\frac{1}{6}$
41. $.0482 \div .0002$
42. $59.68\frac{1}{2} \div .0005$
43. $11.25 \div .003\frac{3}{4}$
44. $131 \div .008\frac{1}{3}$
45. $45.75 \div .001\frac{2}{3}$

SOME FUNDAMENTAL PRINCIPLES OF ARITHMETIC

243. Every problem in arithmetic is based on one or more fundamental principles or laws. A knowledge of these principles and of their application to the solution of problems is essential to success in arithmetical calculations.

Instead of stating the principles of notation, addition, subtraction, etc., in connection with these topics, it has been deemed advisable to group them in one chapter, in order that the proper emphasis may be placed upon them.

It is suggested that special attention be given to these principles, as frequent reference will be made to them.

PRINCIPLES OF NOTATION AND NUMERATION

244. 1. *Each removal of any significant figure* one place to the left toward or from the decimal point multiplies its value by ten.*

2. *Each removal of any significant figure one place to the right toward or from the decimal point divides its value by ten.*

3. *Each cipher annexed to an integer multiplies its value by ten.*

4. *Each cipher prefixed to a decimal divides its value by ten.*

PRINCIPLES OF ADDITION

5. *Only like numbers or parts of like numbers can be added.*

6. *The sum is composed of the same kind of units as are the addends.*

PRINCIPLES OF SUBTRACTION

7. *Only like numbers or parts of like numbers can be subtracted.*

8. *The remainder is composed of the same kind of units as are the minuend and subtrahend.*

9. *The minuend is equal to the sum of the subtrahend and the remainder.*

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* A significant figure is a figure that possesses a value of its own. Thus, all the figures, 1, 2, 3, etc., except 0, are significant figures. 0 has no value of its own.

PRINCIPLES OF MULTIPLICATION

10. *The multiplicand may be either an abstract or a concrete number.*

11. *The multiplier is an abstract number.*

12. *The multiplicand and the product are like numbers.*

13. *The multiplier and the multiplicand may be interchanged without affecting the value of the product. (This principle is called the commutative law of multiplication.*

14. *The product of two factors divided by either of them will give the other one. (If there are more than two factors in any product, the product divided by any one of the factors will give a quotient equal to the product of all the other factors.)*

PRINCIPLES OF DIVISION

15. *The dividend is equal to the product of the divisor and the quotient (plus the remainder, if any).*

16. *The dividend and the remainder are like numbers.*

17. *Multiplying the divisor divides the quotient by the same number.*

18. *Dividing the divisor multiplies the quotient by the same number.*

19. *Multiplying the dividend multiplies the quotient by the same number.*

20. *Dividing the dividend divides the quotient by the same number.*

21. *Multiplying or dividing both divisor and dividend by the same number does not change the value of the quotient.*

REMARK. Some of the above principles are already familiar to the student and are applied unconsciously to the solution of problems. Others (Nos. 11, 12, 14, 15, 16, 17, 18, 19, 20, and 21) are not so familiar, and need emphasis. In the number of its practical applications, Principle No. 14 is the most important.

APPLICATION OF THE FUNDAMENTAL PRINCIPLES OF ARITHMETIC TO THE SOLUTION OF PROBLEMS

245. The solution of problems requires a knowledge of three things: (1) The student must know how to add, subtract, multiply, and divide; (2) he must know in what order these operations are to be performed; and (3) he must be able to make the right combinations of numbers in any given problem.

The order of performing the operations and the proper combining of numbers depend upon a knowledge of the fundamental principles of arithmetic.

The following illustrations show the practical application of the most important principles to the solution of problems.

Very many problems are reducible to one or more of three type forms. As type problems, the following may be taken as examples:

Type 1. $\frac{3}{4}$ of \$ 48 = ?

Type 2. $\frac{5}{8}$ of what number = \$ 45?

Type 3. What part of \$ 90 = \$ 75?

In the first example given, the operation is obviously one of multiplication, and the product is \$ 36 (Prin. 12).

In the second example one factor and a product are given. The product is \$45, and the given factor is $\frac{5}{8}$. The missing factor is indicated by the words "what number." By Principle 14, if \$ 45 is divided by $\frac{5}{8}$, the other factor will be obtained. \$ 45 ÷ $\frac{5}{8}$ = \$ 72.

Many problems are reducible to this type form. It is worthy of the careful attention of the student.

In the third example there are also given a product, \$ 75, and a factor, \$ 90. The product divided by the given factor gives the other factor. \$ 75 ÷ \$ 90 = $\frac{75}{90}$ = $\frac{5}{6}$. $\frac{5}{6}$ is the missing factor.

In the second example the missing factor was the multiplicand (Prin. 12). In the third example it was the multiplier (Prin. 11).

Practice in reducing problems to their appropriate type form will be found helpful. Following are a few illustrations and explanations:

1. A farmer had 48 cd. of wood and sold $\frac{3}{4}$ of it. How many cords did he sell?

First, after reading the problem, note the question, "How many cords did he sell?" The problem answers this question by saying that he "sold $\frac{3}{4}$ of it." What is "it"? "It" is 48 cd. Now put the number in the place of "it," and you have $\frac{3}{4}$ of 48 cd. = number of cords sold. The problem is now reduced to the form of the first type problem (Prin. 12).

2. A street is paved for a distance of 960 yd., which is $\frac{4}{5}$ of its length. What is the length of the street?

Read the problem. Note the question to be answered. Is the street all paved? What part of it is paved? How many yards are paved? How do

"$\frac{4}{5}$ of the length of the street" and "960 yd." compare? (They are alike.) Therefore write,

$\frac{4}{5}$ of the length of the street = 960 yd. or

$\frac{4}{5}$ of ? yards = 960 yd.

960 yd. is the product obtained by multiplying some number by $\frac{4}{5}$. Therefore,

960 yd. ÷ $\frac{4}{5}$ = 1200 yd., length of street. (Prin. 14, Type 2.)

3. A man's annual income is $ 2500, and his expenses for the year are $ 1500. What part of his income does he save?

Read the problem till you know what it means. Note the question carefully.

The question is, "What part of his income did he *save?*" He saved the difference between $ 2500 and $ 1500, which is $ 1000. He saved $ 1000. His income was $ 2500.

Now write the question in the shortest way, using numbers instead of words where possible.

? part of $ 2500 = $ 1000? (Prin. 14, Type 3.)

Reduce each of the following problems to its appropriate type form.

1. A merchant bought 480 yd. of muslin and sold $\frac{5}{8}$ of it. How many yards did he sell?

2. A man's total expenses for one year were $ 1280, of which amount $ 400 was spent for board. His board bill was what part of all his expenses?

3. A manufacturing concern pays profits amounting to $\frac{3}{16}$ of its capital. If the profits are $ 15,000, what is the capital?

4. A man sold $\frac{3}{8}$ of his farm for $ 4612$\frac{1}{2}$. At the same rate how much was the whole farm worth?

5. A dealer bought goods for $ 12$\frac{1}{2}$, and sold them for $ 15. The gain was what part of the cost? The gain was what part of the selling price?

6. If a hatter buys a hat for $ 6, and has to sell it for $ 4$\frac{1}{2}$, he loses what part of the cost? of the selling price?

7. The first of these lines is 1$\frac{1}{2}$ in. long, the second 1$\frac{1}{4}$ in. long.

1909.________________________

1910.____________________

If the first line represents a merchant's sales in 1909, and the second one, his sales in 1910, the amount of sales in 1910 was what part of the sales in 1909? The sales in 1910 were how much (fractional part) less than in 1909? The sales in 1909 were how much greater than in 1910?

PROBLEM ANALYSIS

246. The analysis of a problem consists in stating the reason for each successive step in the solution.

1. How long will it take a man earning $15 a week to pay for a motor cycle costing $200, if his expenses are $7 a week?

ANALYSIS. If he earns $15 a week and spends $7 a week, he can save $8 a week. It will take as many weeks to pay for the motor cycle as $8 is contained times in $200, or 25 times. Therefore, it will take 25 weeks to pay for the motor cycle.

2. If $3\frac{1}{2}$ pounds of sugar cost 21¢, how much will 12 pounds cost?

ANALYSIS. If $3\frac{1}{2}$ pounds cost 21¢, 1 pound will cost as many cents as $3\frac{1}{2}$ is contained times in 21, or 6 times. If 1 pound of sugar costs 6¢, 12 pounds will cost 12 × 6¢, or 72¢.

REVIEW PROBLEMS IN FRACTIONS

MENTAL

247. Find the results as indicated:

1. $\frac{1}{2}+\frac{1}{3}$; $\frac{1}{2}-\frac{1}{3}$; $\frac{1}{2}\times\frac{1}{3}$; $\frac{1}{2}\div\frac{1}{3}$.
2. $\frac{4}{5}+\frac{1}{2}$; $\frac{4}{5}-\frac{1}{2}$; $\frac{4}{5}\times\frac{1}{2}$; $\frac{4}{5}\div\frac{1}{2}$.
3. $\frac{3}{7}+\frac{3}{8}$; $\frac{3}{7}-\frac{3}{8}$; $\frac{3}{7}\times\frac{3}{8}$; $\frac{3}{7}\div\frac{3}{8}$.
4. $4\frac{1}{2}+2\frac{1}{4}$; $4\frac{1}{2}-2\frac{1}{4}$; $4\frac{1}{2}\times 2\frac{1}{4}$; $4\frac{1}{2}\div 2\frac{1}{4}$.
5. $7\frac{1}{3}+3\frac{2}{3}$; $7\frac{1}{3}-3\frac{2}{3}$; $7\frac{1}{3}\times 3\frac{2}{3}$; $7\frac{1}{3}\div 3\frac{2}{3}$.
6. If $\frac{3}{4}$ is subtracted from a given fraction, the remainder will be $\frac{1}{6}$. What is the fraction?
7. $\frac{5}{8}$ added to a certain fraction gives $1\frac{7}{24}$. What is the fraction?

Analyze as many of the following problems as the teacher directs:

8. If a man spends, on the average, $$\frac{5}{8}$ a day for cigars and liquor, in what time will he spend $100?

9. A boy bought a bicycle for $45, and sold it for $\frac{8}{9}$ of what he gave for it. How much less did he receive than he paid?

10. Two men bought a motor boat for $450, each paying half. Later each sold one third of his share at cost. How much had each invested then, and what part of the boat did each own?

11. I bought $11\frac{3}{8}$ lb. of tea at $\$\frac{1}{2}$ a pound and 6 lb. of coffee at $\$\frac{3}{8}$ a pound. How much change should I receive from a ten-dollar bill?

12. A young lady bought $7\frac{1}{2}$ yd. muslin at $8\frac{1}{2}$¢ a yard. What change ought she to receive from a half dollar and a quarter?

13. A woman bought $10\frac{1}{2}$ yd. of silk at $\$2\frac{1}{2}$ a yard, and $3\frac{1}{2}$ yd. lining at $.25 a yard. She gave in payment three ten-dollar bills. What change should she receive?

14. I sold a horse for $75, which was $\frac{5}{6}$ of what it cost. How much did I lose?

15. A crate containing 10 doz. oranges cost $1.75. If the oranges sell at the rate of 2 for 5¢, what is the gain?

16. A bunch of 8 doz. bananas cost $1. If one dozen spoil and the rest are sold at the rate of 3 for 10¢, what is the gain?

17. Two railroads carry merchandise 450 mi., for $3.60. If the first road carries it 200 mi., how much should each receive?

18. Two men engage to do a piece of work for $157.50. The first works 18 da. and the other works 27 da. How shall the money be divided?

19. $3\frac{1}{2}$ doz. eggs at $\$\frac{1}{4}$ a dozen will pay for how many pounds of rice at $8\frac{3}{4}$¢ a pound?

20. I sold $\frac{3}{8}$ of my apples for $90. At that rate, how much were they all worth?

21. A merchant by selling cloth at 42¢ a yard gained $\frac{1}{6}$ of the cost. Find the cost.

22. I bought a quantity of cloth for $180. By selling it at $2 a yard, I gained $\frac{1}{3}$ of the cost. How many yards were there?

23. A buying agent charges $\frac{1}{16}$ of the net cost of goods as his commission. If his commission for one month amounts to $240, what is the value of the goods bought?

24. I sold $\frac{3}{7}$ of my land for $1500. If I sold 75 A., how much land have I left, and how much is it worth at the same rate?

25. A man picked $\frac{3}{4}$ of a bushel of pears and sold $\frac{1}{2}$ of them for $.45. At that rate, how much would a bushel cost?

26. A man had $\$37\frac{1}{2}$, and spent $\$12\frac{1}{2}$. What part of his money did he spend?

27. A clerk earns $\$87\frac{1}{2}$ a month, and spends \$30 for board and \$20 for other expenses. What part of his money does he save?

28. In a certain factory 56 men were employed. During a dull season 24 of them were laid off. What part of the men continued working?

29. In an orchard there are 120 trees; 96 of them are apple trees, 16 are pear trees, and the remainder are cherry trees. What part of all is each kind of trees?

30. Three railroads carry a piece of freight 540 mi. The first carries it 180 mi., the second 300 mi., and the third the remainder of the distance. What fractional part of the distance does each carry it?

PROBLEMS

(Solve mentally when possible. Analyze as directed by the teacher.)

248. **1.** $\frac{3}{8}$ of a pole 32 ft. long is decayed. How many feet are good?

2. A tree 76 ft. high breaks 19 ft. from the ground. What part is broken off?

3. A man has $\frac{1}{2}$ of his money in one bank, $\frac{1}{3}$ of it in another, and the remainder in a third bank. If the amount in the first bank is \$660, how much is there in each of the other banks?

4. A man sold a horse for \$150, which was $\frac{3}{4}$ of what he paid for it. Find the loss.

5. The wholesale price of Rio coffee at New York increased from $7\frac{1}{4}$¢ in August, 1909, to $8\frac{3}{4}$¢ in August, 1910. What fractional part of the price in 1909 was the increase?

6. A farmer sold $\frac{3}{5}$ of his sheep for \$360. At the same rate what is the value of all his sheep? What is the value of those he did not sell?

7. A boy spent $\frac{3}{5}$ of his money for a bicycle, and had \$24 left. How much money had he at first?

8. A man had $\frac{1}{2}$ of his sheep in one pasture, $\frac{1}{4}$ of them in another pasture, and the rest of them, which was 33 sheep, in a third pasture. How many sheep had he?

9. A gentleman's estate is so divided that $\frac{3}{8}$ of it is woodland, $\frac{1}{4}$ of it is water, and the remaining 600 A. are under cultivation. How many acres are there in the estate?

10. A house and lot are worth $ 6300. If the lot is worth $\frac{4}{5}$ as much as the house, how much is each worth?

11. A gentleman left his son $ 6500, which amount was $2\frac{1}{6}$ times as much as he left to his daughter. How much did the daughter receive?

12. A owned $\frac{4}{7}$ of a farm and B the remainder. If A had $24\frac{3}{4}$ A. more than B, how many acres had each?

13. A house and a barn are valued at $ 14,805. Find the value of each if the barn is worth $\frac{2}{5}$ as much as the house.

14. Find the cost of each, if a house and lot together cost $ 14,000, the lot costing .4 as much as the house.

15. A gentleman left his estate to his three sons. The eldest received .40 of it, the next son .35, and the youngest, $ 7500. Find the value of the estate.

16. The cost of furnishing two rooms is $ 480. If the cost of one is $\frac{7}{9}$ as much as the cost of the other, what is the cost of each?

17. The income from a certain business for two years was $ 27,500. Find the income for each year if the second year's income was .20 greater than the first.

18. What number increased by $\frac{3}{4}$ of itself is 651?

19. If goods are sold for $ 90, and a profit of $\frac{1}{5}$ of the cost is made, find the cost.

20. If the profit on eggs is 3¢ per dozen, and the eggs sell for 30¢ a dozen, what part of the cost is gained?

21. The profit on a pair of shoes is $ .75. If the profit is .20 of the cost, for how much per pair do the shoes sell?

22. What number decreased by $\frac{3}{7}$ of itself is equal to 56?

23. Find the cost of an article sold for $ 36, if $\frac{1}{7}$ of the cost was lost.

24. A hatter marked all his straw hats down $\frac{1}{4}$. What was the marked price of a hat that he now sells for $ 4.50? If he made a profit of $\frac{1}{2}$ of the cost *before* marking the hat down, what part of the cost does he make *after* marking it down?

25. If a man buys $868\frac{1}{2}$ yd. of cloth and sells $\frac{3}{8}$ of it at $14\frac{3}{4}$¢ a yard, and the remainder at $12\frac{3}{4}$¢ a yard, how much does he receive for all of it?

26. A field of $9\frac{1}{2}$ A. was bought for $ 380. The first year's wheat crop averaged 20 bu. to the acre. If wheat was worth 90¢ per bushel, the value of the crop was what part of the purchase price?

27. Find the total value of the following articles: $1\frac{3}{4}$ doz. buttons at 15¢; $2\frac{3}{4}$ yd. ribbon at $9\frac{1}{2}$¢; $3\frac{1}{2}$ yd. lace at 65¢; $\frac{3}{4}$ yd. velvet at 79¢.

28. I bought 360 sheep for $ 1200. I sold $\frac{2}{3}$ of them at $ 4 per head, and $\frac{1}{4}$ of the remainder died. At what price per head must the rest of them be sold to gain $\frac{1}{6}$ of the cost of all?

29. A fruit dealer buys 10 doz. oranges for $ 2.30. If 2 doz. spoil, at what price per dozen must he sell the good ones to gain $\frac{1}{4}$ of the cost?

30. From the sum of $36\frac{3}{4}+47\frac{1}{8}+18\frac{1}{8}$ take the sum of $31\frac{2}{3}$ and $24\frac{3}{8}$.

31. Multiply the sum of $27\frac{1}{8}$ and $34\frac{3}{5}$ by the difference between $18\frac{2}{3}$ and $16\frac{1}{2}$.

32. Divide the product of $83\frac{13}{16}\times 79\frac{1}{12}$ by the product of $18\frac{1}{4}\times 9\frac{5}{16}$.

33. A young man buys $13\frac{3}{4}$ A. of land at $ 36.50 per acre. If he works for $27\frac{1}{2}$¢ an hour, 8 hr. a day, and spends $ 6.60 a week for board and other expenses, how long will it take him to pay for the land?

34. If eggs are bought at 28¢ a dozen and sold at the rate of 10 eggs for 25¢, find the gain on 500 doz. eggs.

35. On the first Saturday of each month of the fiscal year ending June 30, 1910, the wholesale weekly price of rubber per pound at New York City was $ 1.45, $ 1.87, $ 1.92, $ 2.03, $ 1.84, $ 1.75, $ 1.76, $ 1.84, $ 2.06, $ 2.80, $ 2.47, $ 2.28, respectively. Find (1) the average price for the year; (2) what fractional part the average price is greater than the first price given.

NOTE. The average of several numbers is found by dividing their sum by the number of numbers.

36. Using the form of pay roll given on page 91, make a pay roll from the following data: The rate is for a week of 48 hr., with ¼ extra for overtime. Clerk #1 worked 9, 8½, 10, 10½, 8, and 9¼ hr. at $16.50; #2 worked 8, 8, 11½, 9, 7½, and 10 hr. at $15; #3 worked 9¼, 8½, 8, 8¾, 8, and 8 hr. at $20; #4 worked 8½, 8, 8, 9, 10, and 9¼ hr. at $18; #5 worked 8, 8, 8, 9, 8, and 8 hr. at $14.50; #6 worked 8, 8, 9, 9, 8, and 10 hr. at $16; #7 worked 8, 9, 8, 10, 8½, and 10 hr. at $17; and #8 worked 8½, 9, 9, 8, 9, and 8½ hr. at $19.50.

The following facts are taken from the weekly reports of a cheese factory as given to a dairyman who supplies part of the milk. Sales are made weekly, and the proceeds returned to the milk producers. Sale #7 is worked out in detail. Find the missing items in the other sales.

		37.	**38.**	**39.**	**40.**	**41.**
	SALE #7	SALE #8	SALE #9	SALE #10	SALE #11	SALE #12
From	May 25	June 1	June 8	June 15	June 22	June 29
To	May 31	June 7	June 14	June 21	June 28	July 5
Total Milk . .	59572 lb.	53781 lb.	63114 lb.	63639 lb.	63256 lb.	61247 lb.
Total Cheese .	5767 lb. @ 11½¢	5070 lb. @ 11⅝¢	6119 lb. @ 11¾¢	6084 lb. @ 11⅛¢	5997 lb. @ 10¾¢	5847 lb. @ 11⅜¢
Total Money .	$663.20					
Total Expense	$74.59	$67.13	$78.99	$78.55	$77.46	$75.59
Pounds Milk to 1 lb. Cheese .	10.3298					
Ratio	$.98806					
Your Milk . .	1209 lb. $11.95	1018 lb.	1165 lb.	1141 lb.	1150 lb.	1074 lb.
Less Cheese . .	4 lb. .46				3½ lb.	
Your Money .	$11.49					

The total money is the product of the total number of pounds of cheese by the price per pound. The number of pounds of milk to a pound of cheese is found by dividing the total amount of milk by the total amount of cheese. The ratio is the net return per 100 lb. of milk, found by dividing the net proceeds of the sale by the total milk in hundredweight. Thus, $663.20 − $74.59 = $588.61, net proceeds. $588.61 ÷ 595.72 = $.98806, the ratio. 1209 lb. multiplied by the ratio per 100 gives $11.95, from which is deducted the value of the cheese used by the dairyman, leaving $11.49.

249. The following is a form used in making an annual statement of receipts and disbursements by the business manager of a high school paper.

Tenth Annual Financial Statement of the Business Manager of *The High School News* for the Year ended June, 1914.

	RECEIPTS								DISBURSEMENTS						BALANCE	
	Sales		Ads		Mdse.		Total		Printing		Mdse.		Total			
Balance															275	90
September	98	60	15		12	50	126	10	110	60			110	60	15	50
October	102	30	25	75	60	40	188	45	112	70	42	25	154	95	33	50
November	127	40	35	50	62	30	225	20	121	50	35	20	156	70	68	50
etc.																

1. Prepare a form similar to the above and tabulate the receipts and disbursements for the football game between the Commerce High School and the Central Business College. There were sold to members of the athletic association 1528 tickets at 25 ¢; to other students 732 tickets at 35 ¢; and at the gate 2406 tickets at 50 ¢. The expenses were for printing and advertising, $ 45; for the use of the athletic field, one third of the total receipts from ticket sales; miscellaneous expenses, $ 28.75.

2. The same general form may be used for making a statement of receipts and disbursements in your school lunch room.

DOMESTIC PARCEL POST

250. The Parcel Post Law, which became effective January 1, 1913, provides:

> "That hereafter fourth-class mail matter shall embrace all other matter, including farm and factory products, not now embraced by law in either the first, second, or third class, not exceeding eleven pounds in weight, nor greater in size than seventy-two inches in length and girth combined, nor in form or kind likely to injure the person of any postal employee or damage the mail equipment or other mail matter and not of a character perishable within a period reasonably required for transportation and delivery.
>
> "That the rate of postage on fourth-class matter weighing not more than four ounces shall be one cent for each ounce or fraction of an ounce; and on such matter in excess of four ounces in weight the rate shall be by the pound, as

hereinafter provided, the postage in all cases to be prepaid by distinctive postage stamps affixed."

For the purpose of carrying this law into effect the United States is divided into zones with different rates of postage applicable to each, as follows:

WEIGHT	1ST ZONE		2D ZONE, 50 to 150 MILES. *Rate*	3D ZONE, 150 TO 300 MILES. *Rate*	4TH ZONE, 300 TO 600 MILES. *Rate*	5TH ZONE, 600 TO 1,000 MILES. *Rate*	6TH ZONE, 1,000 TO 1,400 MILES. *Rate*	7TH ZONE, 1,400 TO 1,800 MILES. *Rate*	8TH ZONE, ALL OVER 1,800 MILES. *Rate*
	Local rate	Zone rate, 50 miles							
1 pound	$0.05	$0.05	$0.05	$0.07	$0.08	$0.09	$0.10	$0.11	$0.12
2 pounds	.06	.06	.06	.12	.14	.16	.19	.21	.24
3 pounds	.06	.07	.07	.17	.20	.23	.28	.31	.36
4 pounds	.07	.08	.08	.22	.26	.30	.37	.41	.48
5 pounds	.07	.09	.09	.27	.32	.37	.46	.51	.60
6 pounds	.08	.10	.10	.32	.38	.44	.55	.61	.72
7 pounds	.08	.11	.11	.37	.44	.51	.64	.71	.84
8 pounds	.09	.12	.12	.42	.50	.58	.73	.81	.96
9 pounds	.09	.13	.13	.47	.56	.65	.82	.91	1.08
10 pounds	.10	.14	.14	.52	.62	.72	.91	1.01	1.20
11 pounds	.10	.15	.15	.57	.68	.79	1.00	1.11	1.32

The local rate is applicable to parcels intended for delivery at the office of mailing or a rural route starting therefrom. On Aug. 15, 1913, the limit of weight for first and second zones was increased to 20 lb.

Procure from your Post Office a parcel post map of the United States. It will show the territory included in the different zones.

251. Using your zone map and the table of rates given above, find the cost of sending by parcel post, the following from your home town to the places mentioned:

1. A package weighing 3 lb. to Buffalo, N.Y.
2. A package weighing 4 lb. to St. Louis, Mo.
3. A package weighing 8 lb. to Savannah, Ga.
4. A package weighing 2 lb. to New Orleans, La.
5. A package weighing 10 lb. to San Francisco, Cal.
6. A package weighing 9 lb. to Chicago, Ill.
7. A package weighing 11 lb. to Denver, Colo.
8. A package weighing 6 lb. to Boston, Mass.
9. A package weighing 5 lb. to Portland, Ore.
10. A package weighing 7 lb. to Galveston, Tex.

REMARK. Let students compare the parcel post rates with express rates.

EXAMINATIONS

SPEED TEST

252. Minimum time, thirty minutes; maximum time, one hour. Suggestion for marking: Allow 100 credits if test is completed correctly in minimum time. Deduct one credit for each minute required beyond minimum time.

1. Add the following:

241693798	$693.05
473185459	7.847
391533768	78.98
427936857	1489.86
819348673	.75
473925165	974.98
274639827	6.
315987352	87.87
675431298	9748.098
897316984	7.96
931258369	75.899
687316984	17.

2.

From	Take	Difference
752361	582834	
1021314	987476	
7934.19	986.74	
784	95.9876	

3. Find the total value of the following:

222^1 yd. @ 3^2¢
166^2 yd. @ 4^1¢
265^1 yd. @ 7^1¢
163^2 yd. @ 9^2¢
254^3 yd. @ 8^1¢
212 yd. @ 4^3¢

4. Complete the following bill:

CHICAGO, ILL., July 1, 1910.

H. P. JONES

Bought of

D. C. ROY.

3 bbl. Sugar,	640# @ 6¼¢		
5 chests Tea,	680# @ 75¢		
3 chests Tea,	498# @ 66⅔¢		
6 bbl. Flour,	1176# @ 4¢		
Total	. . . . .		

5. Find the total value of the following:

3290 lb. @ $2.75 per hundredweight.
4332 lb. @ 2.75 per hundredweight.
3570 lb. @ 1.87½ per hundredweight.
5792 lb. @ 22.50 per 1000 lb.
2396 lb. @ 16.50 per 1000 lb.
32690 lb. @ 12.80 per 1000 lb.

6. Add 873 to 428 continuously till the sum equals or exceeds 10,000. (Show all the work.)

7. Divide 25,473 by 2237 by continued subtraction. (Show all the work.)

8. Reduce to equivalent fractions having the least common denominator:

(*a*) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{12}$, $\frac{7}{8}$. (*c*) $\frac{9}{16}$, $\frac{5}{8}$, $\frac{5}{24}$, $\frac{3}{32}$.
(*b*) .75, $\frac{5}{6}$, $\frac{7}{9}$, $\frac{5}{18}$. (*d*) .80, $\frac{2}{3}$, $\frac{4}{15}$, .3.

9. Reduce to common fractions in their lowest terms:

.065 .0625 .225 .00125 .

10. Perform the operation indicated:

$15\frac{3}{4} \times 16\frac{1}{3}$ $426\frac{3}{4} \div 12\frac{1}{2}$ $48.72 \times .00125$ $54.7325 \div .001$

SPEED TEST

253. 1. Add horizontally and vertically. Check by adding the totals:

$36,968	$ 7,634	$14,262	$——
22,635	28,329	37,257	——
9,564	82,378	6,295	——
49,877	69,764	34,472	——
Grand total			$——

2. Find the gain on each of the following items and the total gain:

Selling Price	Cost	Gain
$40,000.00	$37,624.37	$——
9,437.83	5,819.24	——
4,341.85	2,838.96	——
57,640.33	48,924.29	——
89,754.00	68,947.25	——
Total		$——

Perform the following indicated operations:

3. $425 \div \frac{5}{6} =$
$234 \div .33\frac{1}{3} =$
$.25 \times \frac{8}{9} =$
$9\frac{1}{3} - 4\frac{3}{16} =$
$\frac{1}{3} + \frac{1}{6} + \frac{1}{9} =$

4. $9 \times 12 \times 14 \div 42 =$
$95 \times 8 \times 3 \div 19 =$
$52 \times 8 \times 7 \div 91 =$
$20 \times 21 \times 22 \div 154 =$
$(48 \times 36 \times 24) \div (16 \times 12) =$

5. Find the value of each of the following:

$$\frac{\frac{3}{4}+\frac{5}{6}-\frac{1}{3}}{\frac{3}{4}}\times 12\tfrac{1}{2}= \qquad \frac{\frac{13}{15}}{\frac{7}{5}}\times\frac{15}{16}\times\frac{14}{65}=$$

6. Reduce each of the following groups to equivalent fractions having the least common denominators:

(*a*) $\frac{9}{24}$ $\frac{13}{18}$ $\frac{7}{12}$ $\frac{11}{36}=$ (*c*) $\frac{4}{15}$ $\frac{5}{21}$ $\frac{4}{27}$ $\frac{2}{63}=$
(*b*) $\frac{3}{5}$ $\frac{4}{7}$ $\frac{11}{14}$ $\frac{9}{10}=$ (*d*) $\frac{5}{12}$ $\frac{8}{21}$ $\frac{11}{28}$ $\frac{13}{42}=$

7. Multiply and prove:

76897 × 328
45723 × 126

8. Divide and prove:

$6806\frac{1}{4} \div 412\frac{1}{2}$

9. Find the total cost of the following:

250 bu.	@	$ .75 =	$ ——
340 bu.	@	.80 =	——
1360 bu.	@	1.12½ =	——
1840 bu.	@	1.25 =	——
1480 bu.	@	.62½ =	——
	Total	.	$ ——

10. Find the total interest at 6 % on the following:

$ 500.00	for 90 da. =	$ ——
750.00	for 80 da. =	——
1000.00	for 30 da. =	——
1200.00	for 45 da. =	——
895.50	for 20 da. =	——
	Total . .	$ ——

WRITTEN TEST

254. 1. A company employs 17 laborers at $1.50 per day; 28 mechanics at $ 3.60 per day; 9 teamsters at $2.25 per day; 2 bookkeepers at $15 a week; and a superintendent at $1560 a year. What is the weekly pay roll of the company?

2. A man sold a farm of 248½ A. at $45¾ per acre, and with the proceeds bought another farm of 125 A. What price per acre did he pay for the new farm?

3. A person owned $\frac{5}{8}$ of a mine and sold $\frac{3}{4}$ of his share for $1710. What was the value of the mine?

4. Find the net cost of the following bill of hardware: 12 kegs cut nails, 8*d.*, 1200 lb. @ 3½ ¢ per pound; 5 doz. handsaws, 26 in., @ $15 per dozen; 4 doz. carpet stretchers at $ 2.87½ per dozen; 20 doz. garden rakes @ $ 3.25 per dozen; and 12 doz. try squares @ $ 5.40 per dozen. Less 20 % on the entire bill.

5. Find the total cost of the following: 8 pieces broadcloth, 56^1, 58^3, 57, 59^2, 55^1, 60^1, 61^2, 58^3 yd. at $ 3.12½; 5 pieces corduroy, 47^2, 44^2, 43^3, 46^2, 45^1 yd. at $ 0.57½; and 7 pieces storm serge, 43^1, 47^2, 48^3, 44, 45^2, 48^2, 47^3 yd. at $ 0.62½.

6. A man had on hand in the morning cash in the safe amounting to $ 206.20, and in the bank $ 1379.30. During the day he received cash in currency amounting to $ 909.24 and checks amounting to $ 489.36. Cash deposited in the bank, $ 604.37. Checks drawn on the bank account, $ 3.97, $ 47.86, $ 396.25, $ 49.83, $ 246.97. Cash paid out in bills and coin, $ 49.86. Show the condition of the cash and bank accounts in the evening.

7. Find the total cost of 64 bags of sugar, each containing 12½ lb. at 5½ ¢ a pound; 19 bbl. of flour at $ 5.50 a barrel; 1740 eggs at 33 ¢ a dozen; 375 pineapples at $ 12.75 per hundred; 42 qt. of walnuts at 8½ ¢ a quart.

8. A stenographer gets $ 83⅓ per month. How long will it take him to pay for a house and lot costing $ 2300, if his yearly expenses average $ 625.60? Reduce time to years, months, and days.

9. Find the cost of the following: 348 eggs at 14 ¢ per dozen; 643 lb. sugar at 6½ ¢ per pound; and 7750 lb. coal at $ 5.87 per ton.

10. In a hospital having 1048 inmates there were consumed the following articles: Meat, fresh, 287,138 lb., average price, $ .082; meat, smoked, 7614 lb., average price, $ 0.12; meat, canned, 7239 lb., average price, $ .066; poultry, 4128 lb., average price, $ .167. Find the total cost and the cost per capita.

PRACTICAL MEASUREMENTS

TIME

255. The time between two dates may be expressed in three ways; viz. compound time, exact time, and bankers' time.

256. Compound time is expressed in years, months, and days. It is determined by the method of compound subtraction. The time from Dec. 14, 1912 to Oct. 11, 1915 is reckoned thus:

1915	10	11
1912	12	14
2	9	27

The compound time is 2 yr. 9 mo. 27 da. Compound time is used in the majority of the ordinary business transactions involving time, especially if the period is more than a year.

257. The method is based on a 360-da. year — 12 mo. of 30 da. each. "One half a month" is fifteen days; one quarter of a year is three calendar months; and one half of a year is six calendar months.

258. A month from a given day in any month is the same day in the next month except in those months in which there is no day to correspond with the given day in the month from which the time is reckoned. For instance, 1 mo. from Jan. 14 is Feb. 14, and 5 mo. from Feb. 1 is July 1. But 1 mo. from Jan. 29, 30, or 31 is Feb. 28 (29th in leap year), and 3 mo. from Aug. 31 is Nov. 30, etc.

259. There is a discrepancy of 5 da. per annum between compound time and exact time, but business men generally are willing to forego the difference on account of the facility of reckoning compound time.

260. Exact time is expressed in days, or years and days. It is determined by counting the exact number of days between two dates. The exact time from May 3 to Sept. 28 is reckoned thus:

May, 28 days remaining.
June, 30 days.
July, 31 days.
Aug., 31 days.
Sept., 28 days.
148 days.

The first date, May 3, is omitted; the last date, Sept. 28, is included. For exceptions, see Art. 454, note 1.

261. Exact time is used chiefly for periods of time less than one year, by the government in making interest calculations, by bankers in reckoning bank discount, and by some business men.

262. **Bankers' time** is expressed in days, or months and days. It is reckoned by counting months for the whole months and exact days for any remaining part of a month. To illustrate: From Jan. 20 to June 15, is 4 mo. and 26 da. 4 mo. from Jan. 20 is May 20, and from May 20 to June 15 is 26 da.

This method is used by bankers in reckoning interest on bonds, mortgages, notes, etc., for fractional periods of time.

PROBLEMS

263. Find the compound time between:

1. May 15, 1912 and Aug. 12, 1915.
2. Apr. 4, 1908 and Jan. 3, 1914.
3. Dec. 26, 1910 and Nov. 13, 1914.
4. Oct. 13, 1911 and June 1, 1914.
5. Sept. 1, 1910 and July 30, 1915.
6. Sept. 28, 1912 and Feb. 14, 1915.
7. Nov. 20, 1908 and Jan. 1, 1915.
8. Dec. 31, 1913 and May 1, 1915.
9. June 30, 1911 and May 15, 1914.
10. July 4, 1911 and Dec. 25, 1915.

Find the exact time and the bankers' time between:

11. May 4 and Oct. 9.
12. Apr. 24 and Dec. 1.
13. Feb. 15 and Nov. 10.
14. June 30 and Nov. 20.
15. Jan. 15 and July 1.
16. Feb. 18 and June 12.
17. May 13 and Dec. 10.
18. June 28 and Nov. 4.
19. Apr. 16 and Sept. 5.
20. Mar. 30 and Sept. 8.

Find the compound time and the exact time between:

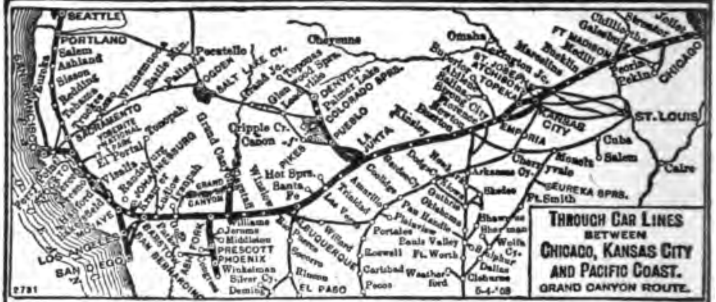
21. Apr. 1, 1913 and Jan. 30, 1915.
22. Mar. 30, 1914 and Feb. 28, 1915.
23. Dec. 26, 1909 and Jan. 1, 1914.
24. Nov. 20, 1913 and July 21, 1915.
25. Oct. 18, 1912 and July 8, 1915.
26. Sept. 17, 1915 and Aug. 12, 1916.

27. July 16, 1912 and June 1, 1915.

28. June 14, 1914 and May 5, 1916.

264. This is a condensed time-table of the Atchison, Topeka and Santa Fe Railway, extending from Chicago to San Francisco.

Santa Fe Santa Fe



Chicago. Kansas City and Pacific Coast.

(Grand Canyon Line.) WEST.

Mls.	STATIONS	3 California Limited	By Days						
0	Lv Chicago..........Santa Fe	8.00 PM	Sun.	Mon.	Tues.	Wed.	Thu.	Fri.	Sat.
458	Ar Kansas City .... "	9.00 AM	Mon.	Tues.	Wed.	Thu.	Fri.	Sat.	Sun.
......	Lv St. Louis.......Burlington	11.28 PM	Sun.	Mon.	Tues.	Wed.	Thu.	Fri.	Sat.
......	" St. Louis...........C. & A.	11.28 PM	"	"	"	"	"	"	"
......	" St. Louis.......Mo. Pacific	11.30 PM	"	"	"	"	"	"	"
......	" St. Louis .....Rock Island	10.02 PM	"	"	"	"	"	"	"
......	Lv St. Louis..........Wabash	11.31 PM	"	"	"	"	"	"	"
458	Lv Kansas City .....Santa Fe	9.10 AM	Mon.	Tues.	Wed.	Thu.	Fri.	Sat	Sun.
586	" Emporia.......... "	12.30 PM	"	"	"	"	"	"	"
660	" Newton.......... "	2.25 PM	"	"	"	"	"	"	"
827	" Dodge City...... "	5.35 PM	"	"	"	"	"	"	"
1029	" La Junta .... "	11.20 PM	"	"	"	"	"	"	"
1245	" Las Vegas .... "	7.00 AM	Tues.	Wed.	Thu.	Fri.	Sat.	Sun.	Mon.
1377	Lv Albuquerque .. "	1.00 PM	"	"	"	"	"	"	"
1663	Ar Winslow......... "	9.00 PM	"	"	"	"	"	"	"
1755	Ar Williams.......... "	12.50 AM	Wed.	Thu.	Fri.	Sat.	Sun.	Mon.	Tues.
1819	Ar Grand Canyon..G. C. Ry.	9.00 AM	* "	"	"	"	"	"	"
1819	Lv Grand Canyon.. "	8.30 PM	Tues.	Wed.	Thu.	Fri.	Sat.	Sun.	Mon.
1778	Ar Ash Fork ....Santa Fe	1.52 AM	Wed.	Thu.	Fri.	Sat.	Sun.	Mon.	Tues.
1835	Ar Prescott .... S.F.P.&P.	10.12 AM	Wed.	Thu.	Fri.	Sat.	Sun.	Mon.	Tues.
1972	Ar Phœnix .... "	4.40 PM	"	"	"	"	"	"	"
1955	Ar Needles ....Santa Fe	6.35 AM	"	"	"	"	"	"	"
2124	Ar Barstow .... "	12.30 PM	"	"	"	"	"	"	"
2205	Ar San Bernardino. "	3.40 PM	"	"	"	"	"	"	"
2215	" Riverside........ "	4.00 PM	"	"	"	"	"	"	"
2214	" Redlands .... "	4.37 PM	"	"	"	"	"	"	"
2255	" Pasadena. .... "	5.31 PM	"	"	"	"	"	"	"
2265	" Los Angeles ..... "	6.00 PM	"	"	"	"	"	"	"
2347	Ar San Diego .... "	7.00 AM	Thu.	Fri.	Sat.	Sun.	Mon.	Tues.	Wed.
2265	Lv Los Angeles ..So. Pacific	7.30 PM	Wed.	Thu.	Fri.	Sat.	Sun.	Mon.	Tues.
2369	Ar Santa Barbara "	*11.15 PM	"	"	"	"	"	"	"
2124	Lv Barstow ....Santa Fe	1.15 PM	"	"	"	"	"	"	"
2195	Lv Mojave........... "	3.10 PM	"	"	"	"	"	"	"
2265	Ar Bakersfield ...... "	7.00 PM	"	"	"	"	"	"	"
2375	" Fresno........... "	11.50 PM	"	"	"	"	"	"	"
2498	" Stockton .... "	4.10 AM	Thu.	Fri.	Sat.	Sun.	Mon.	Tues.	Wed.
2577	" Oakland .... "	7.08 AM	"	"	"	"	"	"	"
2576	Ar San Francisco . "	7.35 AM	"	"	"	"	"	"	"

For equipment, see table A, page 13. * Passengers may remain in Pullman until 7.00 AM

265. Use Central Time from Chicago to Dodge City; Mountain Time from Dodge City to Grand Canyon, and Pacific Time from Needles to San Francisco.

1. Find the average speed of the "California Limited" from Chicago to Kansas City; from Kansas City to Albuquerque; from Chicago to Grand Canyon; from Kansas City to Los Angeles; from Albuquerque to Needles; from Needles to San Francisco; from Chicago to San Francisco.

2. Find the running time from Chicago to Dodge City; from Chicago to La Junta; from Chicago to San Diego; from Chicago to Oakland; from Kansas City to Ash Fork; from Needles to Los Angeles; from Kansas City to San Francisco.

266. Counting forward or backward from a given date.

1. Count forward 76 da. from June 23.

76 da.
7 da. remaining in June.
69
31 da. in July
38
31 da. in August
7 September.
Therefore, September 7.

There are 7 da. left in June after June 23. Subtract 7 da. from 76 da., obtaining 69 da. to count forward after June 30. Subtract 31 da. for July, which leaves 38 da. to count forward after July 31. Subtract 31 da. for August, which leaves 7 da. to count into September, or September 7.

2. Count back 83 da. from May 10.

83 da.
10 da. back in May.
73
30 da. in April
43
31 da. in March
12 da. back into Feb.
Feb. 28 – 12 da. =
Feb 16.

In counting backward from May 10, note that 10 da. are subtracted for May, leaving 73 da. to count back from Apr. 30. Subtract 30 da. for April, and 31 da. for March. The 12 da. remaining are to be deducted from the 28 da. in February, which leaves February 16.

Note. Fractional parts of a day are not recognized in law. Hence, as soon as it is past midnight on May 9, it is May 10, and the whole day is considered, in law, to be gone. Therefore, in counting back from the 10th, there are 10 days to count back through to count back out of May.

PROBLEMS

267. Count both forward and backward:

1. 95 da. from July 10.
2. 110 da. from June 30.
3. 57 da. from Apr. 15.
4. 132 da. from June 1.
5. 160 da. from June 30.
6. 173 da. from Oct. 15, 1914.
7. 208 da. from Sept. 1, 1913.
8. 142 da. from Dec. 30, 1914.
9. 111 da. from Jan. 10, 1914.
10. 131 da. from Feb. 1, 1915.
11. 168 da. from Mar. 31, 1914.
12. 174 da. from Feb. 28, 1915.
13. 60 da. from July 1, 1914.
14. 90 da. from May 1, 1915.
15. 89 da. from May 31, 1913.
16. 101 da. from Nov. 1, 1913.
17. 201 da. from July 31, 1914.
18. 179 da. from Aug. 19, 1915.
19. 120 da. from Apr. 30, 1914.
20. 229 da. from Dec. 1, 1914.

PAINTING, PLASTERING, PAPERING, CARPETING

268. Painting and plastering are estimated by the square yard.

There is no definite rule regarding allowances for openings. It is a matter for special agreement, and should be mentioned in the contract when made.

269. Papering is estimated by the roll.

270. Most American wall papers are 18 in. wide, a single roll being 24 ft. long, and a double roll, 48 ft. long. Imported papers vary in width and length.

271. As there is always more or less waste in cutting and matching wall paper, it is better to use double rolls. The exact number of rolls cannot always be determined in advance.

272. In estimating the number of rolls for a room, paper hangers generally deduct the total width of openings from the perimeter of the room. Then they reckon the number of strips required to cover the remaining surface. The number of rolls is found by dividing the total number of strips required by the number of whole strips that can be cut from 1 roll.

The spaces over and under the windows, over the doors, etc., are covered with the parts of strips left from each roll.

273. Carpet is estimated and sold by the linear yard. Linoleum is generally sold by the square yard.

274. Carpets vary in width, the more common widths being 1 yd. for Ingrain, and $\frac{3}{4}$ yd. for Axminster, Brussels, Moquette, Velvet, and Wilton.

275. Carpet may be bought in any *length* desired, but only in *whole* strips; hence, in order to determine the number of yards for a floor, the number of strips must be known.* The number of yards is then found by multiplying the yards in 1 strip by the number of strips.

276. A room is 16 ft. long, 14 ft. wide, and $8\frac{1}{2}$ ft. high. It has 3 windows each 3 ft. by 6 ft. and 1 door 3 ft. by 7 ft. Find the total cost of plastering the walls and ceiling at 50¢ per square yard, allowing for the openings; of carpeting the floor with Brussels carpet at $1.75 per yard; and of papering the walls at $1.60 per double roll.

PLASTERING

$2 \times (16 \text{ ft.} + 14 \text{ ft.}) = 60$ ft., perimeter of room.

$8\frac{1}{2} \times 60$ sq. ft. $= 510$ sq. ft. in the walls.

16×14 sq. ft. $= 224$ sq. ft. in the ceiling.

510 sq. ft. $+ 224$ sq. ft. $= 734$ sq. ft. in ceiling and walls.

$3 \times 3 \times 6$ sq. ft. $= 54$ sq. ft. in the windows.

3×7 sq. ft. $= 21$ sq. ft. in the door.

54 sq. ft. $+ 21$ sq. ft. $= 75$ sq. ft. in all openings.

734 sq. ft. $- 75$ sq. ft. $= 659$ sq. ft., net area of ceilings and walls.

$$\frac{659 \times \$.50}{9} = \$36.61, \text{ cost of plastering.}$$

CARPETING

$14 \text{ ft.} \div 2\frac{1}{4} \text{ ft.} = 6\frac{2}{9} = 7$, no. strips.

$$\frac{7 \times 16}{3} = 37\frac{1}{3}, \text{ no. yd. of carpet.}$$

$37\frac{1}{3} \times \$1.75 = \$65.33$, cost of carpeting.

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*If any whole number of strips exactly covered a floor, and there were no waste in matching the pattern, the number of yards for a floor could be found by dividing the area of the floor by the area of a yard of carpet, but these conditions occur so rarely that the statement above may be taken as correct.

PAPERING

2 × (16 ft. + 14 ft.) = 60 ft., perimeter of room.
4 × 3 ft. = 12 ft., total width of openings.
60 ft. − 12 ft. = 48 ft., net perimeter.
48 ft. ÷ $1\frac{1}{2}$ ft. = 32 strips required.
48 ft. ÷ $8\frac{1}{2}$ ft. = 5 whole strips from 1 roll.
32 strips ÷ 5 strips = $6\frac{2}{5}$ = 7, no. rolls.
7 × $1.60 = $11.20, cost of papering.
$36.61 + $65.33 + $11.20 = $113.14, total cost.

Plastering. The area of the walls is the perimeter of the room multiplied by the height. The ceiling is a rectangle whose area is 224 sq. ft., making the total area of the walls and ceiling equal to the sum of 510 sq. ft. and 224 sq. ft., or 734 sq. ft. The windows and the door are rectangles whose total area is 75 sq. ft. Since allowance is made for the openings, the net area to be plastered is found by deducting the 75 sq. ft. from 734 sq. ft., leaving 659 sq. ft. Dividing by 9 sq. ft. to the square yard and multiplying $.50 by the quotient, gives the cost of plastering as $36.61.

Carpeting. Unless otherwise specified, carpet is generally considered to be laid with strips running lengthwise of the room. Hence, to find the number of strips, divide the width of the floor by the width of the carpet. 14 ft. ÷ $2\frac{1}{4}$ = $6\frac{2}{9}$, the number of strips, but as carpet has to be bought in whole strips, 7 strips are necessary. Each strip is 16 ft. long. The number of yards, therefore, is $\frac{1}{3}$ of 7 times 16, or $37\frac{1}{3}$. At $1.75 per yard, the cost is $65.33.

Papering. From the perimeter of the room deduct the total width of the windows and the door, leaving 48 ft. as the net perimeter to be papered. There are as many strips of paper required as $1\frac{1}{2}$ ft., the width of one strip, is contained times in 48 ft., or 32. The number of strips from one roll is found by dividing the length of a roll, 48 ft., by the length of one strip, $8\frac{1}{2}$ ft., which gives 5 strips from a roll. Dividing 32 strips by 5 strips gives $6\frac{2}{5}$, or 7 rolls, which, at $1.60 a roll costs $11.20. The total cost is found by adding the costs of the several kinds of work.

277. In writing the dimensions of rooms it is customary to write the length first, then the width, then the height. It is also usual to write 10 ft. thus, 10′, and 8 in., 8″. Hence a room 15 ft. long, 13 ft. wide, and 8 ft. 4 in. high, may be written "a room 15′ × 13′ × 8′ 4″."

PROBLEMS

278. Five rooms have dimensions and openings as follows:

1. 18′ × 16′ × 9′; 2 windows 3′ × 6′ 6″; 2 doors 3′ 6″ × 7′.
2. 16′ × 12′ × 8′ 6″; 3 windows 4′ × 5′ 6″; 1 door 3′ × 6′ 6″.
3. 26′ × 16′ × 10′ 6″; 4 windows 4′ × 7′ 6″; 3 doors 4′ × 8′.
4. 22′ 6″ × 18′ × 9′ 6″; 3 windows 4′ × 7′; 2 doors 3′ 6″ × 8′.
5. 19′ 6″ × 15′ × 9′; 3 windows 5′ × 6′; 2 doors 3′ × 7′.

For the first room find the cost of plastering walls and ceiling at 60¢ per square yard, allowing for one half the openings, of papering the walls at $1.20 per double roll, and of carpeting with Wilton carpet at $1.40 per yard.

For the second room find the cost of plastering the walls and ceiling (no allowance for openings) at 70¢ per square yard, of papering the walls at 90¢ a double roll, and of painting and varnishing the floor at 75¢ per square yard.

For the third room find the cost of kalsomining the ceiling at 80¢ per square yard, of papering the walls at $2.50 per double roll, and of carpeting the floor with velvet carpet at $2.25 per yard.

For the fourth room find the cost of plastering walls and ceiling at 65¢ per square yard (no allowance), of papering walls and ceiling at $1.75 per double roll, and of carpeting the floor with Moquette carpet at $1.80 per yard.

For the fifth room find the cost of plastering the walls and ceiling at 55¢ a square yard (100 sq. ft. for openings), of papering the walls at $1.30 per double roll, and of carpeting the floor with Ingrain carpet at $1.10 per yard.

The diagram on the following page is the first-floor plan of a modern farmhouse.

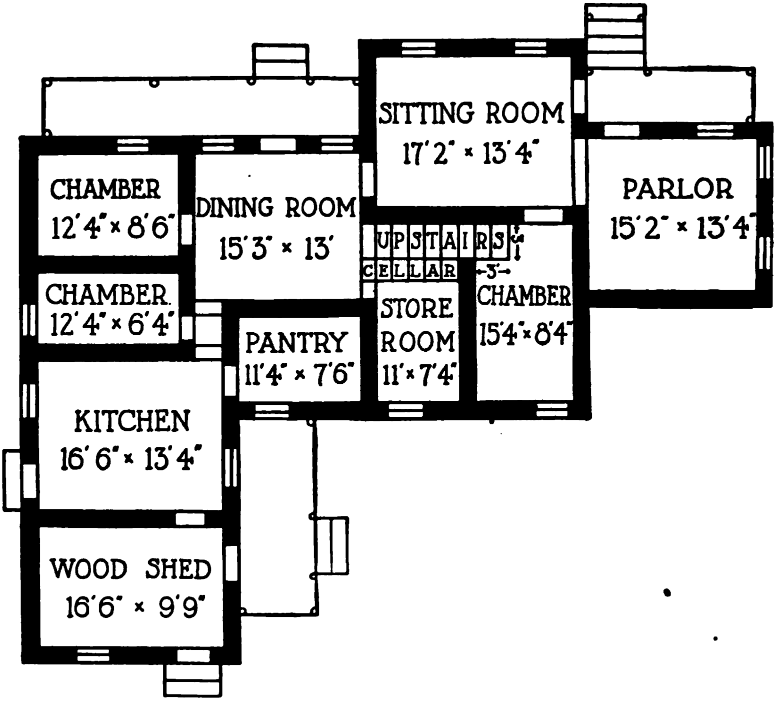
The rooms have a uniform height of 8′ 6″. The dining room and kitchen each have a wainscot 3′ high. All others have a base board 9″ wide. All windows are 5′ 9″ × 3′. Outside doors are 7′ × 3′ 4″; inside doors are 7′ × 3′ 2″. The double doors between the sitting room and the parlor are 7′ × 5′ 8″. (Measurements of windows and doors include casings.)

6. Find the number of square yards of plaster required for the walls of the sitting room and parlor. Make allowance for openings and baseboard.

7. How many square yards of plaster are there in the walls and ceilings of the three chambers? Make all allowances.

8. How many square yards of linoleum will cover the floors of the kitchen and the pantry?

9. Find the cost of ingrain carpet at $.90 a yard for the dining room (carpet to be laid most economical way).



10. How much will it cost to paper the dining room at 60¢ a double roll?

11. If the kitchen walls are painted above the wainscot, find the the cost at 10¢ per square yard.

12. Find the cost of papering the walls and the ceiling of the parlor at $1.25 per double roll.

13. How many double rolls of paper are required for the three chambers, allowing for one foot of waste for each strip in matching the pattern?

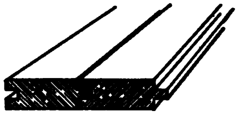
14. How many yards of Brussels carpet will cover the floor of the sitting room?

15. At $1.35 per yard, find the cost of Wilton carpet for the parlor.

FLOORING

279. Flooring is estimated by the square or by the thousand board feet.

280. When lumber is "tongued" and "grooved," or matched, as it is called, there is some waste, as lumber dealers always measure the lumber at its full width before it is matched. The amount of waste depends upon the width of the boards. Carpenters generally allow one fifth for waste. That is, for 1000 sq. ft. of floor space, 1200 sq. ft. (board feet) of flooring is needed.



Matched Lumber

281. How many square feet of flooring are required for the parlor represented on page 124?

$15' \, 2'' = 15\frac{1}{6}'$; $13' \, 4'' = 13\frac{1}{3}'$;

$$\frac{91}{\cancel{6}} \times \frac{\overset{8}{\cancel{40}}}{3} \times \frac{\cancel{6}}{\cancel{5}} = \frac{728}{3} = 242\frac{2}{3},$$

Ans. 243 sq. ft.

Reduce feet and inches to feet and multiply the length of the room by the width. The allowance for waste is best reckoned by multiplying by $\frac{6}{5}$. Using the cancellation form of solution, the result is readily obtained.

PROBLEMS

282. 1. A floor is $18' \, 6'' \times 14' \, 4''$. How many square feet of lumber are required to build it?

2. Find the total amount of lumber required for all the floors in the diagram on page 190, except the woodshed. How much will it cost at $55 per 1000?

3. Find the cost of a mosaic floor $60' \times 32'$ if the tile cost $1.25 per square foot, and the cost of laying is $8 per square.

ROOFING

283. Roofing is generally estimated by the square of 100 square feet.

284. The most common roofing materials are shingles, slate, and tin.

285. Shingles are estimated as having an average width of 4 in., and are laid 4 in., 4½ in., 5 in., or 5½ in. to the weather, depending on the pitch of the roof. The steeper the pitch the greater the length exposed to the weather. The usual estimate per square is shown in the following table:

TABLE

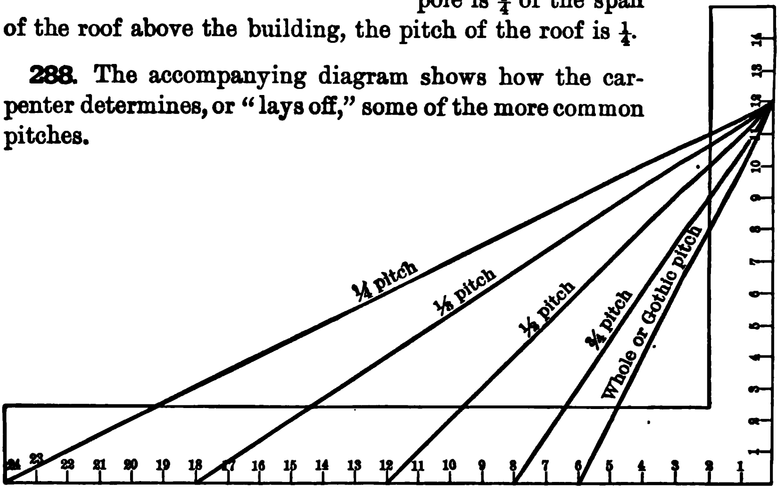
LENGTH OF SHINGLE EXPOSED TO THE WEATHER	NUMBER OF SHINGLES PER SQUARE
4 inches	1000
4½ inches	900
5 inches	800
5½ inches	700

286. A bunch contains 250 shingles. A part of a bunch is not sold. Estimates, therefore, have to be made in whole bunches.

287. The pitch of a roof is numerically expressed by dividing the height by the span.

When the height of the ridge-pole is ¼ of the span of the roof above the building, the pitch of the roof is ¼.

288. The accompanying diagram shows how the carpenter determines, or "lays off," some of the more common pitches.



PROBLEMS

289. 1. Using a span of 36 ft., make a diagram illustrating one third pitch; one fourth pitch; five twelfths pitch; five eighths pitch; equilateral or true pitch (60°).

2. If the foot of the rafters projects 14″ over the side of the building, find the length of the rafters for each of the pitches in 1.

The rafter is the hypotenuse of a right triangle.

3. A roof is 60 ft. long and 30 ft. wide on each side. How many shingles laid 5 in. to the weather must be bought to cover it?

$$\frac{2 \times 60 \times 30}{100} = 36 \text{ squares.}$$

$$36 \times 800 \text{ shingles} = 28{,}800 \text{ shingles} = 29{,}000 \text{ shingles.}$$

Each side of the roof is a rectangle 60 ft. by 30 ft. The area of both sides divided by 100 sq. ft. in a square, gives 36 squares. It takes 800 shingles for 1 square, and for 36 squares it takes 36 times 800 shingles or 28,800 shingles, making it necessary to buy 29,000 shingles.

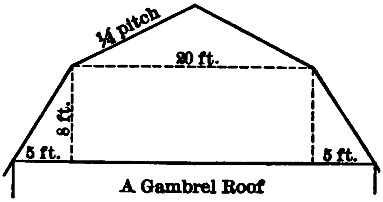
4. A building is 36 ft. wide. The pitch of the roof is one half and the rafters project 15 in. How many shingles laid $4\frac{1}{2}$ in. to the weather are required if the ridgepole is 50 ft. long?

5. How many slates 6 in. wide, 5 in. to the weather, are required for a building 24 ft. wide, the pitch of the roof being five eighths, the ridgepole 36 ft. long, and the rafters projecting 15 in.?

6. A roof 44 ft. long and 18 ft. wide on each side is covered with tin. If each sheet is 18″ × 27″, how many sheets are there?

7. How many slates are needed to cover a roof 32′ × 16′ on a side if the slates are 6″ wide and are exposed 8″ to the weather?

8. The diagram shows part of the end of a barn having a gambrel roof. If the projection of the rafters is 18″ and the ridgepole is 59 ft. long, how many shingles are required for the roof if they are laid $5\frac{1}{2}$ in. to the weather on the lower half of the roof and 4 in. to the weather on the upper half?



A Gambrel Roof

PAVING

290. Paving is estimated by the square foot or by the square yard.

PROBLEMS

291. 1. Find the cost of paving a street 8 rd. long and 30 ft. wide, at $1.75 per square foot.

2. How many paving blocks 4″ × 8″ are required for a section of street 80 rd. long and 36 ft. wide? Find the cost of paving at $14 per square yard.

3. If paving stones are 5″ × 9″, how many will it take to pave a street 1 mi. long and 40 ft. wide?

4. A man who owns a corner lot with a frontage of 75 ft. and a depth of 100 ft., builds a cement walk 6 ft. wide on the side and front. If he puts the walk on his lot, find the cost at $3.75 per square yard.

PAPER AND BOOKS

292. Paper is of various kinds, and has many uses, some of which are:

1. Wrapping — made of straw, manila hemp, or wood pulp. It is used for wrapping bundles, making paper bags, etc. It is one of the cheapest kinds of paper.
2. News — made of wood pulp. It is of a higher grade than wrapping paper, and is used in making newspapers, and cheap books and magazines. It is sold chiefly in large rolls.
3. Book — made of wood pulp and cotton cloth (called "paper rags"). It is better than news and is used in making the better class of books and magazines. It is sold chiefly in large unfolded sheets; also in large rolls like news.
4. Flat — made of wood pulp and cotton or linen cloth, mixed with an animal size, or glue. The sizing produces a firm smooth surface and prevents the ink from spreading in the paper when written upon with a pen. It is used for correspondence, and for records and documents in which permanency is desired. It is sold wholesale in large unfolded sheets. For retail purposes it is cut into various-sized sheets dependent upon the use to which it is to be put.

293. Paper is sold in large quantities by weight; in small quantities, by the quire or ream. In some cases a ream consists of 500 sheets instead of 480.

294. COMMON SIZES OF PAPER

BOOK	FLAT	
25″ × 38″	Cap	14″ × 17″
28″ × 42″	Demy	16″ × 21″
30½″ × 41″	Folio	17″ × 22″
32″ × 44″	Medium	18″ × 23″
35″ × 46″	Super Royal	20″ × 28″

The 25″ × 38″ size is the basis from which the weights of other sizes of book paper are reckoned. A ream 25″ × 38″ weighs 40 lb., 50 lb., 60 lb., etc., according to the quality of the paper.

295. Formerly the terms folio, quarto, 8vo, 12mo, etc., indicated the number of times a sheet was folded in making a book. For instance, folio was the name given to a book made of sheets which had been folded but once, thus making two leaves or four pages from each sheet; a quarto indicated a book made of sheets folded twice, making four leaves or eight pages; etc.

At the present time the terms folio, quarto, octavo, or 8vo, have reference to the size of the page, regardless of the number of times a sheet is folded.

296. The following sizes are the commercial standard board (cover) measurements of various kinds of books:

Quarto	$12\frac{5}{8}'' \times 8\frac{1}{4}''$	16mo	$6\frac{7}{8}'' \times 4\frac{1}{4}''$
8vo	$9\frac{3}{4}'' \times 6''$	18mo	$6\frac{1}{8}'' \times 3\frac{7}{8}''$
12mo	$7\frac{5}{8}'' \times 4\frac{3}{4}''$	32mo	$4\frac{5}{8}'' \times 3\frac{1}{8}''$

In practice any of the above sizes may run large or small; hence it is impossible to tell the exact size of a book by the terms folio, quarto, octavo, etc.

297. Most books, except very large ones, are printed in 16's or 32's; that is, each sheet has 16 or 32 pages on each side. The sheets are then folded and cut into two or four sections called "signatures." Each signature contains 16 pages. Hence a sheet is folded to make either 32 or 64 pages.

298. 1. If a sheet is folded into 4 signatures, each page $7\frac{1}{2}'' \times 4\frac{7}{8}''$, which of the above sizes of paper is the most economical to use?

4 signatures = 64 pages = 32 pages on one side of a sheet.
32 pages = 8 pages by 4 pages.
$4 \times 7\frac{1}{2}'' = 30''$, width of sheet.
$8 \times 4\frac{7}{8}'' = 39''$, length of sheet.
Hence the size $30\frac{1}{2}'' \times 41''$ is best.

2. A book containing 448 pages is printed in 32's. If the pages are $7\frac{3}{4}'' \times 5\frac{1}{4}''$, what size paper would be used? How many reams (500 sheets) would be required to print 5000 copies of the book?

32's means 32 pages on one side of a sheet.
32 pages = 8 pages by 4 pages.
$4 \times 7\frac{3}{4}'' = 31''$, width of sheet.
$8 \times 5\frac{1}{4}'' = 42''$, length of sheet.
Hence, use sheets $32'' \times 44''$.
448 pages ÷ 64 pages (on 1 sheet) = 7, no. sheets in 1 book.
$\frac{5000 \times 7}{500} = 70$, no. reams.

3. If 70 sheets of strawboard 26″ × 38″ weigh 50 lb., what will be the cost of 1000 sheets of the same grade of strawboard $8\frac{1}{2}$″ × 17″ at $28 per ton?

First cause : Second cause : : First effect : Second effect.
70 sheets : 1000 sheets
26 in. : $8\frac{1}{2}$ in. : : 50 lb. : ? lb.
38 in. : 17 in.

$$\frac{1000 \times 8\frac{1}{2} \times 17 \times 50}{70 \times 26 \times 38}\text{ lb.} = 104\tfrac{1}{2}\text{ lb.}$$

$28 ÷ 2000 = $.014 per pound.

$104\frac{1}{2} \times \$.014 = \$1.46$, cost.

By the "cause and effect" method of stating the proportion, it is readily seen that the first cause is the 70 sheets 26″ × 38″, and that the corresponding effect is 50 lb. The second cause is the 1000 sheets $8\frac{1}{2}$″ × 17″, leaving the second effect to be found. Dividing the product of the means by the given extreme gives the other extreme, or $104\frac{1}{2}$ lb., the weight of 1000 sheets $8\frac{1}{2}$″ × 17″. Multiplying by the price per pound gives $1.46, the required cost.

With a little practice the student should be able to write the above statement directly in the form for solution, thus:

$$\frac{50 \times 8\frac{1}{2} \times 17 \times 1000 \times \$28}{70 \times 26 \times 38 \times 2000} = \$1.46$$

reasoning in this way: If 70 sheets 26″ × 38″ weigh 50 lb., one sheet will weigh $\frac{1}{70}$ of 50 lb., and 1 sq. in. of 1 sheet will weigh as much as 26 × 38 is contained times in the weight of one sheet. Thus far the statement would be $\frac{50}{70 \times 26 \times 38}$ = weight of 1 sq. in. If 1 sq. in. of 1 sheet weighs that much, one sheet $8\frac{1}{2}$″ × 17″ will weigh $8\frac{1}{2} \times 17$ times that, and 1000 sheets will weigh 1000 times that result. To this point the statement is $\frac{50 \times 8\frac{1}{2} \times 17 \times 1000}{70 \times 26 \times 38}$. Having the weight, divide by 2000 lb. to a ton, and multiply by the price per ton. The complete statement is as given above.

PROBLEMS

299. **1.** If two signatures are printed from one sheet, what size paper is best for a book whose pages are $11\frac{1}{4}$″ × $8\frac{1}{2}$″?

2. A 24-signature book is printed in 32's. If the pages are $6\frac{3}{4}$″ × 5″, what size of sheet would be best to use? How many volumes could be printed from 100 reams (480 sheets) of paper?

3. If 1 ream (500 sheets) of book paper 26″ × 38″ weighs 50 lb., find the cost of 70 reams of the same grade 28″ × 42″ at $85 per ton.

4. A ton of flat paper 17″ × 22″ costs $.15 a pound and is made into 10¢ notebooks 5½″ × 8½″, of 64 pages each. If 12 sheets of paper weigh 1 lb., and the cost of manufacturing the books including the covers is $1 per hundred, find the profit.

5. If a ream of Demy weighs 50 lb., and a firm is charged for 1075 lb. at 22¢ per pound for 80 reams 8″ × 10½″, how much too great is the charge? (A discrepancy of 1 lb. in 20 lb., or 5%, is not considered an overcharge in the paper trade.)

The regulation size of letter paper is 8″ × 10″; of note paper is 5″ × 8″. (Commercial note is 5½″ × 8½″.)

6. How many reams of paper, letter size, can be cut from 2000 sheets 16″ × 21″?

7. If 24 reams 17″ × 22″ (500 sheets to a ream) are cut into commercial note size and sold at $1.10 per ream (480 sheets to a ream), what is the amount received?

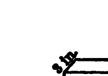
8. How many pads, 80 sheets each, 5¾″ × 9″, can be made from 4000 sheets 18″ × 23″?

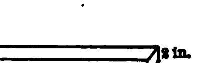
9. How many reams (500 sheets to a ream) Super Royal are required for 500 ledgers of 320 pages each, if the page is 10″ × 14″?

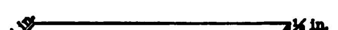
10. A publisher has an order for 10,000 copies of a book containing 512 pages, each page 8½″ × 5½″. He prints in 32's, and the paper costs $100 per ton. If the same grade of paper 25″ × 38″ weighs 50 lb. to the ream, find the cost of the paper for the order.

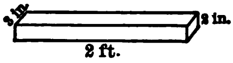
LUMBER

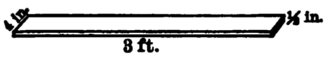
300. The unit of lumber measure is the **board foot.** A board foot is a square foot of board one inch (or less) thick. Each of these illustrations represents a board foot of lumber.











The volume of a board foot of lumber is thus seen to be 144 cu. in., except when the lumber is less than 1 in. thick, in which case a board foot is equal to a square foot of surface of the board.

301. How many board feet are there in 48 scantlings 2″ × 4″ × 16′ (2 in. × 4 in. × 16 ft. long)?

$$\frac{\overset{4}{\cancel{48}} \times 16 \times 2 \times 4}{\cancel{12}} = 512, \text{ no. of board feet in 48 scantlings.}$$

When there is more than one piece of lumber, the simplest method is to find the total length of all the pieces, and then multiply by the width and by the thickness, in inches, and divide the product by 12, using cancellation, as shown in the solution.

LUMBER YARD PRACTICE

302. To find the number of board feet in one piece of lumber

4 inches wide, take ⅓ of the length.
6 inches wide, take ½ of the length.
8 inches wide, take ⅓ less than the length.
9 inches wide, take ¼ less than the length.
10 inches wide, take ⅙ less than the length.
12 inches wide, take the length.
14 inches wide, add ⅙ to the length.
15 inches wide, add ¼ to the length.
2 in. by 4 in. (2 × 4 = 8) take ⅓ from the length.
2 in. by 8 in. (2 × 8 = 16) add ⅓ to the length.
8 in. by 8 in. (8 × 8 = 64) take 5⅓ times the length, etc.

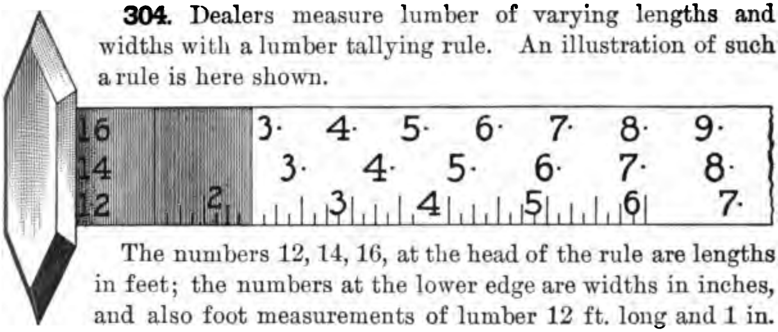
Note. When no thickness is mentioned, lumber is understood to be one inch thick.

PROBLEMS

303. Find, mentally when possible, the number of board feet in the following:

1. 20 pcs. 3″ × 8″ × 14′
2. 44 pcs. 4″ × 4″ × 18′
3. 60 pcs. 12″ × 1″ × 16′
4. 90 pcs. 2″ × 4″ × 16′
5. 160 pcs. 6″ × 8″ × 12′
6. 16 pcs. 10″ × 12″ × 18′
7. 120 pcs. 9″ × 11″ × 16′
8. 75 pcs. 4″ × 6″ × 14′
9. 49 pcs. 8″ × 8″ × 22′
10. 200 pcs. 4″ × 8″ × 10′
11. 14 pcs. 3″ × 5″ × 14′
12. 18 pcs. 4″ × 9″ × 16′
13. 28 pcs. 6″ × 12″ × 18′
14. 38 pcs. 12″ × 12″ × 16′
15. 560 pcs. 10″ × 1″ × 16′
16. 960 pcs. 8″ × 1″ × 14′
17. 760 pcs. 6″ × 1″ × 18′
18. 1000 pcs. 5″ × 1″ × 16′

304. Dealers measure lumber of varying lengths and widths with a lumber tallying rule. An illustration of such a rule is here shown.



The numbers 12, 14, 16, at the head of the rule are lengths in feet; the numbers at the lower edge are widths in inches, and also foot measurements of lumber 12 ft. long and 1 in. thick. The other lines of numbers opposite 14 and 16 are the number of board feet in 14 ft. and 16 ft. lumber, respectively, for varying widths. For instance, a board 16 ft. long and 1 in. thick contains 8 board ft. if it is 6 in. wide, as shown by the 8 above the 6 in the lower line. As lumber is measured, the number of board feet is called off and recorded on a tally sheet. The numbers at the top of the tally sheet are foot measurements. The tallies in the columns are the number of pieces containing the number of board feet indicated at the top of the column. The total number of board feet in any column is the product of the number at the top by the number of tallies in the column.

TALLY SHEET

10	12	14	16	18	20	22	24
卌 卌 卌 卌 卌 卌 111	卌 卌 卌 卌 卌 111	卌 卌 卌 卌 卌 卌 卌 卌 1	卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 11	卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌	卌 卌 卌 卌 卌 卌 卌 卌 卌 111	卌 卌 卌 卌 卌 11	卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 11

Problems of this sort are an excellent training in alertness and accuracy. This is essentially a class exercise, and it is suggested that teachers provide themselves with original problems for class use.

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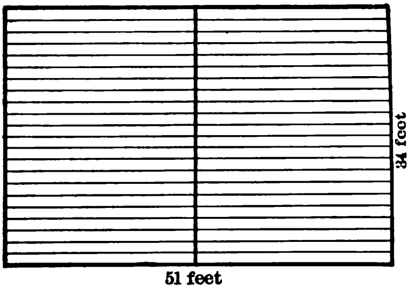
* The tallies are grouped in fives for convenience in counting. The oblique line through the group of four is the fifth tally in the group.

305. This diagram shows the framework of the ground floor plan of a house 34′ × 51′. The sills are 6″ × 8″. The side sills are spliced 5 ft. in the middle. The joists are 3″ × 8″, and are 18″ apart from center to center. Find the cost of the lumber at $18 per M.



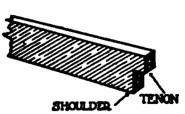
3 sills 6″ × 8″ × 34′ = 408 ft
4 sills 6″ × 8″ × 28′ = 448 ft.
42 joists 3″ × 8″ × 26′ = 2184 ft.
Total, 3040 ft.

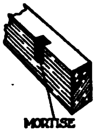
3.040 × $18 = $54.72 cost.

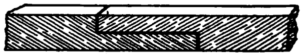
($18 per M = $.02 − $.002 per foot.)

The three cross sills are each 34 ft. long, making 408 board feet. The side dimensions are so long as to make splicing necessary. The length of each piece of timber is found by adding the length of the splice to the length of the sill and dividing into two equal parts. (5 + 51) ÷ 2 = 28, no. ft. The four pieces contain 448 board feet.

Since the sills are 6″ thick, the width inside the sills is 33′. To find the number of joists divide 33′ by 18″, and subtract 1 from the quotient. 33′ ÷ 18″ = 22; 22 − 1 = 21. (The number of spaces is 22, but the number of joists is one less. Prove it by counting spaces and joists in the diagram.) As one joist reaches only to the middle sill, there are 2 times 21, or 42, joists in all. They are mortised into the sills 2″ at each end. The total width of the three cross sills is 18″, which, deducted from 51′, leaves 49′ 6″, net length of house exclusive of the sills. One half of 49′ 6″ equals 24′ 9″, length of joist exclusive of the tenon, — 2″ at each end, — which makes total length of a joist 25′ 1″ (24′ 9″ + 2″ + 2″). But as lumber is sawed in even foot lengths, a joist 26′ long must be bought. 42 joists 3″ × 8″ × 26′ = 2184 board feet.







HOW SILLS ARE SPLICED

The total amount of lumber is 3040 ft., which at $18 per M is worth $54.72.

Note. Lumber is practically never cut in an odd number of feet lengths, as 13′, 15′, 19′, 25′, but in even numbers, as 10′, 12′, 16′, 22′, etc. Hence, when the actual length required in building is an odd or fractional number, the nearest length in *even* number of feet greater is purchased.

PROBLEMS

306. 1. Estimate the cost of the lumber, at $22 per M, for a similar plan 25′ 4″ × 40′. The sills are 8″ × 8″, those on the sides being spliced 4 ft. The joists are 3″ × 8″, and are 16″ from center to center.

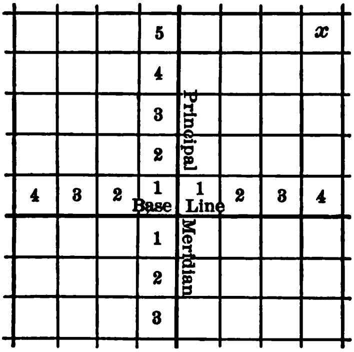
2. A house is 25 ft. by 30 ft. The sills are 6″ × 6″. A sill extends across the middle of the plan as in the preceding diagram. The joists are 3″ × 6″ and are 16″ from center to center. How many feet of lumber are required for the sills and joists?

3. Make a diagram of an original plan similar to those of the preceding problems, and estimate the amount of lumber necessary to construct.

LAND

307. Land areas are estimated by the acre of 160 sq. rd.

308. In laying out public lands, surveyors select a north and south line as a **principal meridian**, and an east and west line as a **base line.**



Townships, 6 miles square

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

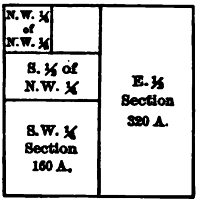
A Township divided into Sections

Other lines 6 mi. apart are run parallel to the principal meridian and to the base line, thus dividing the land into **townships**, as shown in the diagram.

Rows of townships north and south are called **ranges**, and are numbered both east and west from the principal meridian. The townships are also numbered north and south of the base line. Thus, Township *x* is 5 north, in the 4th range east of the principal meridian.

Each township is again divided into squares by parallel lines 1 mi. apart. Each square is called a **section**, and contains 640 A.

The sections in a township are always numbered as shown in the diagram. If the diagram of a township on p. 204 represents Township *x* of the preceding diagram, section 17 would be designated as section 17 of township 5 north, in the 4th range east of the principal meridian.



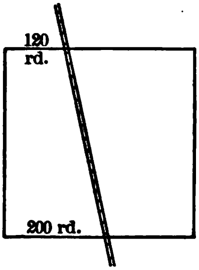
A Section

Sections are divided into halves, quarters, half-quarters, quarter-quarters, etc.

PROBLEMS

309. 1. A man owned the southeast quarter of a section of land. He bought the south half of the northeast quarter, and the northeast quarter of the northeast quarter of the same section. Make a diagram of his original farm, adding thereto the areas bought, showing number of acres in each parcel.

2. A man owning the north half of a section bought the east half of the southeast quarter, and the west half of the southwest quarter of the same section. How many acres of land has he? Draw a diagram of his farm.



3. A railroad extends through a section of land as shown in the diagram. It occupies a strip 4 rd. wide. Find the land damage for the right of way at $40 an acre.

4. Find the cost at $.65 a rod of building a fence around a section of land; a quarter section of land.

5. Your deed shows that you own the southerly half of section 8 in Clark Co., S. Dak. Draw a township plan, and locate your land. Locate a house in the southeastern corner of your land. Locate a haystack in the northwestern corner of your land. Find distance in a direct line from the house to the haystack.

6. How many rods of fence are required to inclose each of the three parcels in problem 1?

7. How many rods of fence will inclose the combined tracts mentioned in problem 2?

310. An **estimate** is an approximate result obtained by making a rough calculation of a given problem. Estimates are often made by inspection, and in advance of careful, accurate solution.

1. Estimate the quantity of lumber required to build a porch 8 ft. wide along the front of a house 25 ft. wide. The flooring is of matched lumber and is laid on joists 2″ × 6″. The joists are 17 in number and are supported by sills 8″ × 8″. The porch is sealed with matched lumber. The ceiling is nailed to 11 joists each 2″ × 4″. There are 13 rafters 2″ × 4″. The roof boards are 6″ wide.

2. Estimate the quantity of lumber required to build shelves on one side of a room 40 ft. long, if there are 7 shelves one above the other 14″ apart, and 12″ deep. The shelves are supported by board partitions 30″ apart.

3. Reckon the cost of painting (2 coats) a house 28′ × 36′, the eaves being 18 ft. from the top of the cellar wall. Use local prices of paint and labor. One gallon of paint will cover 400 sq. ft. first coat, and 500 sq. ft. second coat. One man will spread 3 gal. of paint first coat and 2 gal. second coat. Add 15 % for contractor's profit.

4. Make a list of articles necessary for furnishing a 5-room apartment. Estimate the cost for each room, the total cost not to exceed $1000.

5. Assume that your house has been destroyed by fire. Make an insurance inventory of the articles burned with the values thereof, as a basis of settlement with the insurance company.

6. A quarter section of land in the form of a square is to be divided into 4 fields of 40 acres each, in such manner as to require the least amount of fencing. Make a diagram of the quarter section showing the position of the fences. Estimate the quantity of lumber required for a board fence inclosing the four fields with 6 in. boards, if the fence is 4 boards high. If the posts are 8 ft. apart, how many posts are needed?

7. A man owns a city lot 100 ft. deep and 75 ft. front. He desires to build a picket fence on the two sides and across the back end of the lot. Consult local builders and lumbermen for prices and materials, and estimate the cost of the fence.

8. Compare the cost of the picket fence in problem 7 with the cost of a moderately priced iron fence. Consult a hardware dealer for prices and lasting qualities, and determine which is the more economical, the picket or the iron fence.

9. Investigate and find the cost of a thorn hedge across the front of the lot mentioned in problem 7. Make allowance for gateways for a walk to the house and for a driveway leading to the garage.

10. Consult local contractors and determine the relative costs of covering a roof 36 ft. long having 20 ft. rafters, with #1 cedar shingles and with slate. Which is the more economical, considering durability?

11. A man desires to inclose his front porch with portable wire screens for summer use, and with portable glass sash for use as a sun parlor in winter. If the porch is 28 ft. long and 12 ft. wide, estimate the cost of the screens and sash.

12. It is estimated that there are 825,000 automobiles in the United States. Assuming that each automobile requires, on the average, six new tires a year, and that the average price of tires is $35, what is the approximate value of the tire industry in the United States per annum?

FARM PROBLEMS

311. 1. A field of potatoes is one quarter of a mile long. If the rows are 3 ft. 8 in. apart, how many rows make an acre?

2. If the field in problem 1 yields 10 bu. to the row, how many bushels will there be from 10 acres?

3. If potatoes shrink 1 % a month for five months, what will be the loss in weight on the above crop of potatoes from October to March?

4. If the price of potatoes in October was 70¢ and in March it was 80¢, did it pay to hold the crop till March before marketing? How much was gained or lost by holding the crop?

5. During the month of June one cow gives an average of 3 gal. of milk a day which tests 3.6 % of butter fat; another cow gives an average of 2½ gal. of milk testing 4 % butter fat. Milk weighs 8⅝ lb. per gallon. If butter is worth 32 ¢ a pound, find the value of the butter produced by each of the cows.

6. Complete the following ticket:

TICKET SHOWING WEIGHTS OF HAY

GROSS WEIGHT	WEIGHT OF WAGON	NET WEIGHT	VALUE AT $16.50 A TON
3674	1467	—	—
3860	1490	—	—
4135	1505	—	—
Totals			

7. Prepare a ticket like the above for 6 loads of coal weighing as follows: Gross weights, 4530, 4654, 4850, 5260, 4960, 5120; Weights of wagons: 1650, 1642, 1528, 1470, 1465, 1640. Coal is worth $6.10 per ton.

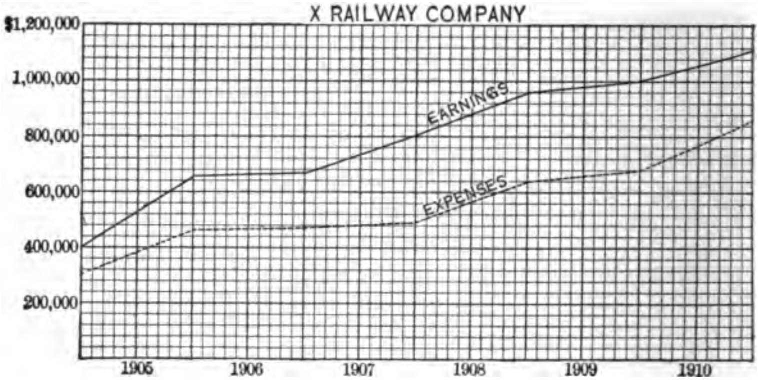
8. A farmer's corn field is 40 rd. long. If the rows are 3 ft. 8 in. apart each way, how many rows lengthwise of the field make an acre? How many hills of corn are there to the acre?

9. Compare two crops from the field in problem 8, under the following conditions: (1st) 3 stalks to the hill, each stalk bearing one good ear of corn, 100 ears making a bushel, and the corn selling for 70 ¢ a bushel; (2d) 5 stalks to the hill, each stalk bearing a small ear of corn, 160 ears being required for a bushel, and the corn selling at 65 ¢ a bushel. Which is the better crop, and how much better per acre?

10. Estimate the number and size of pieces of lumber required to construct a corncrib 24 ft. long, 5 ft. wide at the bottom, 8 ft. wide at the top, and 6 ft. high to the foot of the rafters. The ridge of the roof rises 3 ft., and the rafters project 1 ft. Also estimate the number of board feet of lumber, and the number of shingles required.

GRAPHS

312. A **graph** is a diagram illustrating some relationship. In the illustration here given it shows the relation of earnings and expenses of the X Railway.



YEARLY EARNINGS AND EXPENSES

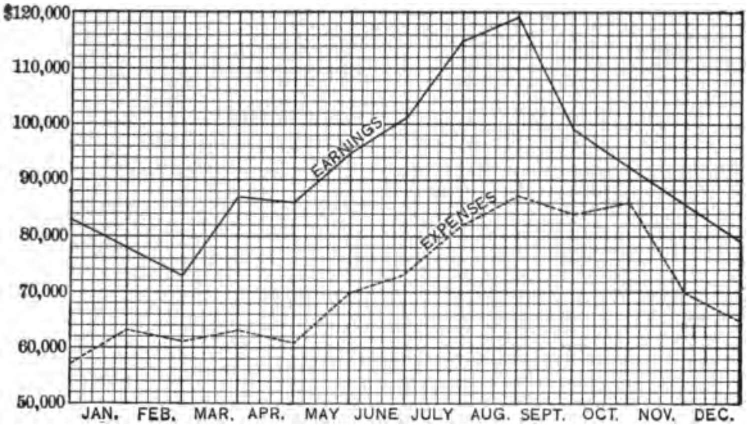
From 1905 to 1910 inclusive, the annual earnings and expenses of the X Railway were as follows:

	Expenses	Earnings
1905	$460,000	$ 660,000
1906	470,000	670,000
1907	485,000	800,000
1908	640,000	960,000
1909	680,000	1,000,000
1910	850,000	1,120,000

The monthly earnings and expenses of the same railway for 1910 were as follows:

	Earnings	Expenses
January .	$ 78,000	$63,000
February .	73,000	61,000
March . .	87,000	63,000
April . .	86,000	61,000
May . .	95,000	70,000
June . .	101,000	73,000
July . .	115,000	82,000
August .	119,000	87,000
September	98,500	84,000
October .	92,000	86,000
November	85,500	70,000
December	79,000	65,000

313. 1. The wheat crop of the Canadian Northwest for twenty years is indicated by the following figures (in nearest 100,000 bu.): 1889, 7,200,000 bu.; 1893, 15,600,000 bu.; 1897, 18,300,000 bu.; 1902, 67,000,000 bu.; 1906, 110,600,000 bu.; 1909, 168,400,000 bu. Illustrate graphically.



MONTHLY EARNINGS AND EXPENSES-1910

2. Illustrate graphically: During 1910 the monthly earnings and expenses of a given railway are as follows:

Earnings		Expenses	
Jan.	$19,000*	Jan.	$20,500†
Feb.	17,000	Feb.	18,500
Mar.	22,000	Mar.	19,000
April	28,000	April	20,500
May	25,000	May	24,000
June	24,000	June	23,500
July	25,500	July	24,500
Aug.	25,000	Aug.	23,000
Sept.	24,000	Sept.	22,500
Oct.	26,000	Oct.	25,500
Nov.	20,500	Nov.	20,000
Dec.	26,000	Dec.	25,000

* Increase of $1000 over Dec., 1909. † Decrease of $1000 from Dec., 1909.

3. Show by diagram the relative earnings and expenses of a railway whose quarterly earnings and expenses for 1910 were as follows:

Earnings		Expenses	
Jan.-Mar.	$ 3,800[*]	Jan.-Mar.	$3,200[†]
Apr.-June	8,200	Apr.-June	8,900
July-Sept.	15,500	July-Sept.	8,200
Oct.-Dec.	5,400	Oct.-Dec.	5,800

4. Make a diagram showing the production of beet sugar in the United States for five consecutive years (in nearest 1000 tons).

	Long Tons
1903-1904	215,000
1904-1905	216,000
1905-1906	279,000
1906-1907	432,000
1907-1908	414,000

5. Illustrate graphically the production of cotton in the United States since 1865, as shown by the following (numbers of bales given to the nearest 100,000):

	Bales (500 lb. Each)
1865	2,100,000
1875	4,300,000
1885	6,400,000
1895	7,100,000
1905	10,800,000
1908	13,200,000

6. The following table shows (to the nearest 1000) the number of cattle imported into and exported from the United States for five years, beginning 1905. Show these facts graphically.

	Imports	Exports
1905	28,000	568,000
1906	29,000	584,000
1907	32,000	423,000
1908	92,000	349,000
1909	139,000	208,000

7. The total corn crop of the United States for 1909 was 2,772,000,000 bu. Of this amount, seven states produced as follows:

Illinois	370,000,000
Iowa	290,000,000
Missouri	214,000,000
Indiana	197,000,000
Nebraska	194,000,000
Kansas	154,000,000

---

[*] Decrease from $ 5200, Oct.-Dec., 1909.

[†] Increase from $ 4400, Oct.-Dec., 1909.

8. Find the per cent of the entire crop for each of the seven states, and for "all the other states." Show these results graphically.

EXAMINATIONS

SPEED TEST

Minimum time, thirty minutes; maximum, one hour. Deduct one credit for every minute required beyond minimum time.

314. **1.** Perform the following operations:

$$\frac{\$1.25 \times 28 \times 77 \times 35}{4 \times 11 \times 7 \times 25} = ?$$

$12' \times 16'' \times 2\frac{1}{4}''$ = board feet

20 planks $16' \times 9'' \times 2\frac{1}{2}''$ @ \$18 per M = \$

2. Find the total cost:

7,480 lb. bran	@	\$22 per ton.
15,870 lb. hay	@	16 per ton.
12,456 lb. meal	@	24 per ton.
9,498 lb. straw	@	12 per ton.

3. Find the number of yards of carpet $\frac{3}{4}$ yd. wide, for rooms,

16′ by 14′
13′ 6″ by 12′ 4″

4. How many double rolls of paper are required for rooms,

$16' \times 12' \times 8'\ 6''$, openings 11 ft. wide?
$18' \times 14' \times 9'$, openings 15 ft. wide?

5. How many paving blocks $12'' \times 5''$ are required for streets as follows:

$1\frac{1}{4}$ mi. long, 60 ft. wide?
$\frac{7}{8}$ mi. long, 56 ft. wide?

6. Find the compound time from Aug. 29, 1912, to July 1, 1914.

7. Find the exact time from May 17, 1913, to Feb. 28, 1914.

8. Count back 150 days from Dec. 1, 1914.

9. A rectangular field 320 rd. long and 240 rd. wide has a board fence around it. If the boards are 16 ft. long and 6 in. wide, and the fence is 4 boards high, how many feet of lumber are there in the fence?

10. Find the gain on each, and the total gain:

Cost	Selling Price	Gain
$ 1.75	$ 2.50	—
.88	1.45	—
135.40	163.50	—
27.75	36.40	—
	Total Gain	

WRITTEN TEST

315. **1.** Find the total cost of:

1350 ft. of pine @ $ 32.50 per M.
6240 ft. of hemlock @ 24.50 per M.
3650 cedar posts @ 9.50 per C.
4260 lb. of coal @ 7.25 per ton.

2. A railway train runs 240 rd. in $\frac{6}{7}$ of a minute. Find its velocity in miles per hour.

3. Find the cost of 16 pieces of timber 28 ft. long, 14 in. wide, and 9 in. thick, at $ 28.50 per 1000.

4. Find the cost of plastering the walls and ceiling of a room 18 ft. long, 14 ft. wide, and 8 ft. high, at $ .62½ per sq. yd. Allow 100 sq. ft. for openings and baseboard.

5. Make a diagram, and estimate the quantity of lumber required for the framework of a ground floor plan of a house 44 ft. long and 28 ft. wide. The sills are 6 inches square, those on the sides being spliced 4 ft. The joists are 3 in. × 6 in. and are 18 in. apart.

6. A man had a yard 38 ft. long by 27 ft. wide; he reserved two grass plots each 8 ft. square, and had the rest paved with stone at $ .45 a square yard. How much did the paving cost?

7. Find the value of a rectangular field 24½ rd. long and 16¼ rd. wide, at $ 47.50 per acre.

8. Around the field in problem 7 is a wire fence. If the fence is 5 wires high and the posts average 7 ft. apart, how many feet of wire and how many posts are needed for the fence?

9. A park is 80 rd. long and 56 rd. wide. Around it is a brick walk 8 ft. wide. The exposed surface of each brick is 4″ × 8″. Find the cost of the brick at $ 7.50 per 1000.

PERCENTAGE

316. The term *percentage* includes those subjects in arithmetic in the computation of which $\frac{100}{100}$ is taken as the basis of comparison. All comparisons of values or quantities are expressed in hundredths. Instead of the term *hundredths*, the equivalent Latin expression *per cent* is used.

317. Per cent means by the hundred. The per cent sign (%) is generally used instead of the words per cent.

Observe these equations:

$$\frac{1}{4} = \frac{25}{100} = .25 = 25\ \%$$

$$\frac{1}{8} = \frac{12\frac{1}{2}}{100} = .12\frac{1}{2} = 12\frac{1}{2}\ \%$$

$$\frac{2}{3} = \frac{66\frac{2}{3}}{100} = .66\frac{2}{3} = 66\frac{2}{3}\ \%$$

$$\frac{5}{13} = \frac{38\frac{6}{13}}{100} = .38\frac{6}{13} = 38\frac{6}{13}\ \%$$

Note that in each case a common fraction has first been reduced to hundredths, and that next the fraction is in its decimal form. The last step shows the fraction in its "per cent" form. Note also that the per cent form of the fraction is identical with its decimal form except that the *per cent sign is used instead of a decimal point.*

Percentage problems are, therefore, only a continuation of problems in fractions under a new name.

318. Nearly all problems in percentage can be reduced to one of the three type forms explained on page 102. In the language of percentage problems, the type forms would read thus:

1. Find 75 % of \$ 48.
2. $62\frac{1}{2}$ % of what number equals \$ 90 ?
3. \$ 75 is what per cent of \$ 90?

When reduced to their simplest arithmetical form, the types appear thus:

1. 75 % of $ 48 = ?
2. $62\frac{1}{2}$ % of ? = $ 45.
3. ? % of $ 90 = $ 75?

In each of these examples there are three elements — the multiplicand, the multiplier, and the product. The corresponding percentage terms are **base**, **rate**, and **percentage**.

319. The **base** is the number or quantity represented by 100 %, and is the basis of comparison in any given problem.

320. The **rate** is the number of hundredths or per cent, and shows the *ratio* of the *percentage* to the *base*.

321. The **percentage** is the *product* of the base multiplied by the rate. It is the number or quantity whose ratio or relation to the base is expressed in the rate.

322. What number is 24 % of 300?

24 % of 300 = 72.

300 is the base because it is the number with which comparison is made.

24 % is the rate because it shows the ratio of the percentage to the base.

72 is the percentage because it is the product of the base and the rate. It is the number that is compared with the base.

323. If the percentage is added to the base, the sum is called the **amount.**

324. If the percentage is subtracted from the base, the result is called the **difference.**

325. Since any common fraction can be reduced to an equivalent decimal or per cent form, conversely, any per cent form of a fraction can be reduced to an equivalent decimal or common fraction form.

$\frac{1}{2} = \frac{50}{100} = .50 = 50\,\%$, and conversely, $50\,\% = .50 = \frac{50}{100} = \frac{1}{2}$.

$\frac{5}{8} = \frac{62\frac{1}{2}}{100} = .62\frac{1}{2} = 62\frac{1}{2}\,\%$, and conversely, $62\frac{1}{2}\,\% = .62\frac{1}{2} = \frac{62\frac{1}{2}}{100} = \frac{5}{8}$, etc.

The student must recognize the difference between $62\frac{1}{2}\%$ and $.62\frac{1}{2}\%$. The first expression equals $.62\frac{1}{2} = \frac{62\frac{1}{2}}{100} = \frac{5}{8}$, and the second expression equals $.0062\frac{1}{2} = \frac{62\frac{1}{2}}{10000} = \frac{5}{800}$.

326. When the per cent sign is used, it takes *the place* of the decimal point, or it is equivalent to moving the decimal point *two places to the right.*

To change a decimal to a rate per cent, move the point two places to the right and annex the per cent sign, and, conversely,

To change a rate per cent to a decimal, remove the per cent sign and move the point two places to the left.

$.045 = 4.5\% = 4\frac{1}{2}\%$ $\quad 15\% = .15$

$1.25 = 125\%$ $\quad 1\frac{1}{4}\% = .01\frac{1}{4} = .0125$, etc.

327. Reduce each of the following to the form indicated:

To the per cent form:

1. .18
2. .25
3. .56
4. .98
5. .05
6. .01
7. $.33\frac{1}{3}$
8. $.62\frac{1}{2}$
9. $.56\frac{1}{4}$
10. .125
11. .025
12. .0025

To both the decimal and the per cent forms:

13. $\frac{1}{4}$
14. $\frac{1}{5}$
15. $\frac{3}{8}$
16. $\frac{1}{16}$
17. $\frac{5}{6}$
18. $\frac{1}{7}$
19. $\frac{7}{9}$
20. $\frac{5}{12}$
21. $\frac{7}{10}$
22. $\frac{3}{2}$
23. $\frac{9}{8}$
24. $\frac{12}{8}$

To the decimal form:

25. 15%
26. 29%
27. 4%
28. 72%
29. 125%
30. $1\frac{1}{4}\%$
31. $12\frac{1}{2}\%$
32. $66\frac{2}{3}\%$
33. 37.5%
34 $\frac{1}{2}\%$
35. $\frac{5}{8}\%$
36. $\frac{3}{8}\%$

To both the decimal form and the common fraction form in its lowest terms:

37. 10%
38. 30%
39. 45%
40. 75%
41. $87\frac{1}{2}\%$
42. $93\frac{3}{4}\%$
43. $\frac{1}{8}\%$
44. $\frac{1}{6}\%$
45. $\frac{9}{16}\%$
46. 580%
47. $137\frac{1}{2}\%$
48. $101\frac{1}{2}\%$

328. In the chapter on Aliquot Parts many of the aliquot or fractional parts of one dollar were learned.

The aliquot or fractional parts of 100% are the same numerically as the like parts of a dollar.

329. In the following table of fractional equivalents, related rates are grouped, the more important rates being in heavy-faced type.

TABLE OF FRACTIONAL EQUIVALENTS

HALF QUARTERS EIGHTHS	SIXTEENTHS	THIRDS SIXTHS TWELFTHS	SEVENTHS NINTHS	MISCELLANEOUS RATES
$50\% = \frac{1}{2}$	$6\frac{1}{4}\% = \frac{1}{16}$	$33\frac{1}{3}\% = \frac{1}{3}$	$14\frac{2}{7}\% = \frac{1}{7}$	$1\frac{1}{4}\% = \frac{1}{80}$
	$18\frac{3}{4}\% = \frac{3}{16}$	$66\frac{2}{3}\% = \frac{2}{3}$	$28\frac{4}{7}\% = \frac{2}{7}$	$1\frac{2}{3}\% = \frac{1}{60}$
$25\% = \frac{1}{4}$	$31\frac{1}{4}\% = \frac{5}{16}$		$42\frac{6}{7}\% = \frac{3}{7}$	$2\frac{1}{2}\% = \frac{1}{40}$
$75\% = \frac{3}{4}$	$43\frac{3}{4}\% = \frac{7}{16}$	$16\frac{2}{3}\% = \frac{1}{6}$	$57\frac{1}{7}\% = \frac{4}{7}$	$3\frac{1}{3}\% = \frac{1}{30}$
	$56\frac{1}{4}\% = \frac{9}{16}$	$83\frac{1}{3}\% = \frac{5}{6}$	$71\frac{3}{7}\% = \frac{5}{7}$	$4\% = \frac{1}{25}$
$12\frac{1}{2}\% = \frac{1}{8}$	$68\frac{3}{4}\% = \frac{11}{16}$		$85\frac{5}{7}\% = \frac{6}{7}$	$5\% = \frac{1}{20}$
$37\frac{1}{2}\% = \frac{3}{8}$	$81\frac{1}{4}\% = \frac{13}{16}$	$8\frac{1}{3}\% = \frac{1}{12}$	$11\frac{1}{9}\% = \frac{1}{9}$	$6\frac{2}{3}\% = \frac{1}{15}$
$62\frac{1}{2}\% = \frac{5}{8}$	$93\frac{3}{4}\% = \frac{15}{16}$	$41\frac{2}{3}\% = \frac{5}{12}$	$22\frac{2}{9}\% = \frac{2}{9}$	$7\frac{1}{7}\% = \frac{1}{14}$
$87\frac{1}{2}\% = \frac{7}{8}$		$58\frac{1}{3}\% = \frac{7}{12}$	$44\frac{4}{9}\% = \frac{4}{9}$	$9\frac{1}{11}\% = \frac{1}{11}$
		$91\frac{2}{3}\% = \frac{11}{12}$	$55\frac{5}{9}\% = \frac{5}{9}$	$10\% = \frac{1}{10}$
			$77\frac{7}{9}\% = \frac{7}{9}$	
			$88\frac{8}{9}\% = \frac{8}{9}$	

330. The labor involved in solving percentage problems is materially diminished by using the common fractional equivalents of rate per cents wherever possible.

331. Problems in percentage are solved in accordance with the fundamental principles of multiplication and division of simple numbers, page 101. In the language of percentage they are stated thus:

PRINCIPLES: 1. Base multiplied by the rate = percentage.
2. Percentage divided by rate = base.
3. Percentage divided by base = rate.

332. To find the percentage, the base and rate being given.

1. Find 43% of $360.

$$\begin{array}{rl} \$360 & \text{Base} \\ .43 & \text{Rate} \\ \hline 1080 & \\ 1440 & \\ \hline \$154.80 & \text{Percentage} \end{array}$$

43% of $360 means .43 of $360. $360 multiplied by .43 equals $154.80.

2. Find $16\frac{2}{3}\%$ of \$540.

$$\begin{array}{r} \$540 \\ \underline{.16\frac{2}{3}} \\ 360 \\ 3240 \\ \underline{540} \\ \$90.00 \end{array} \qquad \text{Or,} \qquad \begin{array}{l} 16\frac{2}{3}\% = \frac{1}{6} \\ \frac{1}{6} \text{ of } \$540 = \$90. \end{array}$$

Multiplying \$540 by $.16\frac{2}{3}$ gives \$90. Or, since $16\frac{2}{3}\% = \frac{1}{6}$, take $\frac{1}{6}$ of \$540, which equals \$90.

3. Find $\frac{1}{8}\%$ of \$1250.

$$\begin{array}{l} 8\underline{)\$12.50} \\ \quad \$1.5625 \equiv \$1.56 \end{array}$$

$\frac{1}{8}\%$ means $\frac{1}{8}$ of 1%. 1% of \$1250 is \$12.50; and $\frac{1}{8}$ of \$12.50 = \$1.56.

4. What number is $33\frac{1}{3}\%$ greater than \$480?

$$\begin{array}{rcr} 100\% & \equiv & \$480 \\ \underline{33\frac{1}{3}\%} & = & \underline{160} \\ 133\frac{1}{3}\% & = & \$640 \end{array}$$

The base is always equal to 100%. Hence \$480 is equal to 100%. The required number is $33\frac{1}{3}\%$, or $\frac{1}{3}$, of \$480 greater than \$480. $33\frac{1}{3}\%$ of \$480 is \$160. By adding \$480 and \$160, the desired result, \$640, is obtained.

Note. It should be observed that the base and the percentage are like numbers.

333. Performing all operations mentally where possible, find:

1. 21% of 200.
2. 35% of 500.
3. $12\frac{1}{2}\%$ of 640.
4. 25% of 700.
5. 26% of 700.
6. $66\frac{2}{3}\%$ of 950.
7. $87\frac{1}{2}\%$ of 1200.
8. 13% of 148.
9. 17% of 263.
10. 6% of 475.
11. 24% of 300.
12. $16\frac{2}{3}\%$ of 372.
13. $8\frac{1}{3}\%$ of 744.
14. $18\frac{3}{4}\%$ of 640.
15. 64% of 298.
16. 78% of 725.
17. 93% of 800.
18. 125% of 960.
19. 80% of 1200.
20. 75% of 1500.
21. $6\frac{2}{3}\%$ of 1800.
22. $14\frac{2}{7}\%$ of 2149.
23. $22\frac{2}{9}\%$ of 1881.
24. $31\frac{1}{4}\%$ of 8000.
25. $56\frac{1}{4}\%$ of 3200.
26. 250% of 440.
27. 375% of 560.
28. $2\frac{1}{2}\%$ of 1600.
29. $2\frac{1}{3}\%$ of 1260.
30. $1\frac{1}{4}\%$ of 888.
31. $\frac{3}{4}\%$ of 84.
32. $\frac{5}{8}\%$ of 500.
33. $1\frac{1}{2}\%$ of 64.
34. 100% of 128.
35. 150% of 630.
36. 300% of 25.
37. 3% of 6172.
38. $12\frac{1}{2}\%$ of 8176.
39. $58\frac{1}{3}\%$ of 7392.
40. $6\frac{1}{4}\%$ of 6560.
41. $62\frac{1}{2}\%$ of 6560.
42. $3\frac{1}{3}\%$ of 7230.
43. $33\frac{1}{3}\%$ of 7230.
44. $333\frac{1}{3}\%$ of 7230.
45. $\frac{1}{10}\%$ of 8400.

Short Method

334. Find 28 % of \$ 7500.

¾ of \$ 2800 = \$ 2100. By the commutative law of multiplication (page 102) the product of 28 times 75 equals the product of 75 times 28. Therefore, 28 % of \$ 7500 equals 75 % of \$ 2800. 75 % = ¾, and ¾ of \$ 2800 = \$ 2100.

335. In like manner find, mentally, the value of each of the following:

1. 16 % of \$ 1250.
2. 18 % of \$ 333⅓.
3. 26 % of \$ 2500.
4. 44 % of \$ 2500.
5. 56 % of \$ 5000.
6. 96 % of \$ 125.
7. 160 % of \$ 3750.
8. 180 % of \$ 750.
9. 32 % of \$ 375.
10. 36 % of \$ 75.
11. 240 % of \$ 2500.
12. 320 % of \$ 625.
13. 32 % of \$ 3125.
14. 124 % of \$ 6250.
15. 39 % of \$ 666⅔.
16. 42 % of \$ 166.66⅔.
17. 132 % of \$ 2500.
18. 12 % of \$ 25.
19. 36 % of \$ 87.50.
20. 52 % of \$ 75.
21. 48 % of \$ 375.

PROBLEMS

336. 1. In a certain school the attendance in 1908 was 425. What was the attendance in 1909 if there was an increase of 20 %?

2. A house and lot cost \$ 5000. If taxes, repairs, and other expenses amount to \$ 180 per annum, what rent per month must the owner receive in order to clear 6 % on his investment?

3. If a plant and machinery, valued at \$ 75,000, depreciates at the rate of 5 % of its value each year, find its value at the end of the fifth year.

4. The capital stock of a manufacturing company is \$ 150,000. If the gross receipts amount to 32 % of the capital stock, and the total expenditures amount to 75 % of the receipts, find the profit for the year.

5. If a clerk's salary is \$ 600 the first year of service, and he gets a 10 % increase each year for 5 yr., what will be his salary the sixth year?

6. Chemical analysis shows that "Pennsylvania" Portland cement contains the following ingredients in the proportion here stated:

Silica	22%
Iron and Alumina	10.98%
Lime	61.50%
Magnesia	2.47%
Sulphur Anhydride	1.70%
Combustible materials	.85%

Find the amount by weight of each of the ingredients in 5 T. of cement.

7. Merchandise costing $\$.37\frac{1}{2}$ sells so as to gain $27\frac{1}{2}\%$. At what price does it sell?

8. On June 1, 1906, the city of Cleveland, Ohio, covered an area of 25,620 A. If the area annexed since that time is equal to 14.727% of its former area, how many acres does the city cover now?

9. If a young man earns $900 a year and saves $22\frac{1}{2}\%$ of it, how long will it take him to save enough to pay for 3 lots valued at $337.50 each?

10. A fruit dealer buys 40 doz. oranges at 24¢ a dozen. Before he can sell them, 20% of them spoil. At what price per dozen must the remaining oranges be sold to make a profit of $16\frac{2}{3}\%$?

11. At $\frac{1}{10}\%$ premium, what is the cost of insuring a factory for $15,000?

12. A dealer bought 860 gross (long) tons of coal at $3.86 a ton f. o. b. Freight was 6¢ a cwt., and the cost of hauling was $321.60. Find the selling price a net (short) ton to gain 22% on entire cost.

337. To find the rate, the base and percentage being given.

What is the ratio of 6 to 24?

6 is what part of 24?

6 is what per cent of 24? or

What per cent of 24 is 6?

In each of these examples, 24 is the basis of comparison, or it is the base. 6 is the number compared with 24; hence it is the percentage.

The ratio of 6 to 24 is expressed as $\frac{6}{24}$ or $\frac{1}{4}$. In each of the examples, the desired relation between the numbers is determined by dividing 6 by 24. $6 \div 24 = \frac{1}{4} = 25\%$. That is, the percentage divided by the base gives the rate (Prin. 14, page 101).

338. **1.** A man's income is $1800 a year. If he saves $630 a year, what per cent of his income does he save?

$? \% \text{ of } \$1800 = \$630.$

$\$630 \div \$1800 = .35 = 35\%.$

The problem is of the third type form as shown in the first equation. Dividing the percentage, $630, by the base, $1800, gives the rate, 35%.

2. $1500 is what per cent greater than $1200?

$\$1500 - \$1200 = \$300$, amount which $1500 is greater than $1200.

$\$300 = ? \%$ of $\$1200$?

$\$300 \div \$1200 = \frac{300}{1200} = \frac{1}{4} = 25\%.$

First find how much $1500 is greater than $1200. The problem then becomes, $300 is what per cent of $1200? which, as shown in the solution, is 25%.

To the Student. Reduce each problem (either mentally or in writing) to its proper type form. Its solution will then be entirely clear to you.

Use the fractional form of division, when possible; reduce the fraction to its lowest terms mentally; and place the result equal to its equivalent rate per cent.

339. Solve, mentally when possible, each of the following:

What per cent of

1. 12 is 6?
2. 18 is 12?
3. 200 is 150?
4. 72 is 56?
5. 25 is 30?
6. 14 is 21?
7. 450 is 225?
8. 4 yd. is 6 ft.?
9. 2 bu. is 3 pk.?
10. $24 is $2.40?
11. $1.75 is $.75?
12. $15 is $.90?

What per cent greater than

13. 54 is 63?
14. 300 is 390?
15. 560 is 630?
16. 270 is 300?
17. $3000 is $4200?
18. 3 pk. is 1 bu.?
19. 1 lb. Troy is 1 lb. avoir.?
20. 1 oz. avoir. is 1 oz. Troy?

What per cent less than

21. 20 is 18?
22. 30 is 20?
23. 50 is 40?
24. 1 bu. is 3 pk.?
25. 1 oz. Troy is 1 silver dollar?
26. A long ton is a short ton?
27. $\frac{1}{2}$ is $\frac{1}{3}$?
28. $\frac{1}{3}$ is $\frac{1}{4}$?

PROBLEMS

340. 1. If in a school there are 160 boys and 192 girls, what per cent of the enrollment are boys? girls?

2. A man asked $180 for his horse. If he accepted $160, what per cent less than his asking price did he accept?

3. A building worth $12,500 is insured for $7500. For what per cent of its value is it insured?

4. In a spelling lesson of 60 words a pupil spelled 3 words incorrectly. What per cent of the words were spelled correctly?

5. Find the per cent of increase in the population of the following cities from 1900 to 1910:

	Population, 1900	Population, 1910
Akron, Ohio	42,728	69,067
Bridgeport, Conn.	70,996	102,054
Camden, N. J.	75,935	94,538
District of Columbia	278,718	331,069
Grand Rapids, Mich.	87,565	112,571
Houston, Tex.	44,633	78,800
Reading, Pa.	78,961	96,071
St. Paul, Minn.	163,065	214,744

6. A merchant fails, having liabilities amounting to $28,500. If his total resources amount to $19,000, what per cent of his debts can he pay? He owes Thos. Higbee $1846.50. How much will Mr. Higbee receive?

7. A man bought a house for $8000. To pay for it he borrowed $4000 on bond and mortgage at 6%. The other $4000 he paid from his own money. If the house rents for $800 per annum, and he pays $200 a year for taxes, repairs, insurance, etc., besides the interest on the mortgage, what rate per cent does the investment pay him?

8. The following table shows the number of organized wage earners (males) in New York State employed one or more days during the first quarter of the year 1910. Find, to the nearest tenth per cent, the percentage for each limit of time. Prove the work.

Period	Number Employed
1–29 da.	15,497
30–59 da.	63,695
60–79 da.	189,756
80 da. or more	60,029

9. In May, 1910, there arrived at the port of Portland, Ore., 74 vessels having a total net registry of 82,809 net tons, as compared with 56 vessels of 51,087 net tons in May, 1909. Find the average tonnage of each vessel, and the per cent of increase in the average for the year.

10. The first five months of 1910 there were shipped from Chicago 61,855,300 bu. of grain. Find the average per month, and compare, in terms of per cent, with the shipments for May, when 13,244,700 bu. were shipped.

11. The imports of drugs, chemicals, and dyes into Canada for the fiscal year 1908–1909 were from the following countries. Find the per cent from each country.

From	Amount
United States	$5,837,545
United Kingdom	2,016,629
Germany	259,064
France	254,485
All other countries	765,409

12. In the fiscal year 1908–1909, Canada imported watches, cases, and movements, as follows. Show the result graphically.

From	Amount
United States	$538,336
United Kingdom	22,374
Switzerland	172,299

13. In 1909, Brazil imported dried fruits valued at $332,164, of which amount $7097 worth were from the United States. What per cent came from the United States?

341. **To find the base, the percentage and the rate being given.**

18 is $\frac{1}{4}$ of what number?
18 is 25% of what number?
18 is $.25 \times$?

The equation is in the form of the second type problem. The product (percentage) and one of the factors (rate) are given to find the other factor (base). $18 \div .25 = 72$, the base.

To find the base, divide the percentage by the rate.

342. 48 is $18\frac{3}{4}\%$ of what number?

$$18\tfrac{3}{4}\% \times ? = 48.$$
$$48 \div .18\tfrac{3}{4} = 256. \quad \text{Or,}$$
$$18\tfrac{3}{4}\% = \tfrac{3}{16}.$$
$$\tfrac{3}{16} \text{ of the number} = 48.$$
$$\tfrac{1}{16} \text{ of the number} = \tfrac{1}{3} \text{ of } 48 = 16.$$
$$\tfrac{16}{16} \text{ of the number} = 16 \times 16 = 256.$$

On reducing the problem to its simplest type form we have $18\frac{3}{4}\% \times ? = 48$. Since a product and one factor are given, the other factor (base) is found by dividing the product, 48, by the given factor, $18\frac{3}{4}\%$. The result is 256. Or,

By the use of the fractional equivalent of $18\frac{3}{4}\%$, the solution can be made mentally. $18\frac{3}{4}\% = \frac{3}{16}$.

If $\frac{3}{16}$ of the number is 48, $\frac{1}{16}$ of the number is $\frac{1}{3}$ of 48, or 16, and $\frac{16}{16}$ of the number, or the number itself, is 16 times 16, or 256.

To THE STUDENT. Wherever possible, use this fractional method of solution. It is far shorter and quicker than the solution first explained. The first method will have to be used in those problems containing rates that do not have convenient fractional equivalents.

343. Find the number of which

1. 42 is 6 %.
2. 18 is 2 %.
3. 25 is 20 %.
4. 43 is 1 %.
5. 86 is 43 %.
6. 72 is $12\frac{1}{2}\%$.
7. 90 is $6\frac{2}{3}\%$.
8. 92 is 25 %.
9. 106 is $33\frac{1}{3}\%$.
10. 125 is $16\frac{2}{3}\%$.
11. 130 is 50 %.
12. 240 is 75 %.
13. 280 is $62\frac{1}{2}\%$.
14. 224 is $87\frac{1}{2}\%$.
15. 378 is $56\frac{1}{4}\%$.
16. 663 is 13 %.
17. 374 is 17 %.
18. 836 is 19 %.
19. 1236 is 6 %.
20. 360 is $112\frac{1}{2}\%$.
21. 900 is $128\frac{4}{7}\%$.
22. 755 is 250 %.
23. 848 is $266\frac{2}{3}\%$.
24. 833 is $116\frac{2}{3}\%$.
25. 48 is $\frac{1}{4}\%$.
26. 321 is $\frac{1}{2}\%$.
27. 424 is $\frac{8}{9}\%$.
28. 375 is $\frac{5}{8}\%$.
29. 15 is $\frac{1}{20}\%$.
30. $5\frac{1}{4}$ is $\frac{1}{8}\%$.
31. $3\frac{2}{3}$ is $\frac{1}{3}\%$.
32. $7\frac{1}{2}$ is $\frac{3}{4}\%$.
33. $4\frac{1}{2}$ is $\frac{3}{8}\%$.

PROBLEMS

344. 1. A young woman, by saving $31\frac{1}{4}\%$ of her earnings, paid for a house and lot costing $1875 in 6 yr. How much did she earn a year?

2. A parlor suite sold for $156, which was 30 % more than it cost. How much did it cost?

3. A bookkeeper receives an increase of 10 % in his salary each year for four years. His salary for the fifth year is $2196.15. What was his salary the first year?

4. A house rents for $900 a year. The expenses are $250 a year. If the net income is 6½ %, what was the cost of the house?

5. A house was insured for 80 % of its value at ⅜ %. If the premium paid was $22.50, find the value of the house.

6. Of a consignment of eggs, 6 % were broken. For the remainder, $45.12 was received at 32 ¢ a dozen. How many dozen eggs were in the consignment?

7. A merchant's sales for October were $28,460.50, for November were $35,827.43, and for December were $46,921.07. If the sales for these three months were 35 % of the sales for the entire year, find the average monthly sales for the remainder of the year.

8. A bankrupt can pay 72 % of his debts. If his resources amount to $13,475.28, what are his liabilities? How much will a creditor receive whom he owes $2485?

9. In an orchard 40 % of the trees are winter apples; 10 % are fall apples; 5 % are harvest apples; 30 % are pear trees; and the remaining trees are peach. If there are 30 peach trees, how many trees are in the orchard?

10. A house and lot cost $6400. The house cost 66⅔ % more than the lot. Find the cost of each.

11. A druggist's sales increased 15 % the second year, 20 % the third year, and 25 % the fourth year. If the fourth year's sales were $29,023.13, what were the sales for the first year?

12. A merchant's sales increased 16⅔ % the second year, 20 % the third year, 25 % the fourth year, and 14²⁄₇ % the fifth year. If the sales for the five years amount to $213,244.25, what were the sales the first year?

13. A salesman's sales increased 25 % in November over October's sales, and 20 % in December over November's sales. His January sales decreased 40 % from those of December. If January's sales amounted to $3501, what were they for October?

14. In 1910 the population of Muskogee, Okla., was 25,278, an increase of 427 % in ten years. What was the population in 1900?

REVIEW

345.

RAPID DRILL EXERCISE

1. What is 25 % of 36?
2. What is 36 % of 25?
3. What is 12½ % of 64?
4. What is 64 % of 12½?
5. What is 8⅓ % of 72?
6. What is 72 % of 8⅓?
7. What is 75 % of 68?
8. What is 68 % of 75?
9. What is 10 % less than 30?
10. What is 30 % less than 10?
11. What is 18¾ % less than 48?
12. What is 48 % less than 18¾?
13. What is 20 % more than 44?
14. What is 44 % more than 20?
15. What is 33⅓ % more than 66?
16. What is 66 % more than 33⅓?
17. Of what number is 20, 25 %?
18. Of what number is 25, 20 %?
19. 40 is 10 % of what number?
20. 10 is 40 % of what number?
21. 30 is 15 % of what number?
22. 15 is 30 % of what number?
23. 60 is 75 % of what number?
24. 75 is 60 % of what number?
25. 50 is what per cent of 60?
26. 60 is what per cent of 50?
27. 75 is what per cent of 25?
28. 25 is what per cent of 75?
29. 20 is what per cent of 50?
30. 50 is what per cent of 20?
31. 20 is what per cent less than 30?
32. 30 is what per cent more than 20?
33. 25 is what per cent less than 30?
34. 30 is what per cent more than 25?
35. 40 is what per cent more than 25?
36. 25 is what per cent less than 40?

346. PERCENTAGE DRILL

	a	b	c	d	e	f	g
1.	$ 15	$ 18	$ 10	25%	10%	$ 6	$ 3
2.	16	22	14	20%	$12\frac{1}{2}$%	3	4
3.	21	24	15	40%	30%	4.20	6.30
4.	24	30	20	20%	$33\frac{1}{3}$%	2	8
5.	32	40	28	$16\frac{2}{3}$%	25%	6	12
6.	40	45	32	$12\frac{1}{2}$%	$11\frac{1}{9}$%	10	8
7.	48	56	36	$33\frac{1}{3}$%	25%	12	20
8.	50	60	45	25%	30%	15	18
9.	60	75	54	$16\frac{2}{3}$%	25%	20	18
10.	72	81	60	$33\frac{1}{3}$%	$16\frac{2}{3}$%	24	9
11.	80	96	70	20%	$12\frac{1}{2}$%	30	8
12.	96	120	84	$12\frac{1}{2}$%	$8\frac{1}{3}$%	24	16
13.	108	120	96	$16\frac{2}{3}$%	5%	9	36
14.	120	144	100	$37\frac{1}{2}$%	50%	20	12
15.	150	175	120	25%	$14\frac{2}{7}$%	50	40
16.	180	210	160	$66\frac{2}{3}$%	$12\frac{1}{2}$%	40	60
17.	240	270	210	$12\frac{1}{2}$%	25%	60	30
18.	300	360	250	20%	25%	25	100
19.	360	420	320	$33\frac{1}{3}$%	$16\frac{2}{3}$%	90	45
20.	450	540	360	20%	$16\frac{2}{3}$%	75	45

1. The numbers in column *b* are what per cent greater than those in column *a*?

2. The numbers in column *c* are what per cent less than those in column *a*?

3. Consider column *a* as cost, and column *d* as rate of gain. Find the selling price.

4. Consider column *a* as cost, and column *e* as rate of loss. Find the selling price.

5. If column *a* is cost and column *f* is gain, find the gain per cent.

6. If column *a* is cost and column *g* is loss, find the loss per cent.

7. If column *b* is cost and column *c* is selling price, find the rate of loss.

8. If column *c* is cost and column *b* is selling price, find the rate of gain.

9. If column *b* is selling price and column *d* is rate of gain, find the cost.

10. If column *b* is selling price and column *e* is rate of loss, find the cost.

PERCENTAGE REVIEW

Mental

347. 1. Two railroads carry freight 200 mi. The first carries it 75 mi., and the second 125 mi. What per cent of the distance does each carry it?

2. A man earns $1500 a year and spends $900. What per cent of his money does he spend?

3. A boy bought a watch for $12 and a chain for $4. Later he sold them both for the cost of the watch. What per cent of the cost was the selling price?

4. During the month of May a farmer sells 500 qt. of milk; during June he sells 600 qt. What is the per cent of increase?

5. Of a pile of potatoes containing 120 bu., one bushel out of five is bad. What per cent are bad? How many bushels are good?

6. From a half chest of Oolong tea containing 50 lb., 15 lb. were sold one day and 20 lb. the next day. What per cent of the tea remained unsold? What per cent was sold each day?

7. A bankrupt's resources are $12,000, and his liabilities are $20,000. What per cent of his debts can he pay? He owes Mr. Brown $600. How much will he receive?

8. Last year my house and lot was worth $4000. This year it is worth $4400. What per cent has it increased in value? If it increases in value at the same rate per cent next year, how much will the property then be worth?

9. In an invoice of 4 doz. crates of oranges, 10% are decayed. If there is an average of 8 doz. oranges to a crate, how many oranges are spoiled?

10. If coffee loses 10 % of its weight in roasting, how many pounds of roasted coffee will there be in 500 lb. of green coffee?

11. A clerk's monthly salary was $80. Owing to hard times, his salary is reduced 5 %. How much does he now receive per month?

12. A dairyman had 1000 qt. of milk a week in July. During August he increased the amount of grain fed to his cows $3 worth per week, and obtained 6 % more milk. If he sold the milk at 8¢ a quart, did it pay to increase the amount of grain fed?

13. If coal has been selling at $5.50 a ton, a 10 % increase will make it sell for how much?

14. A merchant's sales increased 33⅓ % the second year over the first year. If the first year's sales were $72,000, what were the sales the second year?

15. I bought 56 yd. of cloth at $2.40 a yard and sold it to gain 25 %. How much did I receive for all of it?

16. If merchandise cost $360, and increases in value 16⅔ %, how much will it be worth? If later it decreases 16⅔ % in value, how much will it then be worth?

17. In settling with a bankrupt, I received $2400, which was 75 % of my claim. How much did I lose?

18. A man receives $50 a month rent for a house. His expenses for a year are $150. What is the value of the house if it nets him 9 %?

19. During the second year a grocer's sales increased $5000, which was 20 % of the first year's sales. What were the sales each year?

20. If, in problem 19, the profit was 20 % of the cost of the goods, what were his profits each year? What would have been his profits if they were 20 % of the selling price?

21. How much must a young man save so that at 6 % he may have an annual income of $1200?

22. A merchant's average profit is 12½ % of his sales. How much merchandise must he sell to make a profit of $6000?

23. If an enterprise pays 8 %, what is the investment, the income being $1600?

24. A man sold 37½ % of his business for $3600. What was the entire business worth?

25. I sold a house and lot at a gain of $200, which was 5 % of the selling price. How much did the property cost me?

PROBLEMS

348. **1.** If potatoes shrink 5 % in weight during the winter, what is the shrinkage in weight of 1200 bu.?

2. A store is worth $18,000. It rents for $2000. Repairs cost 1 % of the value of the store; taxes cost 1½ %; insurance ¼ % on a policy of $10,000; and other expenses amount to $85. The net income is what per cent of the gross rental? of the value of the store?

3. A bankrupt paid me 68 % of my claim. If I received $1275.46, how much did he owe me?

4. If the duty on certain merchandise was 30 %, and the present duty is 20 % less than formerly, what is the present rate?

5. What per cent of the total area of the following cities is land? What per cent is water?

	LAND AREA	WATER AREA
New Haven, Conn.	11,460 acres	2,880 acres
Syracuse, N.Y.	10,639 acres	309 acres
Scranton, Pa.	12,362 acres	147 acres
Fall River, Mass.	21,862 acres	3,793 acres
Albany, N.Y.	6,914 acres	283 acres

6. In a school there are 400 girls, which is $14\frac{2}{7}$ % more than the number of boys. How many pupils are there in the school?

7. After selling 560 sheep, a shepherd found he had 12½ % of his flock left. How many sheep were there in the flock?

8. What is the difference in pounds between ⅝ of a ton and ⅝ % of a ton?

9. A man had $1460 and spent $365 for a year's board. What per cent of his money did his board cost him? The money he spent is what per cent of the amount he had left?

10. An agent sold three farms for $12,750. The price of the second was 50 % more than the price of the first, and of the third was 30 % less than the price of both the others. Find the price of each.

11. A bushel of wheat weighs 60 lb. $5\frac{1}{4}$ bu. will make a barrel of flour (196 lb.). What per cent of the wheat is flour?

12. The population of the city of New York in 1900 was 3,437,202, in 1905 it was 4,000,413, and in 1910 it was 4,766,883. Find the per cent of increase for each five-year period, and for the ten-year period.

13. On an invoice of merchandise valued at $672, a merchant paid freight $12.60, and cartage $9.80. The charges were what per cent of the value of the goods?

14. Find the sum of $\frac{1}{2}$% of $893.68; $\frac{3}{4}$% of $723.85; $\frac{1}{20}$% of $3487.60.

15. The total amounts of exports of agricultural implements from the United States to France for the six years ending 1907 were as follows:

1902	$2,100,692	1905	$2,817,998
1903	2,789,256	1906	2,895,243
1904	3,063,692	1907	3,724,959

Show in per cent the annual increase or decrease of exports.

16. Out of a total of 52,021 harvesting machines imported into Germany in 1907, 43,300 were from the United States, 6358 from Canada, and 2363 from England. Show by diagram the per cent from each country.

17. A manufacturer sells to a wholesaler at a profit of 20%; the wholesaler to the retailer at a 25% gain; and the retailer to the consumer at a gain of 60%. If the consumer pays $14.40, find the cost to the manufacturer.

18. A house and lot rents for $60 a month. The taxes, insurance, repairs, etc., are estimated at $180 per annum. How much can a purchaser afford to pay for the property so as to have a 6% investment?

19. A creditor receives from a bankrupt firm $735 on a debt of $1960. What per cent of their debt does the firm pay? If their total resources amount to $9474.60, how much are their liabilities?

20. A house and lot cost $6000; the insurance is $15, taxes are $60, and repairs are $45 annually. What rent per month must be received to realize 7 % on the investment?

21. What per cent of 5 lb. avoirdupois is 7 oz.?

22. From a cask containing 45 gal., 2 gal. and 1 qt. are lost by leakage. What per cent is lost by leakage?

23. An agent charged $160 as his commission for selling real estate at $2\frac{1}{2}$ %. For how much was the property sold?

24. A farmer's crop of grain this year is 15 % less than it was last year. What was this year's crop if in the two years he raised 1554 bu. of grain?

25. A drover sold 330 sheep and had 1320 sheep left. What per cent of his flock did he sell?

26. A speculator increased his real estate holdings by 44 %. He then sold 25 % of all his property, and found he had 810 A. left. How many acres had he at first?

27. A man drew 65 % of his money from the bank and with 40 % of it bought a house and lot for $5200. How much money had he remaining in the bank?

28. From his income a young man saved 20 %. Had he saved $200 more, the per cent saved would have been 30 %. What is his income?

29. A merchant having 1848 yd. of cloth, sold $33\frac{1}{3}$ % of it at $1.60 a yard, 25 % of the remainder at $1.65 a yard, and then what was left at $1.55 a yard. How much did he receive for all?

30. Multiply each of the following percentages by its relative weight, and find the general average by dividing the sum of the products by the sum of the relative weights.

PERCENTAGES	RELATIVE WEIGHTS	PRODUCTS
76.5	6	——
93.25	5	——
87	4	——
91.65	12	——
71.81	9	——
	General Average	——

349. The following table gives the average composition of a few common **food products.**

FOOD MATERIALS (AS PURCHASED)	REFUSE	WATER	PROTEIN	FAT	CARBOHYDRATES	ASH	FOOD VALUE PER POUND	AVERAGE COST PER POUND
Animal Foods	%	%	%	%	%	%	Calories	Cents
Beef, ribs	20.8	43.8	13.9	21.2	—	.7	1135	20
Veal, hind quar. .	20.7	56.2	16.2	6.6	—	.8	580	18
Mutton, loin chops	16.0	42.0	13.5	28.3	—	.7	1415	18
Ham, smoked . .	13.6	34.8	14.2	33.4	—	4.2	1635	18
Chicken	25.9	47.1	13.7	12.3	—	.7	765	20
Mackerel	44.7	40.4	10.2	4.2	—	.7	370	12
Oysters	—	88.3	6.0	1.3	3.3	1.1	225	18
Eggs	11.2	65.5	13.1	9.3	—	.9	635	16
Butter	—	11.0	1.0	85.0	—	3.0	3410	25
Whole milk . . .	—	87.0	3.3	4.0	5.0	.7	310	3
Vegetable Foods								
Oatmeal	—	7.7	16.7	7.3	66.2	2.1	1800	4
White bread . . .	—	35.3	9.2	1.3	53.1	1.1	1200	5
Brown bread . . .	—	43.6	5.4	1.8	47.1	2.1	1040	5
Beans, dried . . .	—	12.6	22.5	1.8	50.6	3.5	1520	5
Potatoes	20.0	62.6	1.8	.1	14.7	.8	295	1
Apples	25.0	63.3	.3	.3	10.8	.3	190	1
Almonds	45.0	2.7	11.5	30.2	9.5	1.1	1515	40

350. Energy is produced by fat and the carbohydrates.

351. Protein is the material of which the tissues are built.

352. Using the table given above, find the quantity of refuse and nutrients of the different classes in the following (to the nearest tenth of a pound):

1. 15 pounds of oatmeal.
2. 12 pounds of brown bread.
3. 1 bushel of potatoes (60 lb.).
4. 1 gallon of milk (8.6 lb.).
5. 5 pounds of beef ribs.
6. 6 pounds of chicken.
7. 24 pounds of smoked ham.
8. 18 pounds of mackerel.

The following dietaries have been determined by experiment:

QUANTITIES PER MAN PER DAY

	PROTEIN	FAT	CARBOHYDRATES	ENERGY VALUE
	Ounces	Ounces	Ounces	Calories
Farmers	3½	4½	16½	3415
Football players	8	12½	22½	6590
Lawyers, teachers, etc.	4	4½	15½	3220

Select two kinds of animal food and two vegetables that would be suitable for:

9. A street car motorman.

10. A clerk in an office.

11. A laboring man.

12. An athlete.

Find the quantity of nutrients (protein, fat, and carbohydrates), the energy in calories, and the cost of the following daily dietaries:

13. Beef ribs, 12 oz.; potatoes, 16 oz.; butter, 2 oz.; white bread, 24 oz.; oatmeal, 2 oz.; eggs, 4 oz.

14. Beans, 8 oz.; milk, 16 oz.; mackerel, 16 oz.; oatmeal, 4 oz.; potatoes, 16 oz.; brown bread, 16 oz.; mutton chops, 8 oz.

15. Almonds, 2 oz.; brown bread, 24 oz.; chicken, 16 oz.; potatoes, 16 oz.; apples, 16 oz.; butter, 4 oz.

16. Smoked ham, 6 oz.; white bread, 20 oz.; oatmeal, 4 oz.; beans, 4 oz.; eggs, 4 oz.; potatoes, 16 oz.; butter, 2 oz.

17. Prepare a daily dietary containing nutrients in the ratio (approximately) of protein 1 part, fat 1 part, and carbohydrates 4 parts, the total amount of nutrients not to exceed 24 ounces.

18. Find the amount of energy in calories, and the cost of the dietary prepared in problem 17.

19. Prepare a daily dietary containing nutrients in the ratio (approximately) of protein 1 part, fat 1 part, and carbohydrates 3½ parts, the total amount of nutrients not to be less than 24 ounces nor more than 32 ounces.

20. Find the amount of energy in calories, and the cost of the dietary prepared in problem 19.

TRADE DISCOUNT

353. **Discount** is an amount deducted from a sum owing or to be paid. The amount of discount is indicated by a rate per cent.

If goods marked $4 are sold at a discount of 25%, for how much do they sell?

354. Discounts may be trade discounts, or time discounts.

355. Manufacturers, publishers, and wholesalers have fixed price lists for their merchandise. These price lists are printed in their catalogues. To avoid the necessity of frequently reprinting their catalogues to conform to the changes in the prices of their goods, they print the list price, usually higher than the market price, and then regulate their charges by allowing discounts. If the market price increases, the discount allowed is less; if the price decreases, the discount is increased, usually by allowing a second discount to be deducted after the first discount has been deducted. Discounts of this kind are called **trade** or **commercial discounts.**

356. An additional trade discount is frequently allowed on large orders. Two or more discounts allowed on an invoice are called a **discount series.** The "series" is usually spoken of as a "discount."

357. Dealers doing business on a wholesale basis generally allow terms of credit of 30 da., 60 da., or 90 da. A **time discount** will be allowed, in most cases, for the payment of a bill before the term of credit expires.

A payment within 10 da. is generally considered as "cash" terms.

358. The **terms** of credit and time discount, if any, are printed or written on the billheads of business houses.

359. The **gross amount** of a bill is the amount before any discounts are deducted.

360. The **net amount** of a bill is the amount to be paid after the discounts are deducted.

361. Trade discount rates are frequently aliquot parts of 100%.

362. The order in which the discounts are deducted is immaterial, as illustrated in the following example.

Find the net amount of a bill of $640 less 33⅓%, 25%, and 10%.

3)$640	Or	4)$640	
213.33 = 33⅓%		160 = 25%	
4) 426.67		3) 480	
106.67 = 25%		160 = 33⅓%	
10) 320		10) 320	
32 = 10%		32 = 10%	
$288 Net Amount.		$288 Net Amount.	

The first discount is deducted from the gross amount or list price of the bill. The second discount is taken from the amount remaining after the first discount is deducted. So also the third discount is taken from what is left after subtracting the second. The net amount is $288.

By changing the order of the discounts and deducting the 25% first, the work is made a little shorter as it avoids discounts of dollars and cents, due to the fact that $640 is exactly divisible by 4, and not by 3, and that the remainder, $426.67, is not exactly divisible by 4, while the remainder, $480, in the second solution, is exactly divisible by 3. The net amount in either case is the same.

Practice alone will enable the student to determine when it is advantageous to change the order of the rates.

363. The following bill illustrates the discounts and the terms of the sale:

St. Louis, Mo., July 30, 1910.

Mr. J. C. Flint,
Kansas City, Mo.

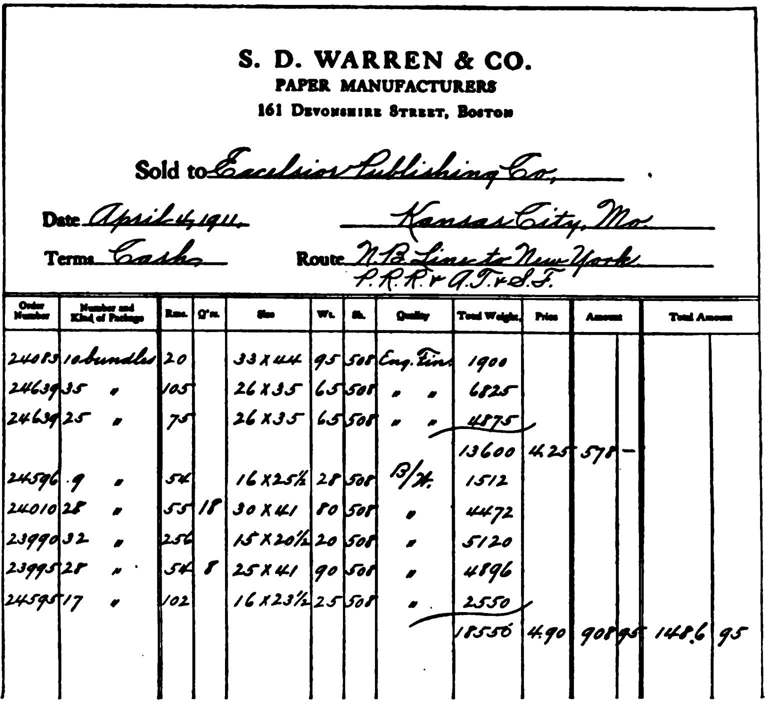
Bought of The Hoyt Hardware Company.

Terms: 60 days; 5% 10 days.

3	Doz. Chisels	$5.60	16	80		
2	" Handsaws	7.50	15			
4	" Hammers	4.80	19	20		
			51			
	Less 25%		12	75	38	25
	Less 5%				1	91
					36	34
	Received payment, Aug. 9, 1910, The Hoyt Hardware Company. Per H.					

Mr. Flint, instead of waiting 60 da. to make payment, has paid the bill within 10 da., thus securing the benefit of the time discount of 5%. Had he waited 60 da. before making payment, he would have paid $38.25.

MODEL INVOICE (PAPER)



S. D. WARREN & CO.
PAPER MANUFACTURERS
161 DEVONSHIRE STREET, BOSTON

Sold to Excelsior Publishing Co.

Date April 4, 1911. Kansas City, Mo.

Terms Cash. Route N.B. Line to New York P.R.R. & A.T. & S.F.

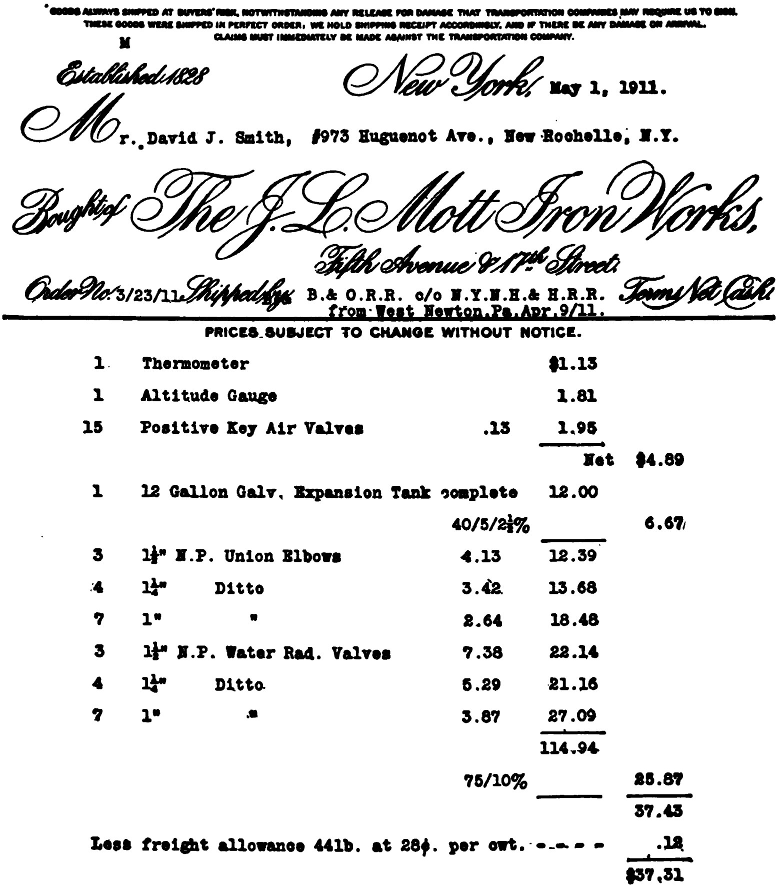
Order Number	Number and Kind of Package	Rms.	Q'rs.	Size	Wt.	Sh.	Quality	Total Weight	Price	Amount		Total Amount	
24085	10 bundles	20		33 x 44	95	508	Eng. Fin.	1900					
24639	35 "	105		26 x 35	65	508	" "	6825					
24639	25 "	75		26 x 35	65	508	" "	4875					
								13600	4.25	578	—		
24596	9 "	54		16 x 25½	28	508	B/H.	1512					
24010	28 "	55	18	30 x 41	80	508	"	4472					
23990	32 "	256		15 x 20½	20	508	"	5120					
23995	28 "	54	8	25 x 41	90	508	"	4896					
24595	17 "	102		16 x 23½	25	508	"	2550					
								18550	4.90	908	95	1486	95

Prove the correctness of the results in the above invoice.

364. Find the net price of each of the following:

LIST PRICE	DISCOUNT	LIST PRICE	DISCOUNT	LIST PRICE	DISCOUNT
1. \$ 8	$12\frac{1}{2}$%	**11.** \$1.25	20%	**21.** \$60	20% and $16\frac{2}{3}$%
2. 10	20%	**12.** 1.50	$16\frac{2}{3}$%	**22.** 75	20% and $33\frac{1}{3}$%
3. 12	25%	**13.** 3.60	25%	**23.** 80	25% and 10%
4. 6	$33\frac{1}{3}$%	**14.** 4.50	10%	**24.** 120	$33\frac{1}{3}$% and 25%
5. 9	50%	**15.** 48	$18\frac{3}{4}$%	**25.** 150	20% and 10%
6. 14	$14\frac{2}{7}$%	**16.** 45	$11\frac{1}{9}$%	**26.** 240	20% and $12\frac{1}{2}$%
7. 16	$6\frac{1}{4}$%	**17.** 72	$8\frac{1}{3}$%	**27.** 540	50% and 20%
8. 20	10%	**18.** 50	4%	**28.** 840	$37\frac{1}{2}$% and 20%
9. 24	$16\frac{2}{3}$%	**19.** 60	3%	**29.** 960	40% and $16\frac{2}{3}$%
10. 30	$6\frac{2}{3}$%	**20.** 75	8%	**30.** 1200	50% and 25%

365. The following illustration shows the form of invoice used by The J. L. Mott Iron Works. Verify the results given.



GOODS ALWAYS SHIPPED AT BUYERS' RISK, NOTWITHSTANDING ANY RELEASE FOR DAMAGE THAT TRANSPORTATION COMPANIES MAY REQUIRE US TO SIGN.
THESE GOODS WERE SHIPPED IN PERFECT ORDER, WE HOLD SHIPPING RECEIPT ACCORDINGLY, AND IF THERE BE ANY DAMAGE ON ARRIVAL,
CLAIMS MUST IMMEDIATELY BE MADE AGAINST THE TRANSPORTATION COMPANY.

M

Established 1828 New York, May 1, 1911.

Mr. David J. Smith, #973 Huguenot Ave., New Rochelle, N.Y.

Bought of The J. L. Mott Iron Works,
Fifth Avenue & 17th Street.

Order No. 3/23/11 Shipped by B.& O.R.R. c/o N.Y.N.H.& H.R.R. from West Newton, Pa. Apr. 9/11. Terms Net Cash

PRICES SUBJECT TO CHANGE WITHOUT NOTICE.

1	Thermometer		$1.13	
1	Altitude Gauge		1.81	
15	Positive Key Air Valves	.13	1.95	
			Net	$4.89
1	12 Gallon Galv. Expansion Tank complete		12.00	
		40/5/2½%		6.67
3	1½" N.P. Union Elbows	4.13	12.39	
4	1¼" Ditto	3.42	13.68	
7	1" "	2.64	18.48	
3	1½" N.P. Water Rad. Valves	7.38	22.14	
4	1¼" Ditto	5.29	21.16	
7	1" "	3.87	27.09	
			114.94	
		75/10%		25.87
				37.43
	Less freight allowance 44lb. at 28¢. per cwt.			.12
				$37.31

366. Make out a bill for each of the following purchases:

1. A. C. Bolton, Cincinnati, Ohio, bought of the Rochester Hardware Co., Rochester, N.H., Sept. 1, 1910, on account 60 da., 3 % 10 da.:

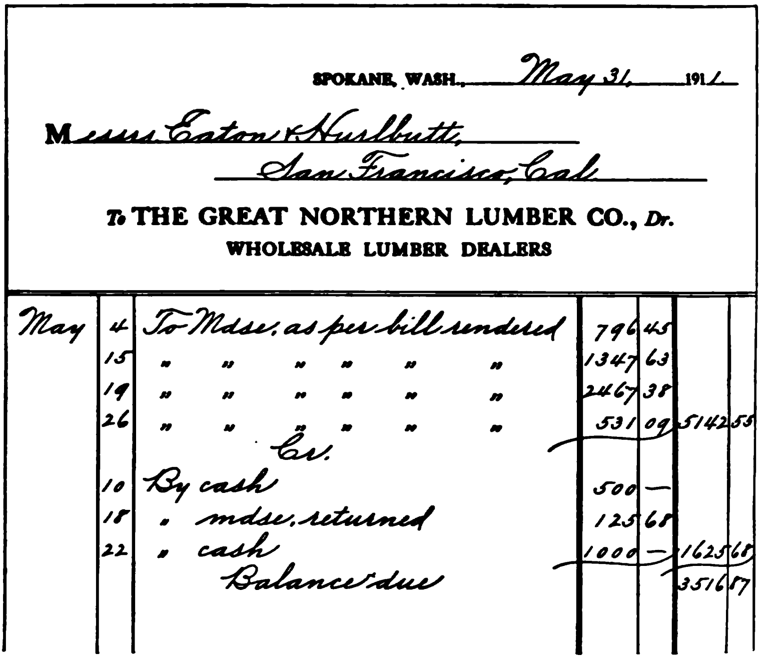
24 doz. C. S. Axes @ $ 10.
12 doz. Handsaws @ $ 14.
Less 20 %.
8 doz. Wrenches @ $ 12.
Less 25 % and 10 %.

2. H. L. Williams, Paterson, N.J., bought of Marshall Field, Chicago, Ill., Dec. 14, 1910, on account 30 da., 1 % 10 da.:

8 Wilton Rugs,	8′ 3″ × 10′ 6″	@	$ 35.
6 Velvet Rugs,	9′ × 12′	@	45.
210 yd. Brussels Carpet		@	1.25.

A discount of 16⅔ % and 10 % is allowed on the entire bill. Find amount due Dec. 24, 1910, and receipt the bill.

MODEL STATEMENT



SPOKANE, WASH. May 31, 1911

Messrs Eaton & Hurlbutt,
San Francisco, Cal.

To THE GREAT NORTHERN LUMBER CO., *Dr.*
WHOLESALE LUMBER DEALERS

May	4	To Mdse. as per bill rendered	796	45		
	15	" " " " " "	1347	63		
	19	" " " " " "	2467	38		
	26	" " " " " "	531	09	5142	55
		Cr.				
	10	By cash	500	—		
	18	" mdse. returned	125	68		
	22	" cash	1000	—	1625	68
		Balance due			3516	87

3. B. S. Shenkman & Co., Wheeling, W. Va., bought of Norfolk Grocery Co., Norfolk, Va., Dec. 1, 1910, on account 90 da., 2 % 30 da., 5 % 10 da.:

20 mats Java Coffee, 1500,	@ 25 ¢.
30 bags Maracaibo Coffee, 3750,	@ 20 ¢.
30 bags Rio Coffee, 3750,	@ 15 ¢.

A discount of 15 % and 10 % is allowed on the above.

25 half-chests Japan Tea,	1875 lb.,	@ 25¢.
30 half-chests Oolong Tea,	1500 lb.,	@ 45¢.
25 half-chests Eng. Breakfast Tea,	1500 lb.,	@ 30¢.

A discount of 20 % and 10 % is allowed on the tea. Find net amount of the bill if paid on Dec. 11, 1910.

4. A. J. Cammeyer, New York, bought of New England Shoe Co., Boston, Mass., Sept. 18, 1910, on account 30 da., 5 % cash:

100 cases* Patent Calf,	3600 pr.,	@ $2.50.
80 cases Tan Calf,	2880 pr.,	@ 2.25.
60 cases Kangaroo,	2160 pr.,	@ 2.25.
120 cases French Kid,	4320 pr.,	@ 3.50.

If a discount of 25 % and 12½ % is allowed on the entire bill, find amount required to pay the bill at once. Receipt the bill.

5. Find the net amount of each of the following:

160 yd. Silk	@ $3.50.
120 yd. Broadcloth	@ 2.25.
240 yd. Duchess Satin	@ 2.50.
80 yd. French Crépon	@ 1.75.

Discount of 33⅓ %, 12½ %, and 5 %.

6. 18 grindstones at $2.25; 24 ice cream freezers at $4.50; 4 doz. jack planes at $7; and 12 doz. mortise locks at $3.50. Discount, 37½ %, 25 %, and 10 %.

7. 24 boxes lemons at $4.50; 30 boxes oranges at $5.40; 18 doz. pineapples at $1.25; 20 bunches bananas at $1.25. Discount 20%, 12½ %.

8. 1800 yd. denim at 12¢; 1400 yd. cashmere at 50¢; 2000 yd. kersey at $1.10; 3000 yd. storm serge at 62½¢. Discount 20 % and 12½ %.

9. 360 yd. Brussels carpet at $1.20; 800 yd. ingrain carpet at 75¢; 1200 sq. yd. linoleum at 62½¢; 900 yd. moquette carpet at $1.25. Discount 22½ % and 5 %.

10. One dealer offers a piano at $650, less 33⅓ % and 20 %. Another dealer offers the same grade of piano for $700 less 37½ %, 20 %, and 5 %. Which is the better offer, and how much?

---

* A case contains 36 pr. of shoes.

11. Which is the better offer, and how much, on a bill amounting to $1480, a discount series of 33⅓ %, 25 %, and 20 %, or a series of 40 %, 20 %, and 12½ %?

12. A bill of $1260.50 is subject to discounts as follows: $450 of the bill to 25 % and 12½ %; $375.60 to 37½ % and 8⅓ %; and the remainder of the bill to a discount of 16⅔ %. Find the net amount of the bill.

13. Find the net amount of a bill of the following amounts: $80 less 25 % and 20 %; $126.50 less 16⅔ % and 12½ %; and $172.75 less 62½ % and 25 %. The whole bill is subject to a cash discount of 5 %.

367. To find a single discount equal to a discount series.

Find a single rate of discount equivalent to discounts of 10 % and 10 %.

$$\begin{array}{l r l}
\text{Let} & 1.00 & = \text{the list price.}\\
\text{then} & \underline{.10} & = \text{the first discount.}\\
 & .90 & = \text{remainder after discount is deducted.}\\
 & \underline{.09} & = \text{the second discount (10 \% of .90).}\\
 & .81 & = \text{the net amount.}
\end{array}$$

1.00 − .81 = .19 = 19 %, the rate of discount equivalent to a discount series of 10 % and 10 %.

$$\begin{array}{l r l}
\text{Or,} & \underline{.10 \quad .10} & 1.00 \text{ list price.}\\
 & .90 \times .90 = & \underline{.81} \text{ net amount.}\\
 & & .19 \text{ single discount equal to a series of 10 \% and 10 \%.}
\end{array}$$

In finding a single rate of discount equivalent to a discount series, 100 % may always be used as the list price or basis of the computation. Deducting the two discounts of 10 % and 10 % gives the net amount of 81 %. The equivalent single rate, then, must be the difference between the list price and the net price; Hence, 100 % − 81 % gives 19 %, the single rate desired.

In the second solution, the several rates of the series are written above a horizontal line and each is deducted from 100 %, the difference being written below the line. Multiplying the differences (.90 and .90) together gives .81 or 81 %, as the net amount. Deducting 81 % from 100 % gives 19 %, the single rate equal to the series 10 % and 10 %.

This method may be used with any number of discounts. For its application to the solution of problems, see page 173.

368. Find a single rate of discount equal to each of the following series:

1. 33⅓ % and 25 %.
2. 40 % and 16⅔ %.
3. 37½ % and 20 %.
4. 25 % and 20 %.
5. 33⅓ % and 10 %.
6. 30 % and $14\frac{2}{7}$ %.
7. 30 % and 20 %.
8. 40 % and 10 %.
9 20 % and 20 %.
10. 20 % and 10 %.
11. 20 % and 12½ %.
12. 16⅔ % and 10 %.
13. 50 % and 40 %.
14. 50 % and 30 %.
15. 50 % and 20 %.
16. 50 % and 10 %.
17. 40 % and 40 %.
18. 40 % and 30 %.
19. 40 % and 20 %.
20. 40 % and 5 %.
21. 37½ % and 10 %.
22. 37½ % and 5 %.
23. 35 % add 20 %.
24. 35 % and 10 %.
25. 30 % and 30 %.
26. 30 % and 10 %.
27. 25 % and 25 %.
28. 25 % and 10 %.
29. 12½ % and $14\frac{2}{7}$ %.
30. 15 % and 10 %.
31. 10 % and 10 %.
32. 10 % and 5 %.
33. 20 % and 5 %.
34. 60 % and 12½ %.
35. 60 % and 25 %.
36. 60 % and 10 %.
37. 60 % and 30 %.
38. 60 % and 40 %.
39. 60 % and 50 %.
40. 55 % and 33⅓ %.
41. 11⅑ % and 12½ %.
42. 75 % and 20 %.
43. 25 %, 20 %, and 16⅔ %.
44. 33⅓ %, 20 %, and 6¼ %.
45. 20 %, 12½ %, and $14\frac{2}{7}$ %.
46. 20 %, 20 %, and 12½ %.
47. 25 %, 20 %, and 10 %.
48. 20 %, 10 %, and 5 %.
49. 37½ %, 20 %, and 10 %.
50 40 %, 20 %, and 16⅔ %.

PROBLEMS

369. **1.** Find the net cost of goods listed at $ 480, less 25 %, 20 %, and 16⅔ %.

$$\frac{\$\overset{80}{\cancel{480}} \times 3 \times \cancel{4} \times \cancel{5}}{\cancel{4} \times \cancel{5} \times \cancel{6}} = \$240.$$

By deducting each rate in the series from 100% and reducing the remainders to their fractional equivalents, and placing the resulting fractions as shown in the solution, the problem resolves itself into one of simple cancellation. In many cases this is a very practical solution.

2. Which is the better for the purchaser, and how much per cent, a series of 30 %, 20 %, and 10 %, or a straight discount of 50 %? How much on a bill of $ 325?

3. A bedroom suite is invoiced at $ 420 less 20 % and $14\frac{2}{7}$ %. What is the net price?

4. A gentleman bought an automobile for $2400 less $12\frac{1}{2}$%, 10%, and 5%. What was the net cost?

5. Goods listed at $128 are sold subject to a discount of 75%, $12\frac{1}{2}$%, and 5%. Find the net price.

6. You are offered 400 suits of ready-made clothing at $18 a suit, less 20% and 5%, by A, and at $16 a suit, less 10%, by B. If you buy of A, you pay the freight, $4.50. B offers to pay the freight. Which offer is better, and how much?

7. The list price of plows is $12. If you buy them at that price less $33\frac{1}{3}$% and 10%, and sell them at the same list price less 20% and 5%, how much do you make on 48 plows?

8. I can buy a bathroom outfit of one dealer for $150, less $33\frac{1}{3}$% and 20%. I can get the same outfit of another dealer for $125, less 25% and $12\frac{1}{2}$%. The terms in each case are net 30 da., 2% cash. What is the least amount of ready money for which I can get the outfit?

9. A merchant bought goods at 30% and 10% less than the list price, and sold them at the same list price less 10%. Find his profit on goods listed at $1000.

10. An overcoat in a clothier's windows is marked "Was $27.50. Now $21." What is the rate of discount?

11. The merchants in a certain town gave a ticket of admission to the county fair costing fifty cents to each customer who purchased $3.50 worth of merchandise. This was equivalent to what rate of discount?

12. The fare one way between two cities is $48.75. If a round-trip ticket costs $85, what is the rate of discount?

PROFIT AND LOSS

370. If eggs are bought at 25¢ a dozen and sold at 30¢ a dozen, what is the profit? The profit is what part of the cost? what per cent of the cost?

If oranges cost 30¢ a dozen and are sold to gain $33\frac{1}{3}$% of the cost, how much is gained per dozen? What is the selling price per dozen?

A building lot cost $400. If it is sold so as to gain 25 %, what is the selling price?

A book cost $4. It was damaged by water and was sold at a loss of $\frac{1}{4}$ of the cost. What was the loss? the selling price?

371. The subject of profit and loss has for its object the reckoning of gains, losses, costs, and selling prices, which arise in business transactions.

372. The **net** or **prime cost** of an article is the net amount paid for it.

373. The **gross cost** of an article is the total cost including freight, cartage, insurance, commission, etc.

374. The **gross selling price** is the total amount received for the goods sold.

375. The **net selling price** is the difference between the gross selling price and the sum of all the expenses of the sale paid by the seller.

376. The **profit** is the amount which the net selling price is greater than the gross cost.

377. The **loss** is the amount which the gross cost is greater than the net selling price.

378. In profit and loss,

The cost is the **base**;
The rate of gain, or loss, is the **rate**;
The profit, or loss, is the **percentage**; and
The selling price is the **amount or difference.**

379. Problems in profit and loss are solved in accordance with the principles of percentage. In their special application to the subject the principles of percentage are stated as follows:

Cost × rate of gain = gain.
Cost × rate of loss = loss.
Gain ÷ rate of gain = cost.
Loss ÷ rate of loss = cost.
Gain ÷ cost = rate of gain.
Loss ÷ cost = rate of loss.
Cost × (100 % + rate of gain) = selling price.
Cost × (100 % − rate of loss) = selling price.
Selling price ÷ (100 % + rate of gain) = cost.
Selling price ÷ (100 % − rate of loss) = cost.

380. To find the gain or loss and the selling price, the cost and the gain or loss per cent being given.

Find the gain or loss and the selling price in each of the following:

Cost	Rate	Cost	Rate	Cost	Rate
1. $ .15	20 % gain	11. $ 1.10	20 % gain	21. $215.00	20 % gain
2. .20	25 % gain	12. 1.40	20 % loss	22. 330.00	66⅔ % gain
3. .25	40 % loss	13. 4.00	15 % gain	23. 225.50	40 % gain
4. .24	12½ % loss	14. 5.00	30 % gain	24. 177.60	12½ % loss
5. .30	33⅓ % gain	15. 12.00	16⅔ % gain	25. 129.20	75 % gain
6. .48	6¼ % loss	16. 14.00	20 % loss	26. 836.00	50 % gain
7. .60	20 % gain	17. 16.00	2½ % loss	27. 999.00	10 % loss
8. .75	40 % gain	18. 24.00	5 % loss	28. 750.00	12½ % gain
9. .80	18¾ % gain	19. 160.00	37½ % gain	29. 660.00	1⅔ % gain
10. .96	16⅔ % gain	20. 180.00	6⅔ % gain	30. 880.00	1¼ % loss

PROBLEMS

381. 1. A stock of goods valued at $13,500 was damaged by fire and water so as to cause a loss of 30%. What was the loss, and for how much did the goods sell?

2. A piece of real estate costing $2400 increased in value 41⅔%, when it was sold. Find the gain and the selling price.

3. A vegetable dealer buys potatoes at 60¢ a bushel and sells them at a profit of 66⅔%. What is the gain per bushel, and the selling price per peck?

4. A fruiterer bought a carload of berries for $1380. He was obliged to sell them at a loss of 12½%. One retailer who purchased $375 worth of the berries on credit failed in business and paid his creditors 40¢ on the dollar. Find the wholesaler's total loss.

5. A grocer buys sugar at 5¢ a pound. He puts it up in bags of 3½ lb. each. At what price per bag must he sell it to gain $2\frac{6}{7}$%?

6. A grocer buys 6 bags Rio coffee, 125 lb. in a bag, at 12¢ a pound, and 9 bags Java coffee, 75 lb. in a bag, at 20¢ a pound. If he makes a mixture of the two kinds, and sells it at 90% profit, how much does he receive per pound?

7. Find the gain on goods bought for $46.50 less 25 % and sold to gain 25 %.

8. An invoice of merchandise cost $74.25 less 33⅓ % and 10 %. Freight and cartage were $7.45. Find the gain and the selling price if the goods were sold to gain 37½ %.

9. A shoe dealer buys shoes at $3 less 16⅔ % and 10 % per pair. The expenses of the purchase amount to 5¢ a pair. At what price must he sell them to clear 40 % profit if it costs 28¢ per pair, on the average, to sell them?

382. To find the gain or loss per cent, the cost and the gain or loss being given.

Find the gain or loss per cent in each of the following:

	Cost			Cost			Cost	
1.	$ .15	$ .05 gain	**9.**	$ 3.50	$ .70 loss	**17.**	$603.00	$ 201.00 gain
2.	.20	.05 gain	**10.**	5.00	1.50 gain	**18.**	74.50	14.90 loss
3.	.36	.06 loss	**11.**	3.60	.90 gain	**19.**	38.50	3.85 gain
4.	.45	.05 gain	**12.**	4.50	.90 loss	**20.**	94.50	31.50 gain
5.	.50	.05 loss	**13.**	16.00	8.00 gain	**21.**	18	22.50 s. price
6.	.80	.20 gain	**14.**	28.00	4.00 gain	**22.**	27	33.75 s. price
7.	1.00	.20 loss	**15.**	450.00	75.00 gain	**23.**	40	36.00 s. price
8.	2.50	.40 gain	**16.**	425.00	170.00 gain	**24.**	64	80.00 s. price

383. The following cost and sales prices are from a "monthly statement of sales" in a hardware store. Find the rate of gain in each case.

	Cost Price	Sale Price		Cost Price	Sale Price
1.	$16.38	$20.25	**9.**	$ 8.65	$10.00
2.	22.50	29.45	**10.**	41.50	48.75
3.	12.50	15.00	**11.**	25.00	31.25
4.	35.25	41.50	**12.**	54.00	65.50
5.	5.65	7.50	**13.**	2.50	3.20
6.	4.50	6.00	**14.**	23.50	26.65
7.	15.25	20.00	**15.**	21.50	26.00
8.	12.50	15.00	**16.**	15.00	18.50

PROBLEMS

384. 1. Find the rate per cent of profit if goods are bought at \$4.80 less 16⅔% and 12½%, and sold at \$4.80 less 12½%.

\$4.80 × ⅚ × ⅞ = \$3.50, net cost.
\$4.80 × ⅞ = \$4.20, net selling price.
\$4.20 − \$3.50 = \$.70, gain.
\$ .70 ÷ \$3.50 = .20 = 20%, rate of gain.

\$4.80 less the trade discounts of 16⅔% and 12½% equals \$3.50, the net cost. \$4.80 less 12½% = \$4.20, the net selling price. The difference between \$4.20 and \$3.50 = \$.70, the gain. The gain divided by the cost equals the rate of gain.

2. Find the rate of gain when merchandise is bought for \$6.40 less 25% and is sold at \$6.40 net.

3. A sealskin coat cost \$120, less 30% and 16⅔%. It was marked \$100 by the retailer, but in order to effect a sale he gave a 10% discount. What rate of profit did he make?

4. The cost of a book is \$8, less a trade discount of 33⅓% and 10%. If it is sold at the same list price, less 10%, what rate of profit is made?

5. A retailer purchases hats at \$54 per dozen, less 20% and 16⅔%. If he sells them at \$5 each, what rate of profit does he make?

6. A statement of the merchandise account at the close of the year showed the following: Inventory at the beginning of the year, \$3500; purchases \$22,500; sales \$28,000; inventory at the close of the year, \$6000. Make a statement showing the net gain or loss. Find the gain or loss per cent.

Returns		
Merchandise Sales		28000
Cost		
Merchandise, inv. at beginning	3500	
Merchandise purchases	22500	
Merchandise, total cost	26000	
Less inventory at closing	6000	
Net cost of sales		20000
Merchandise gain		8000

\$8000 ÷ \$20,000 = .40 = 40%, rate of profit.

7. At the beginning of the year a merchant had on hand merchandise amounting to $5400. He bought during the year $38,000 worth, and sold $45,000 worth of merchandise. He had on hand at the close of the year an inventory of $7400. Make a statement showing the gain or loss. Find the rate per cent of gain or loss.

8. Find the gain or loss per cent, and make a statement showing the gain or loss from the following facts: Jan. 1, 1909, inventory $7925; Dec. 31, 1909, purchases for the year $53,796.50; sales for the year $54,837.50; Dec. 31, 1909, inventory $1721.50.

9. A paper dealer buys 6 reams of "flat paper," 17" × 22", 28 lb. to the ream, at 8¢ a pound. He cuts the paper into sheets 8½" × 11", and sells it at $1 a ream. Find his rate per cent of gain.

10. We buy goods at $.245 per pound and sell at $.30625. At what price shall we sell to make the same per cent profit, if we have to pay $.288?

385. To find the cost, the gain or loss and the per cent of gain or loss being given.

If a profit of 10¢ a dozen is made on oranges, and the profit is ¼ of the cost, what is the cost?

If a loss of 3¢ a dozen on eggs is a loss of 10% of the cost, what is the cost?

386. Find the cost and the selling price in each of the following:

Gain	Gain	Loss	Loss	Gain	Gain
1. $ .06	25%	**11.** $ 12	25%	**21.** $73.00	$36\frac{1}{2}\%$
2. .07	20%	**12.** 15	$3\frac{1}{3}\%$	**22.** 70.00	$17\frac{1}{2}\%$
3. .15	$33\frac{1}{3}\%$	**13.** 18	$2\frac{1}{2}\%$	**23.** 38.00	10%
4. .25	$12\frac{1}{2}\%$	**14.** 30	75%	**24.** 56.00	$87\frac{1}{2}\%$
5. .45	$16\frac{2}{3}\%$	**15.** 160	$31\frac{1}{4}\%$	**25.** 25.00	$41\frac{2}{3}\%$
6. .75	$6\frac{1}{4}\%$	**16.** 220	5%	**26.** 36.00	$42\frac{6}{7}\%$
7. 1.25	$14\frac{2}{7}\%$	**17.** 28	$33\frac{1}{3}\%$	**27.** 12.00	$22\frac{2}{9}\%$
8. 2.40	$37\frac{1}{2}\%$	**18.** 29	$14\frac{2}{7}\%$	**28.** 1.25	$2\frac{1}{2}\%$
9. 3.20	50%	**19.** 38	$66\frac{2}{3}\%$	**29.** 2.50	$1\frac{1}{4}\%$
10. 3.60	$56\frac{1}{4}\%$	**20.** 54	9%	**30.** 37.50	$3\frac{1}{3}\%$

PROBLEMS

387. 1. A house rents for $45 per month, and the expenses for taxes, insurance, repairs, etc., are $150 a year. What price can a man afford to pay for it so that he may have a 6½ % investment?

2. A merchant sold a sideboard at a gain of 25 %, which was a gain of $18. When the merchant bought the sideboard, he was allowed discounts of 20 % and 10 % from the list price. Find the net cost and the list price.

3. A stock of goods was damaged 15 % by fire and water. If the loss was $12,000, what was the cost of the goods? What was the selling price?

4. A speculator bought a lot and sold it so as to gain 20 %. He immediately bought a second lot with the selling price of the first, which he afterward sold at a profit of 16⅔ %. If his entire gain was $1400, how much did he pay for each lot?

5. In selling a quantity of oranges and pineapples a huckster gained $12.50, gaining 25 % on the oranges, but losing 10 % on the pineapples. If the loss on the pineapples was 20 % of the net gain, find the cost of each.

6. I bought a house for 10 % below its estimated value and sold it for 10 % more than its estimated value. My profit was $800. How much did I pay for the house? What rate of profit did I make?

7. If you buy at $.485 per pound less 25 % and 12½ %, and sell at the same list price less 16⅔ % and 10 %, what discount can you allow so that you can have the same rate of profit if you have to buy at $.485 less 20 % and 10 %?

388. To find the cost, the selling price and the rate per cent of gain or loss being given.

If an article sells for 25¢, what is the cost if 5¢ is gained? The gain is what part of the cost? What per cent is gained?

If 25¢ is the selling price and 25 % is gained, the selling price is how many per cent *more* than the cost? The selling price is what per cent of the cost? How may the cost be determined?

Let 1.00 of the cost = the cost.
 .25 of the cost = the gain.
then 1.25 of the cost = the selling price.
1.25 × ? = $.25; hence, $.25 ÷ 1.25 = $.20, the cost.

389. Find the cost and the gain or loss in each of the following:

Selling Price	Rate	Selling Price	Rate	Selling Price	Rate
1. $ .36	10 % loss	**13.** $ 6.40	25 % gain	**25.** $ 660	20 % gain
2. .36	20 % gain	**14.** 18.00	$33\frac{1}{3}$ % loss	**26.** 770	$16\frac{2}{3}$ % gain
3. .48	20 % loss	**15.** 22.50	25 % loss	**27.** 1100	$8\frac{1}{3}$ % loss
4. .75	25 % gain	**16.** 37.50	25 % gain	**28.** 1400	$12\frac{1}{2}$ % loss
5. .80	$14\frac{2}{7}$ % gain	**17.** 31.00	$3\frac{1}{3}$ % gain	**29.** 3000	$11\frac{1}{9}$ % gain
6. .90	$12\frac{1}{2}$ % gain	**18.** 39.00	$2\frac{1}{2}$ % loss	**30.** 3200	$11\frac{1}{9}$ % loss
7. 1.05	$16\frac{2}{3}$ % gain	**19.** 59.00	$1\frac{2}{3}$ % loss	**31.** 4250	25 % gain
8. 1.05	$12\frac{1}{2}$ % loss	**20.** 162.00	$1\frac{1}{4}$ % gain	**32.** 1650	20 % gain
9. 1.40	30 % loss	**21.** 182.00	4 % gain	**33.** 1440	$14\frac{2}{7}$ % gain
10. 2.25	50 % gain	**22.** 672.00	$6\frac{2}{3}$ % gain	**34.** 1100	10 % loss
11. 3.75	$16\frac{2}{3}$ % loss	**23.** 680.00	$6\frac{1}{4}$ % gain	**35.** 3800	5 % loss
12. 4.50	20 % gain	**24.** 380.00	$18\frac{3}{4}$ % gain	**36.** 4240	6 % gain

PROBLEMS

390. 1. A real estate dealer sold property for $5075, gaining $16\frac{2}{3}$ %. How much did the property cost?

$$\$5075 \div 1.16\tfrac{2}{3} = \$4350,\ \text{cost, or}\quad 7)\,\$5075 \qquad \begin{array}{r} 725 \\ \hline \$4350\ \text{Cost.} \end{array}$$

The cost is the base, or 100 %. The selling price, being at a gain, is $16\frac{2}{3}$ % *more* than the cost, or $116\frac{2}{3}$ % of the cost. Hence, dividing $5075 by $116\frac{2}{3}$ % gives the cost, or $4350. Or,

Since $16\frac{2}{3}$ % is equal to $\frac{1}{6}$, and the cost is equal to unity, or $\frac{6}{6}$, the selling price is equal to $\frac{7}{6}$ (of the cost). Dividing $5075 by 7 gives $\frac{1}{6}$ of the cost. $\frac{1}{6}$ of the cost ($725) deducted from $\frac{7}{6}$ of the cost ($5075) equals $\frac{6}{6}$ of the cost, or the cost itself.

This method, once mastered, is a great time and labor saver. Compare the amount of work required with that of the actual division in the first solution.

2. A grocer sells eggs at 32 ¢ a dozen and gains $6\frac{2}{3}$ %. How much do the eggs cost him?

3. Find the cost of an article sold for $45, at a loss of $16\frac{2}{3}$ %.

4. A hatter marked a hat $6. To sell it he gave a discount of 20 %. His profit was 20 %. Find the cost.

5. At what price must I buy so that I can mark goods at $7.20, allow discounts of $16\frac{2}{3}$ % and 10 %, and still make a profit of $33\frac{1}{3}$ %?

6. A merchant's selling price of an article is $65, less 20%. If he buys at the same list price ($65), what discount in addition to a discount of 25% must he be allowed in order to clear a profit of 33⅓%?

7. A manufacturer's profit on a bicycle was 20%; the wholesaler's profit was 16⅔%, and the retailer's profit was 25%. If the retailer's selling price is for $35, what was the first cost?

8. Two farms were sold for $4800 each. On one 20% was gained, and on the other 20% was lost. Find the net gain or loss, and the gain or loss per cent.

9. Find the cost per dozen of handkerchiefs that retail at 50¢ each if a profit of 75% is made.

10. What should be the buying list price of an article bought at discounts of 25% and 16⅔%, if when sold at $64 less 25% and 16⅔% a profit of 33⅓% is realized?

391. PROFIT AND LOSS DRILL CHART

	Cost	% Gain	% Loss	Gain	Loss	Selling Price	Selling Price
	1	2	3	4	5	6	7
1.	$ 16.00	.12½	.10	$ 4.00	$ 1.00	$ 18	$ 12
2.	18.00	.16⅔	.11⅑	6.00	2.00	24	15
3.	22.50	.33⅓	.16⅔	3.75	11.25	30	15
4.	28.00	.25	.12½	4.00	7.00	30	20
5.	36.00	.11⅑	.25	12.00	3.00	42	30
6.	48.00	.08⅓	.16⅔	4.00	2.00	64	36
7.	62.50	.20	.10	25.00	12.50	75	50
8.	96.00	.12½	.37½	6.00	24.00	128	80
9.	120.00	.37½	.06⅔	15.00	12.00	160	108
10.	150.00	.33⅓	.04	15.00	7.50	210	141
11.	240.00	.15	.06¼	15.00	24.00	280	228
12.	360.00	.05	.03⅓	54.00	18.00	450	324
13.	500.00	.24	.03	25.00	10.00	575	490
14.	750.00	.18	.12	7.50	45.00	900	720
15.	900.00	.30	.06⅔	135.00	72.00	1080	837
16.	1200.00	.08⅓	.02½	180.00	84.00	1400	1056
17.	1500.00	.06⅔	.06	300.00	45.00	1550	1350
18.	1800.00	.22$\frac{2}{9}$	.02	450.00	90.00	2100	1500

1. Combine columns 1 and 2, and find selling price.
2. Combine columns 1 and 3, and find selling price.
3. Combine columns 1 and 4, and find gain per cent.
4. Combine columns 1 and 5, and find loss per cent.
5. Combine columns 1 and 6, and find gain per cent.
6. Combine columns 1 and 7, and find loss per cent.
7. Combine columns 2 and 4, and find cost and selling price.
8. Combine columns 3 and 5, and find cost and selling price.
9. Combine columns 4 and 6, and find gain per cent.
10. Combine columns 5 and 7, and find loss per cent.

REVIEW OF PROFIT AND LOSS

MENTAL

392. 1. If milk costs 12¢ a gallon, what per cent is gained by selling it at 6¢ a quart?

2. If eggs cost 25¢ a dozen, how many can a grocer sell for 25¢ if he gains 20%?

3. When potatoes cost wholesale 48¢ a bushel, what per cent profit does a retailer make who sells them at 15¢ a basket (8 baskets to the bushel)?

4. An article that cost $4 sells for $5. Will it pay to reduce the selling price 10%, if by so doing there are three times as many sales daily?

5. A dining table cost $20. Find the selling price to gain 20%; 25%; 15%; 10%; 40%.

6. An article cost $12. Find selling price to gain 20% of the selling price; 25% of the selling price; to lose 20% of the selling price; 25% of the selling price.

7. The cost is $48 per dozen. Find selling price of 1 to gain 25%; of 2 to gain 15%; of 3 to gain 20%; of 2 to lose 5%; of 6 to lose 10%.

8. If chairs cost $36 per dozen and 4 of them sell at $4.50 each, and the remainder sell at $3.75 each, what is the rate of profit?

9. You buy an article for $4.50 and sell it for $6. If the cost drops to $4.20, at what price will you sell to make the same rate of profit?

10. A firm has uncollected accounts of $2400. In closing their books for the year, they allow 5% for bad debts. How much do they allow for bad debts?

11. An article which cost $2.50 is sold to gain 20%. Find the gain on 50 such articles.

12. A dealer bought handsaws at $24 a dozen, less 16⅔%. What rate of profit does he make if he sells them at $2.50 each?

13. If coal costs $5 a ton wholesale, and sells at 30% above that price, what is the gain on 1000 T., if it costs 50¢ a ton to deliver it?

14. The raw material in a rocking chair costs $3.50, the labor of making costs $8.50. If the chair sells for 25% profit, for how much does it sell?

15. An American bought a horse in Canada for $130. He brought him to the United States, paying the duty,* $30, and sold him for $200. What per cent did he gain?

16. The man (in problem 15) returned to Canada and bought another horse with the $200 and paid the duty, 25%. At what price must he sell this horse to gain 20% on total cost?

17. If apples cost 90¢ a bushel, and 10% of them spoil, at what price per bushel must the remainder be sold to gain 20% on the entire cost?

18. A grocer buys 30 doz. eggs at 20¢ a dozen. If, by an accident, 20% of them are broken, what per cent does he gain or lose by selling the remainder at 25¢ a dozen?

19. I buy a carpet at $1.20 a yard, less 25%. What rate of profit do I make if I sell it at $1.20 net?

20. A room is 20′ × 15′, and requires 360 board feet of matched lumber to lay the floor. What is the percentage of waste?

21. 10 lb. of 15¢ coffee are mixed with 10 lb. at 20¢. At what price per pound should it sell to gain 20%?

---

* Duty on horses valued at $150 or less, $30 a head; on horses valued at more than $150, 25% of their value. (Tariff Law, 1909.)

22. Equal quantities of 30¢ and 40¢ tea are mixed. Find selling price to gain 40%.

23. A 40¢ tea is mixed with a 70¢ grade in the ratio of 2 to 1. Find the per cent of profit if the mixture is sold at 65¢.

24. A clerk was told to mix Mocha at 24¢ with Java at 40¢, using 3 lb. of Mocha to 1 lb. of Java. By mistake he used 3 lb. of Java to 1 lb. of Mocha. Find the loss or gain on 100 lb. of the mixture, sold at 35¢.

PROBLEMS

393. 1. A merchant bought 440 yd. of cloth for $1100, less 20% and 10%. At what price per yard must he sell the cloth to gain $33\frac{1}{3}\%$?

2. Hats cost $45 a dozen, less 20% and $16\frac{2}{3}\%$. At what price should they retail to gain 40%?

3. If a wine merchant mixes 24 gal. of wine costing $2.25 a gallon with 6 gal. of water, and sells the mixture at $2.50 a gallon, what is his rate of profit?

4. I bought goods for $435, less 20%, $12\frac{1}{2}\%$, 10%. I sold them for $560, less 25% and $16\frac{2}{3}\%$. What per cent of profit did I make?

5. If a desk costs $18 less $16\frac{2}{3}\%$ and 10%, how many desks can be bought for $270?

6. A fruit dealer bought 60 doz. oranges for $12. He sold 40 doz. at 30¢ a dozen. Of those remaining 5 doz. spoiled. At what price must the rest be sold to gain 45% on the cost of all?

7. A man bought a horse and sold it to gain 25%. With the proceeds he bought a second horse and sold it at a loss of 12%. If his gain on both was $22.50, how much did he pay for the first horse?

8. If 75% of a stock of goods is sold for what the entire stock cost, what is the rate of gain? If the remainder of the goods is sold at cost, what is the rate of gain on the whole stock?

9. At what per cent of the list price should goods be marked that are bought at discounts of 25% and 20%, if a profit of $33\frac{1}{3}\%$ is to be realized?

10. An article cost $24, less 20%, $16\frac{2}{3}\%$, and $6\frac{1}{4}\%$. What is the retailer's price mark if he marks it to gain 40%?

11. At what price shall each of the items in the following bill be sold to gain 25 %?

BOSTON, MASS., Aug. 3, 1910.

MR. JAMES DOYLE,
Pittsfield, Mass.

BOUGHT OF WELLS, FOWLER & CO.

Terms: Net 60 days.

6	Roll Top Desks	$45.00	270			
8	Flat Top Desks	25.00	200			
12	China Closets	37.50	450			
9	Sideboards	62.50	562	50		
12	Dining Tables	25.00	300			
12	Sets Dining Chairs	20.00	240			
			2022	50		
	Less 25 % and 20 %		809		1213	50

SUGGESTION. Find at what per cent of the list price the goods must be sold to make the required profit.

12. Find the net amount of the following items, and determine a selling price mark for each, if sold to gain $33\frac{1}{3}$ %.

2 doz. Smoothing Planes @ $10.50, less $14\frac{2}{7}$ %
4 doz. Screwdrivers @ $5.40, less 25 %
200 Carriage Bolts, each $\frac{3}{8}$" × 2", @ $5.60
$\frac{3}{8}$" × $2\frac{1}{2}$", @ $5.80
$\frac{1}{2}$" × $4\frac{1}{2}$", @ $6.25,
less $37\frac{1}{2}$ % and 20 %.
400 Machine Bolts, $\frac{3}{4}$" × 6", @ $8.40,
less 50 % and 10 %.

13. I bought 60 bbl. of apples at $2.25 per barrel. Upon sorting them, I found there were 35 bbl. first grade and 21 bbl. second grade, the rest being decayed. The first grade sold at $3.50, and the second grade at $2.15. If it cost 10 ¢ a barrel to sort, find the gain or loss per cent.

14. By marking goods at $4.50 each, a merchant sold on the average 4 of the articles per day. Could he afford to reduce the price to $4 if by so doing the sales increased to 8 articles a day, the cost being $3.25? What per cent would his profits be increased or decreased each day?

15. Dining tables selling for $16.50 at the rate of 6 a day are reduced to $14, whereupon the sales increase to 10 a day. If the tables cost $10.50, does the merchant gain or lose, and what per cent, by the cut in the price?

16. At a sale the price of a chiffonier was reduced from $28 to $21. The regular sales were 3 a day. During the sale there were 12 sales a day. If the cost was $16.80, were the daily profits increased or decreased, and what per cent?

17. At what price should goods costing $216 be marked to net a profit of 25 % after allowing a discount of 25 %, 20 %, and 10 %?

18. If coal can be bought at $4.75 per long ton and the cost of handling is 80¢ a short ton, what rate of profit is realized if the coal is sold at $6.50 per short ton?

19. At the beginning of a certain year a firm had on hand $16,800 worth of merchandise. They bought during the year $47,500 worth, and paid freight on it amounting to $700. They sold $62,500 worth of merchandise and paid freight on it to the amount of $850. If at the close of the year they had an inventory of $19,500, what rate of profit did they make? Prepare a statement showing all essential facts of the merchandise account.

Find the proprietor's gain or loss per cent in each of the following:

20. PROPRIETOR

1910					1910				
Jan.	1	Liabilities	4876	50	Jan.	1	Resources	18725	
Dec.	31	Profit & Loss	2562	76					

21. PROPRIETOR

1910					1910				
Mar.	1	To withdrawal	1000		Jan.	1	By investment	8000	
Oct.	1	To withdrawal	3000		June	1	By investment	4000	
					July	1	By investment	2000	
					Dec.	1	By profit & loss	3475	

Find the gain or loss per cent in each of the following:

22.	Mdse. purchases	$13,250.00
	Mdse. sales	12,687.50
	Mdse. inventory at closing	4893.80

23.	Mdse. inventory at beginning	$ 3890.00
	Mdse. purchases	10,860.00
	Mdse. sales	9640.00
	Mdse. inventory at closing	6460.00

24.	Jan. 1, 1910. Inventory	$ 3200.00
	Purchases during the year	9600.00
	Sales during the year	10,500.00
	Returned to creditors	450.00
	Returned by debtors	600.00
	Dec. 31, 1910. Inventory	4500.00

MARKING GOODS

394. It is necessary in many lines of business to have the selling price of goods marked on them. This process requires careful, accurate calculation, otherwise merchandise may be marked incorrectly and sold at a loss.

395. Find the selling price of each of the following:

	Cost	Rate of Gain		Cost	Rate of Gain		Cost	Rate of Gain
1.	$12.00	25%	**7.**	$22.50	$33\frac{1}{3}\%$	**13.**	$ 1.20	$16\frac{2}{3}\%$
2.	6.40	$18\frac{3}{4}\%$	**8.**	15.00	20%	**14.**	1.50	20%
3.	4.50	20%	**9.**	4.40	25%	**15.**	2.20	25%
4.	3.30	$33\frac{1}{3}\%$	**10.**	6.60	$16\frac{2}{3}\%$	**16.**	17.00	25%
5.	2.50	20%	**11.**	8.80	$12\frac{1}{2}\%$	**17.**	14.00	20%
6.	1.60	50%	**12.**	1.10	10%	**18.**	19.00	$12\frac{1}{2}\%$

396. Many kinds of goods are bought and sold wholesale by the dozen. The retailer sells them by the piece.

In marking goods, all calculations, as far as possible, should be mental. In dividing the price of a dozen by 12 to find the cost of one article, twelfths of a dollar from $\frac{1}{12}$ to $\frac{11}{12}$ must frequently be reduced to cents. Hence the equivalents of twelfths of a dollar should be memorized. See page 16.

397. Find the selling price per article of each of the following:

Cost per Dozen	Rate of Gain	Cost per Dozen	Rate of Gain	Cost per Dozen	Rate of Gain
1. $12.00	25%	6. $16.00	25%	11. $32.00	12½%
2. 9.60	33⅓%	7. 18.00	33⅓%	12. 33.60	33⅓%
3. 8.40	25%	8. 20.00	25%	13. 34.00	25%
4. 14.00	20%	9. 25.00	40%	14. 45.00	20%
5. 15.00	40%	10. 27.00	33⅓%	15. 56.00	$14\frac{2}{7}$%

398. To find the marked price, having given the cost with discount, the rate of gain or loss, and the selling discount.

A jobber buys bicycles at $ 24, less 12½ %. He sells to gain 33⅓ %. What is his marking price if he allows 20 % discount?

⅛ of $ 24 = $ 3, discount.
$ 24 − $ 3 = $ 21, net cost to the jobber.
⅓ of $ 21 = $ 7, gain.
$ 21 + $ 7 = $ 28, jobber's net selling price.
$ 28 ÷ 80 % (⅘) = $ 35, jobber's marked price.

The discount is ⅛ of $24, or $3, which deducted leaves $21 as the net cost of the bicycle. 33⅓% gain is equal to ⅓ of $21, or $7, making the net selling price $28. Since the jobber allows a 20% discount, he sells for 80% of his marked price. Hence $28 ÷ .80 gives $35, the list price (Prin. 14).

PROBLEMS

399. 1. A sewing machine catalogued at $ 40 is sold to a jobber subject to a discount of 25 % and 20 %. Find the jobber's marked price if he sells to gain 25 % after deducting 25 % and 20 %.

2. A manufacturer sells bathtubs at $ 35, less 40 %. At what price should the retailer mark them to sell at 25 % discount and still make 50 %?

3. A merchant pays $ 10.80, less 25 % and 20 %, a yard for lace. Find his marked price if he sells to gain 25 % after allowing a discount of 10 %.

4. Complete the following invoice, and determine the marking price of each chiffonier to clear 20%. (Fosdick pays cash.)

Cleveland, Ohio, Aug. 10, 1910.

Peter R. Fosdick,
New Haven, Conn.

Bought of OHIO FURNITURE COMPANY

Terms: n/60 3/10.

18	#5287 Oak Chiffoniers @ 22.50 . . . Less 10% and 5%				
	Freight prepaid			13	61

5. Complete the following, and find the retail selling price to gain 30%.

Kalamazoo, Mich., Sept. 14, 1910.

G. A. Hubbard,
Chicago, Ill.

Bought of EDWARDS & CHAMBERLAIN

Terms: Net Cash

2	doz. Shovels	$12.50				
3	" Wheelbarrows	15.00				
60	ft. 2″ Lead Pipe	.25				
	Less 20% and 10%					

6. If hats cost $48, less 20% and 16⅔%, per dozen, what should be the list price to gain 35%, and allow 16⅔% and 10%?

7. When hosiery costs $9, less 20% and 12½% per dozen pairs, how shall they be marked per pair to gain 20%, if 12½% discount is offered?

8. A rug 12′ × 15′ is imported, the invoice price being £5. The duty is 90¢ a square yard, and 40% ad valorem (of the value). If a pound sterling can be bought at $4.8665, at what price must the rug sell to gain 25%?

COMMISSION AND BROKERAGE

400. A **commission merchant** is a person who buys or sells merchandise, or transacts other business, for another person.

401. The person for whom business is transacted by the commission merchant is called the **principal.**

402. Commission merchants usually charge a rate per cent on the volume of business transacted. Such a charge is called **commission** or **brokerage.** In some cases the commission is reckoned at a certain price per unit of quantity of merchandise, as the bushel, barrel, etc.

403. An additional charge called **guaranty** is sometimes made by commission merchants for undertaking to be responsible for sales on credit or for the quality and quantity of goods bought, etc.

404. Merchandise shipped by a principal (also called **consignor**) to be sold by a comission merchant is called (by the consignor) a **shipment.** The commission merchant (also called the **consignee**) calls the merchandise a **consignment.**

405. The **gross proceeds** of a sale or collection is the entire amount received by the commission merchant. The **net proceeds** is the amount remaining after all expenses have been paid.

406. The **prime cost** of a purchase is the first cost, or the amount actually paid for the merchandise. The **gross cost** is the prime cost plus all the expenses of the purchase.

407. An **account sales** is an itemized statement of the sales of merchandise, the charges thereon, and the net proceeds of the sale. The statement is made by the commission merchant and sent to his principal. (For illustration, see problem 22, page 193.)

408. An **account purchase** is an itemized statement of the quantity, quality, and price of merchandise purchased, the expenses incurred, and the gross cost of the purchase. It is made by the purchasing agent and sent to his principal. (For illustration, see problem 26, page 194.)

409. The principles of percentage apply in commission.

The **gross sales** or prime cost is the *base.*

The **rate of commission** is the *rate.*

The **commission** is the *percentage.*

The **net proceeds** is the *difference.*

The **gross cost** is the *amount.*

410. To find the commission, the cost or selling price, and the rate of commission being given.

1. Find the commission and the net proceeds of a sale of merchandise amounting to $4500; commission 5%.

5% of $4500 = $225, commission.
$4500 − $225 = $4275, net proceeds.

The commission is 5% of the gross sales, $4500, which equals $225. The net proceeds is the difference between the gross sales and the expenses.

2. Find the commission and the gross cost of a purchase of merchandise amounting to $3400; commission 4%, guaranty 2%, cartage $20.

4% of $3400 = $136, commission.
2% of $3400 = $68, guaranty.
$3400 + $136 + $68 + $20 = $3624, gross cost.

The commission and the guaranty are found by taking 4% and 2%, respectively, of $3400. The gross cost is the sum of the prime cost, $3400, and all the charges.

411. Find the commission and the net proceeds, or the gross cost, in each of the following:

	Gross Sales	Com.	Guar.	Other Expenses		Prime Cost	Com.	Guar.	Other Expenses
1.	$2400	5%		$15	**11.**	$1030	4%	3%	$8.50
2.	3000	3%	2%	25	**12.**	1150	3%	2%	9.60
3.	1500	6⅔%	1%	2%	**13.**	1250	4%		12.50
4.	1800	6¼%		$20	**14.**	1440	6¼%		2%
5.	2250	3⅓%	3%		**15.**	1880	12½%		50.00
6.	3600	8⅓%		30	**16.**	3360	16⅔%	5%	
7.	4500	10%		28	**17.**	4150	5%	2½%	
8.	1600	2½%	2%	12	**18.**	9600	2½%	2½%	100.00
9.	2700	5%		50	**19.**	12400	10%	2½%	350.00
10.	3200	4%	2%	60	**20.**	16800	12½%	5%	500.00

21. An agent sold 950 bbl. pork at $17.50 a barrel, on a commission of 2%. The other charges were for storage $25, freight $60, and insurance $5. Find the commission and the net proceeds.

22. Find the net proceeds of the following:

ACCOUNT SALES HAY

Albany, N.Y., Oct. 7, 1910.

Sold for the Account of

August Stadlich, Wellsville, N.Y.

By F. A. Mead & Co.

1910							
Sept.	10	25 tons Hay	$15.00				
	14	35 tons Hay	15.50				
	20	40 tons Hay	14.75				
		Charges					
Sept.	6	Freight, $100; Cartage, $75; Insurance, $5		180			
	20	Storage		10			
		Commission, 50¢ a ton					
		Net proceeds					

23. Prepare an account sales from the following data, using your own name as agent, and any name you wish as principal:

On August 15, you receive a consignment of 3000 bu. of corn. The same day you pay freight, 2¢ a bushel; cartage, 3¢ a bushel; insurance, $7.50. August 20, you sell 1200 bu. at 60¢; August 24, 900 bu. at 62¢; and August 28, 900 bu. at 61¢. Your commission is 2¢ a bushel, and the storage charge is $15.

24. A consignment of 460 bbl. of flour was received Nov. 1, 1910. Freight charges were $56; cartage, $14; cooperage, $3.70; advertising, $11.50. November 5, 65 bbl. at $6.40 were sold; November 7, 185 bbl. at $6.45; November 9, 50 bbl. at $6.35; and November 16, the remainder at $6.50. The storage charge was $15.80; commission 2%; guaranty, 1%. Prepare an account sales from the foregoing data.

25. If it requires $5\frac{1}{2}$ bu. of wheat to make 1 bbl. of flour, and the cost of grinding is 15¢ a barrel, and the barrels cost 35¢ each, find the gain or loss per cent to the consignor in problem 24, wheat costing 80¢ a bushel.

26. Find the gross cost of the following:

ACCOUNT PURCHASE

PHILADELPHIA, PA., Aug. 4, 1910.

Purchase for the account of

J. L. COOK, Watkins, N.Y.

BY H. M. DONOVAN & Co.

		25 M White Pine Lumber	$36				
		30 M Georgia Pine Ceiling	45				
		18 M Hemlock Boards	16				
		22 M White Oak	48				
		CHARGES					
		Cartage		125			
		Commission 2½ %					
		Amount charged to your account					

27. Prepare an account purchase from the following facts:

A. B. Carver & Co., Baltimore, Md., bought for the account of Howard Cole of Scranton, Pa., 100 gal. oysters at 75¢; 200 doz. clams at 8¢; 600 lb. bluefish at 5¢; and 800 lb. white fish at 8¢. Charges are 3% commission and drayage $4.50.

COMMISSION REVIEW

MENTAL

412. **1.** An agent sold $5000 worth of clothing on a commission of 5%. What was his commission?

2. I bought for my principal sugar worth $1250 on a commission of 4%. What was the total cost of the sugar?

3. A lawyer collected 75% of a claim of $1600 and charged 10% for collecting. What was his commission?

4. A book agent's commission was 40%. He sold 120 books at $2.50 each. How much did he earn?

5. An auctioneer received 5% of his sales, which amounted to $2400. Find his commission.

6. My agent sold for me 100 tubs of butter averaging 50 lb. each at 25¢ a pound, and charged 2%. How much did I receive for the butter?

Find the proceeds in each of the following:

7. 20 T. hay at $15; commission 8 %; other expenses $25.

8. 12 T. cabbage at $12.50; commission 6 %; other expenses $11.

9. 300 bbl. flour at $6; commission 3 %; storage 3¢ a barrel; freight 20¢ a barrel.

10. 20,000 ft. lumber at $18 per M; commission 2½ %; freight $30.

11. I bought through an agent 300 yd. of carpet at $1.50, paying him 2 %. How much did the carpet cost me?

12. An agent buys 1000 bu. of wheat at $1.10, and charges ½¢ a bushel. What is his commission, and the total cost?

13. Find the gross cost of 4000 brick at $9.50 per M; commission 50¢ a thousand; other charges $2.50.

Find the gross cost of the following purchases:

14. 1200 bu. corn at 75¢; commission 3 %; freight 1½¢ a bushel.

15. 700 T. coal at $5; commission 2 %; freight 50¢ a ton.

16. 800 yd. silk at $2.50; commission ¾ %; freight $2.50.

17. 10 half chests Young Hyson, 700 lb., at 25¢; commission 2 %; freight $1.50.

18. 10 bbl. sugar, 3200 lb., at 5¢; commission 4 %; freight and drayage $1.25 per barrel.

19. A broker bought for me 800 lb. of leather and charged $8, commission being at 4 %. What was the price of the leather, and what was the gross cost including freight at 25¢ a hundred pounds?

20. My agent charges 2 % commission, and 2 % guaranty, for buying coffee at 20¢ a pound. If the commission and guaranty amount to $40, how many pounds were bought, and what was the gross cost?

21. Which is better for the agent, a commission of 1½¢ a bushel or 2 %, when a bushel sells for 80¢?

22. I bought a job lot of muslin consisting of 1500 yd. bleached and 800 yd. unbleached for $120; commission 5 %; and other expenses $9. If I sell the bleached muslin at 7¢ a yard and the unbleached at 5¢ a yard, what is my profit?

23. I have 75 bbl. of flour which cost me $360. I send them to my agent, who sells them at $6.20, and charges 2 % commission, and $5.70 for other expenses. What per cent profit do I make?

24. Find the per cent profit realized by buying potatoes at 40¢ a bushel, and shipping them to New York, if I pay 5¢ a bushel for freight, 3¢ a bushel commission, 2¢ a bushel for other expenses, and the potatoes sell at 60¢.

REVIEW PROBLEMS

413. **1.** An agent remitted to me $247.38 after retaining a commission of 5 % for collection. What sum did he collect? What was the amount of his commission?

2. The gross cost of a purchase of sweet potatoes was $864.55. The expenses were for drayage $12.50, cooperage $3.35, and commission $2\frac{1}{2}$ %. If the potatoes cost $.72 a bushel, how many bushels were bought?

3. An agent charged his principal $106.25 (commission being $2\frac{1}{2}$ %), for buying 5000 bu. of wheat; the freight charges, etc., amounted to $43.75. How much a bushel did the wheat cost the principal?

4. An agent sold 3000 bu. of wheat, and after deducting his commission of $2\frac{1}{2}$ %, sent his principal the proceeds, $2808. For how much a bushel was the wheat sold?

5. A commission merchant sold a consignment of cottonseed oil for $12,500 on a commission of 4 %. Other expenses of the sale amounted to $375. What amount was due the principal?

6. My agent purchased for me 180 bbl. of sugar, averaging 275 lb. each, at $5\frac{1}{4}$¢ a lb. He charged 2 % commission and $71.77 for other expenses. Find the gross cost of the sugar. If it is sold at 6¢ a pound, what rate of gain is realized?

7. An agent sold for his principal 6500 bu. of potatoes at 60¢ a bushel, charging 3¢ a bushel for selling. The freight charges were 2¢ a bushel, and other expenses amounted to $85. He purchased for his principal an invoice of silks for $2475, charging 5% for buying, and remitted the proceeds. What was the amount of the remittance?

8. At what price shall I mark an article that cost me $18, so that I can instruct my agent to allow 25 % discount on the marked price, pay him 15 % commission for selling, allow 5 % of sales for bad debts, and still make a profit of 33⅓ % ?

9. To manufacture a certain style of automobile costs $800. What must be the catalogue price of this machine, so that the manufacturer can give the retailer 25 % discount on the list price, pay an agent 16⅔ % commission for selling, and still make a profit of 25 % ?

10. A broker sold for me 400 bales of cotton (500 lb. in a bale) at 12.52 ¢ a pound on a commission of $5 per hundred bales. He invested the proceeds of the sale in flour at $5 a barrel, charging 2 % for buying. How many barrels were purchased, and what sum was left unexpended in the broker's hands ?

SUGGESTION : — Find gross cost of 1 barrel of flour before dividing.

11. An agent received $7500 to invest in apples at $2.40 a barrel after paying all expenses of the purchase. Charges were as follows: commission 3 %; guaranty 2 %; drayage 5¢ a barrel; and freight 12¢ a barrel. Find the agent's commission, the number of barrels purchased, and the unexpended balance.

12. An agent sells $8000 worth of hardware on a commission of 2½ %. He pays freight charges, $162.50; drayage $68.50; and charges 2½ % for guaranteeing payment. What are the net proceeds of the sale ?

13. You are a commission merchant and have received, Sept. 1, 1910, 840 bbl. of flour to be sold for the Twin City Milling Co. You pay the freight, $131.50; drayage $210; advertising $27.50; and insurance ¼ % of gross sales. September 8, you sell 225 bbl. at $6.75; September 14, 300 bbl. at $6.85; and September 21, the balance at $6.80. Storage charge is $16.74, and your charge for commission is 2¼ %. Prepare an account sales, showing all facts here given, and the net proceeds.

14. You buy through a purchasing agent 42 Ideal refrigerators at $17.50; 24 dining tables at $26.40; and 18 dressers at $32.75. The agent's commission charge is 5 %, and he pays $15 for drayage. Prepare an account purchase using the current date and any name of agent and place you wish.

EXAMINATIONS

SPEED TEST

Minimum time, 30 minutes; maximum, 1 hour. Deduct one credit for each minute over minimum time.

414. 1. Find the selling price of each of the following:

	Cost		Selling Price
(*a*)	$ 0.16	37½% gained	——
(*b*)	6.00	20 % lost	——
(*c*)	56.00	25 % gained	——
(*d*)	4.50	16⅔% lost	——
(*e*)	828.00	¼% gained	——

2. Find in each of the following the number of hours and minutes from the time commenced to time finished:

	Time Commenced	Time Finished	Hours	Minutes
(*a*)	9 : 50 A.M.	4 : 13 P.M.	——	——
(*b*)	9 : 16 A.M.	3 : 55 P.M.	——	——
(*c*)	9 : 45 A.M.	12 : 16 P.M.	——	——
(*d*)	10 : 15 A.M.	1 : 00 P.M.	——	——
(*e*)	9 : 14 A.M.	10 : 02 P.M.	——	——

3. Divide the numbers in column 1 by the numbers in column 2, and express your answer as per cent correct to two figures only:

	Column 1	Column 2	Per Cent
(*a*)	245,326	981,304	——
(*b*)	5,253	30,900	——
(*c*)	12,596	59,923	——
(*d*)	257,515	476,878	——
(*e*)	18,487	55,460	——

4. Multiply each of the following percentages by its relative weight, and find the general average by dividing the sum of the products by the sum of the relative weights:

	Percentages	Relative Weights	Products
(*a*)	97.51	8	——
(*b*)	81.33⅓	12	——
(*c*)	65.	5	——
(*d*)	69.16	7	——
		——	——
	General average		——

5. Perform the following indicated operations:

⅛ of 6876.88 =
$79,487.98 × 6% =
9569.64 ÷ ⅛ =
$6734.17 × .5% =

6. Find the gain per cent in each of the following:

	Cost	Selling Price	Gain %
(*a*)	$4.50	$4.95	——
(*b*)	7,500.00	8,750.00	——
(*c*)	5,280.00	7,040.00	——
(*d*)	9,600.00	10,200.00	——

7. Find the amount to be remitted to the employer in each case:

	Amount Collected	Rate of Commission	Amount Remitted
(*a*)	$3,400	5 %	——
(*b*)	91,500	3⅓%	——
(*c*)	81,600	2½%	——
(*d*)	30,501	⅛%	——

8. Find net cost of each item and total net cost of all:

	List Price	Discount	Net Cost
(*a*)	$270.00	25%	——
(*b*)	480.00	25 % and 20%	——
(*c*)	6.40	12½% and 10%	——
(*d*)	960.00	33⅓% and 25%	——
		Total	

9. Canadian exports for two years; find the total for each year:

Article	1908	1909
Mineral Products	$36,840,044	$38,669,008
Fish	14,435,023	14,863,343
Forest Products	38,504,738	46,716,480
Animal Produce	53,019,843	53,040,391
Agricultural Products	75,883,951	84,921,684
Manufactures	28,892,297	30,807,548
Miscellaneous	54,913	118,756

10. Find the excess of each kind, and the total excess of exports for 1909 over those for 1908 in problem 9.

WRITTEN TEST

415. 1. A retired merchant has an income of $25 per day. If his property is invested at 6%, how much is he worth?

2. Oranges are bought at $1.80 a gross, and sold at the rate of two for 5¢. Find the gain per cent.

3. A certain lot of goods was sold for $189 after discounts of 12½% and 10% were allowed. What was the list price?

4. A man commenced business with $3000 capital. The first year he gained 22½%, which he added to his capital; the second year he gained 30% of the whole sum, and put the gain into his

business; the third year he lost 16⅔% of his entire capital. How much did he make in 3 yr.?

5. An agent receives $1092.42 with which to buy oats at 42¢ a bushel after deducting his commission of 2% on the sum expended. How many bushels can he buy?

6. A man sold 28 yd. of cloth at $1.25 per yard, thereby gaining $3.50. Find the rate per cent of gain.

7. An insolvent merchant's assets are $3027.90 and his liabilities are $4974. How much ought a creditor to receive whose bill against the merchant is $627?

8. A peddler bought 491 yd. of cloth at 81¢ a yard; he spoiled 29 yd., and sold the remainder at 95¢ a yard. Did he gain or lose, and what per cent?

9. A. M. Orton bought of The Excelsior Hardware Co., Dec. 15, 1910, terms 2/10 net 30, 600 pairs barn-door hangers at $2.75, less 25% and 10%; 175 doz. strap hinges at $3.25, less 20% and 12½%; and 15 doz. wrought iron wrenches at $14.50. Find the net cost if paid for Dec. 24, 1910. At what price should these items be sold retail to gain 25%?

10. A company paid $3000 for a hall. Within a given period they rented it 148 evenings at $15 an evening. Expenses were: coal, 15 T., at $6.25; lighting $31.40; repairs $78.60; insurance $15.20; and taxes $47.80. What per cent profit did they make?

WRITTEN TEST

416. 1. Russian cotton mills used in 1908 $125,000,000 worth of raw cotton. $61,365,340 worth was imported, $49,900,925 worth of it being American cotton. What per cent of the total is grown in Russia? What per cent of the imports is from the United States?

2. Exports of leather and leather goods from the United States to Porto Rico for 1907 were $854,017; 1908, $643,295; 1909, $960,672. Compare the exports of 1908 and 1909 with those of 1907, showing the per cent of increase or decrease.

3. In 1909 Germany imported 3,879.3 metric tons of leather, of which amount 249.7 T. were from the United States. What per cent of it came from the United States?

4. Imports of diamonds into the United States were valued at $37,060,237 in 1906; $27,691,877 in 1907; $11,538,130 in 1908; and $36,159,054 in 1909. Show these values graphically.

5. What price must an article costing $20 be marked so as to gain 30% and still allow a discount of 10% on the asking price? If the purchaser becomes bankrupt and pays only 60¢ on the dollar, what per cent of profit or loss will result from the transaction?

6. Render in full an account sales for the following:
Feb. 1, 1914, Parke and Westerleigh sold for R. T. Booker, Lansing, Mich., 8 doz. chairs, No. 143, at $72; 2 doz. rockers, No. 164, at $98; 42 bookcases at $14.25. Charges were: freight, $62.50; insurance, ¼%; and commission, 2½%.

7. Find the net amount of the following bill: 445 yd. Brussels carpet at $1.12½; 675 yd. ingrain carpet at $.67½; and 1376 sq. yd. linoleum at $.56¼. Less 20% and 5%.

8. A commission merchant sold 12,500 bbl. potatoes at $1.62½, charging 1½%. He invested the proceeds in flour at $5.60 on a commission of ¾%. Find the total commission, the number of barrels of flour purchased, and the unexpended balance, if any.

9. At the beginning of a certain period of time a merchant had on hand $5400 worth of merchandise. During the time he purchased $12,500 worth, and returned to creditors goods valued at $700. His sales amounted to $14,700, and there was returned to him $500 worth of merchandise. At the end of the time he found he had on hand an inventory of $7200. What average rate of profit did he make on his sales?

10. An article costing $3.20 and selling for $4.80 sold at the rate of six a day. Upon the price being reduced to $3.75, the sales increased to eighteen a day. Find the increase or decrease per cent in the daily profits.

11. An invoice of ties was purchased as follows: 4½ dozen ties @ $4.50; 2½ dozen ties @ $15.00; 3 dozen ties @ at $10.50; 1½ dozen ties @ $18.00; 1½ dozen ties at $15.75; 3½ dozen ties @ $12.00; 8 dozen ties @ $3.50; 4½ dozen ties @ $5.40. Less 20% and 16⅔%.

Find the total gain if sold at a profit of 33⅓%.

INTEREST

417. If a man lends $1000 for 1 yr. and receives at the end of the year $1060, why is he paid $60 more than he lent?

418. The compensation paid for the use of money is called **interest.**

419. That sum of money for the use of which interest is paid is called the **principal.** In the above illustration, $1000 is the *principal* and $60 is the *interest.*

420. The amount of interest to be paid is reckoned as a certain rate per cent per annum. The most common legal rate is 6 %. That means that for the use of $1 for a year a man pays 6 % of $1, or $.06. For the use of $1 for 2 yr. a man pays 2 times 6 % of $1, or $.12.

421. The **rate of interest** is the fractional part of $1 (expressed as hundredths, or as a per cent) that is paid for the use of $1 for 1 yr.

422. The **unit of time** used as a basis in interest calculations is 1 yr. Hence, the expressions 4 %, 5 %, and 6 % interest mean, respectively, 4 %, 5 %, and 6 % interest for 1 *year's* use of a given principal.

423. The sum of the principal and the interest is called the **amount.**

424. The principles of percentage apply to all solutions of interest problems.

425. Since the rate of interest is a rate for 1 yr., a new element — time — has to be taken into consideration in interest computations. For the purpose of solving problems the rate is equal to the rate per cent of interest multiplied by the time expressed as years. That is, from a percentage standpoint, if the rate of interest is 6 %, and the time is 2 yr. and 6 mo., the rate is 2½ times 6 %, or 15 %. Hence,

The **principal** is equivalent to the *base.*

The **rate of interest** multiplied by the time in years is equivalent to the *rate.*

The **interest** is equivalent to the *percentage.*

The **amount** is equivalent to the *amount.*

426. The legal rate of interest in the several states is determined by their legislatures. Valid contracts to pay more than the legal rate of interest cannot be made, except in certain cases provided for by statute.

427. Charging more than the legal rate is called **usury.** Most, if not all, the states impose a penalty for charging usury.

428. There are several kinds of interest, viz., **ordinary, accurate,*** and **compound.**

429. Ordinary interest is simple interest reckoned on a 360-day-year basis.

METHODS

430. There are a number of good methods of reckoning ordinary interest. One method is explained in this work, viz., the **banker's 60-day method** — that being the best adapted to all kinds of problems.

For a preliminary discussion of this method, see pages 26–29.

431. To find the interest for any number of days.

Find the interest at 6 % on $ 1660 for 45 da.

$$\begin{array}{r|l} \$16 & 60 = \text{int. for 60 da. at } 6\,\%. \\ 4 & 15 = \text{int. for 15 da. at } 6\,\%. \\ \hline \$12 & 45 = \text{int. for 45 da. at } 6\,\%. \end{array}$$

Interest for 60 da. is found by pointing off 2 places in the principal, which gives $16.60. 45 da. is 15 da. less than 60 da.; hence, the interest for 45 da. is ¼ less than the interest for 60 da. ¼ of $ 16.60 is $ 4.15, which, taken from $ 16.60, leaves $ 12.45.

DRILL EXERCISE

432. The student should practice until he can find the total interest in each of the following groups in four minutes or less.

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* For definition, see page 211.

433. Find the total amount of interest at 6 % on:

1. $1400 for 90 da.
$1600 for 75 da.
$1560 for 80 da.
$1300 for 20 da.
$1200 for 24 da.
$ 875 for 50 da.
$ 930 for 65 da.
$ 870 for 40 da.
$1100 for 150 da.
$1370 for 180 da.
$ 920 for 36 da.
$1670 for 42 da.

2. $2900 for 75 da.
$3200 for 45 da.
$2800 for 15 da.
$2400 for 21 da.
$3300 for 25 da.
$4400 for 33 da.
$1800 for 85 da.
$1200 for 55 da.
$1000 for 72 da.
$2000 for 84 da.
$3000 for 80 da.
$4000 for 48 da.

3. $7340 for 6 da.
$8970 for 12 da.
$5980 for 3 da.
$3890 for 9 da.
$9660 for 10 da.
$4520 for 7 da.
$3360 for 5 da.
$2180 for 9 da.
$1890 for 10 da.
$3280 for 18 da.
$4170 for 2 da.
$7320 for 1 da.

4. $1172 for 40 da.
$1260 for 30 da.
$1440 for 8 da.
$1240 for 3 da.
$ 890 for 45 da.
$ 680 for 48 da.
$ 840 for 65 da.
$ 960 for 55 da.
$1020 for 70 da.
$1210 for 90 da.
$1620 for 80 da.
$ 740 for 3 da.

5. $ 720 for 14 da.
$ 840 for 16 da.
$ 960 for 18 da.
$1200 for 21 da.
$1500 for 27 da.
$1800 for 28 da.
$3600 for 36 da.
$7200 for 54 da.
$5400 for 45 da.
$6300 for 75 da.
$4500 for 90 da.
$7500 for 12 da.

6. $ 80 for 15 da.
$ 90 for 60 da.
$ 60 for 90 da.
$ 72 for 45 da.
$ 45 for 72 da.
$ 40 for 45 da.
$ 300 for 20 da.
$ 200 for 30 da.
$ 180 for 21 da.
$ 210 for 18 da.
$ 360 for 40 da.
$ 500 for 12 da.

434. To find the interest for years, months, and days.

1. Find the interest at 6 % on $840.50 for 5 mo. 10 da.

$ 8	40.50	= int. for 2 mo. at 6 %.
16	81 00	= int. for 4 mo. at 6 %.
4	20 25	= int. for 1 mo at 6 %.
1	40 08	= int. for 10 da. at 6 %.
$ 22	41 33	= int. for 5 mo. 10 da. at 6 %.

Interest for 2 mo. is found by pointing off *two places* in the principal, 4 mo. is twice 2 mo., and the interest for 4 mo. is twice $8.405, or $16.81. Similarly, the interest for 1 mo. is one half of $8.405, or $4.2025, and the interest for 10 da. is ⅓ of $4.2025, or $1.4008. On adding, $22.41 is found to be the interest for the entire time.

2. Find the interest at 6% on $1360.75 for 1 yr. 7 mo. 25 da.

$13	60.75	= int. for 2 mo. at 6%.
136	075	= int. for 20 mo. at 6%.
1	1339	= int. for 5 da. at 6%.
$134	9411	= int. for 1 yr. 7 mo. 25 da. at 6%.

1 yr. 7 mo. 25 da. = 19 mo. 25 da. = 20 mo. less 5 da. From the interest for 2 mo., it is easily found for 20 mo. to be $136.075. 20 mo. is 5 da. too much; hence, find the interest for 5 da., $1.1339, and deduct it from $136.075, leaving $134.94, the desired result.

3. Find the interest at 6% on $1500 for 2 yr. 3 mo., 23 da.

$15	00	= int. for 2 mo at 6%.
195	00	= int. for 26 mo. at 6%.
7	50	= int. for 1 mo. at 6%.
5	00	= int. for 20 da. at 6%.
	75	= int. for 3 da. at 6%.
$208	25	= int. for 2 yr. 3 mo. 23 da. at 6%.

The time is 27 mo. 23 da. 13 times the interest for 2 mo. gives the interest for 26 mo. The interest for the remaining 1 mo. is $\frac{1}{2}$ of $15, or $7.50. 20 da. is $\frac{1}{3}$ of 60 da.; hence, the interest is $\frac{1}{3}$ of $15, or $5. For 3 da. the interest is $\frac{1}{10}$ of the interest for 1 mo., or $.75. The total is $208.25.

Note. Keep partial results to the fourth decimal place to insure accurate results.

435. Find the interest at 6% on the following:

	Principal	Time		Principal	Time
1.	$ 38.40	2 mo. 15 da.	14.	$560	45 da.
2.	$126.75	4 mo. 20 da.	15.	$640	29 da.
3.	$248.50	6 mo. 12 da.	16.	$832	24 da.
4.	$360	8 mo. 15 da.	17.	$975	18 da.
5.	$375.60	9 mo. 20 da.	18.	$760	19 da.
6.	$890.50	11 mo. 10 da.	19.	$480	1 mo. 18 da.
7.	$429.60	1 yr. 4 mo. 15 da.	20.	$760	3 mo. 20 da.
8.	$387.50	2 yr. 6 mo. 20 da.	21.	$1840	9 da.
9.	$840	90 da.	22.	$1920	28 da.
10.	$660	100 da.	23.	$1500	42 da.
11.	$930	4 mo.	24.	$7290	48 da.
12.	$740	6 mo.	25.	$4000	54 da.
13.	$425	30 da.	26.	$3000	50 da.

	Principal	Time		Principal	Time
27.	$125	5 mo. 15 da.	**40.**	$582	1 yr. 3 mo. 5 da.
28.	$115.75	9 mo. 13 da.	**41.**	$2135	2 yr. 6 mo.
29.	$825.50	7 mo. 14 da.	**42.**	$4240	10 mo. 11 da.
30.	$1260	3 mo. 23 da.	**43.**	$1325	9 mo. 8 da.
31.	$1340	5 mo. 29 da.	**44.**	$2150	5 mo. 13 da.
32.	$1696	1 mo. 20 da.	**45.**	$6322	8 mo. 15 da.
33.	$1440	70 da.	**46.**	$4178	132 da.
34.	$5000	65 da.	**47.**	$7500	96 da.
35.	$2840	36 da.	**48.**	$8800	84 da.
36.	$1000	75 da.	**49.**	$8100	78 da.
37.	$2375	90 da.	**50.**	$5400	154 da.
38.	$1024	95 da.	**51.**	$4300	150 da.
39.	$2717	7 mo. 15 da.	**52.**	$3300	165 da.

436. To find the interest at any rate per cent.

What is the interest on $4800 for 60 da. at 6%? at 5%? at 4%? at 7%?

(*a*)
$$\begin{array}{r|l} \$48 & 00 \text{ int. at } 6\% \\ 8 & 00 \text{ int. at } 1\% \\ \hline \$40 & 00 \text{ int. at } 5\% \end{array}$$

(*b*)
$$\begin{array}{r|l} \$48 & 00 \text{ int. at } 6\% \\ 16 & 00 \text{ int. at } 2\% \\ \hline \$32 & 00 \text{ int. at } 4\% \end{array}$$

(*c*)
$$\begin{array}{r|l} \$48 & 00 \text{ int. at } 6\% \\ 8 & 00 \text{ int. at } 1\% \\ \hline \$56 & 00 \text{ int. at } 7\% \end{array}$$

In solution (*a*), the interest at 6% is found by pointing off two places. Since 5% is 1% less than 6%, the interest at 5% is found by subtracting the interest at 1% from the interest at 6%. To find the interest at 1%, divide the interest at 6% by 6. On subtracting, the interest at 5%, $40, is found.

In solution (*b*), the interest at 2% is subtracted from the interest at 6% to find the interest at 4%. The interest at 2% is found by dividing $48 by 3. Why?

In solution (*c*), the interest at 1% is added to the interest at 6% to find the interest at 7%; and so on.

Note. The student should observe that in each case the interest is first found for the given time at 6%. Then the interest at 6% is increased or diminished in proportion as the required rate is greater or less than 6%.

437. The interest at rates other than 6% may be found as follows:

To find the interest at:

6½% . . . add $\frac{1}{12}$ to the interest at 6%.
6¾% . . . add $\frac{1}{8}$ to the interest at 6%.
7 % . . . add $\frac{1}{6}$ to the interest at 6%.
7⅕% . . . add $\frac{1}{5}$ to the interest at 6%.
7½% . . . add $\frac{1}{4}$ to the interest at 6%.
8 % . . . add $\frac{1}{3}$ to the interest at 6%.
9 % . . . add $\frac{1}{2}$ to the interest at 6%.
10 % . . . divide the interest at 6% by 6, and move the decimal point one place to the right.
12 % . . . multiply the interest at 6% by 2.
5½% . . . deduct $\frac{1}{12}$ from the interest at 6%.
5¼% . . . deduct $\frac{1}{8}$ from the interest at 6%.
5 % . . . deduct $\frac{1}{6}$ from the interest at 6%.
4⅘% . . . deduct $\frac{1}{5}$ from the interest at 6%.
4½% . . . deduct $\frac{1}{4}$ from the interest at 6%.
4 % . . . deduct $\frac{1}{3}$ from the interest at 6%.
3 % . . . divide the interest at 6% by 2.
2 % . . . divide the interest at 6% by 3.
1 % . . . divide the interest at 6% by 6.

To find the interest at any other rate, divide the interest at 6% by 6, to find the interest at 1%, and then multiply the interest at 1% by the given rate.

438. Find the interest at 6½% and at 7% in Art. 435, problems 1–12 inclusive; at 7½% and at 4½% in problems 13–24 inclusive; at 5½% and at 5% in problems 25–38 inclusive; and at 10% and 4% in problems 39–52 inclusive.

439. Pointing off two places in any principal gives the interest at rates other than 6% as follows:

At 1 % for 1 yr.
At 2 % for 6 mo. = 180 da.
At 3 % for 4 mo. = 120 da.
At 4 % for 3 mo. = 90 da.
At 4½% for . . . . . 80 da.
At 5 % for . . . . . 72 da.

At 7½% for 48 da.
At 8 % for 45 da.
At 9 % for 40 da.
At 10 % for 36 da.
At 12 % for 30 da.
At 15 % for 24 da.

NOTE. The figures here given may be very easily recalled if it is observed and remembered that the number of days in each case is the quotient of 360 da. divided by the rate (expressed integrally).

440. Find the interest on:

1. $875 at 2% for 6 mo.
2. $980 at 3% for 4 mo.
3. $1260 at 5% for 72 da.
4. $1485 at 7½% for 48 da.
5. $3475 at 8% for 45 da.
6. $6890 at 9% for 80 da.
7. $3960 at 10% for 108 da.
8. $5475 at 4% for 180 da.
9. $6400 at 5% for 144 da.
10. $5600 at 4½% for 80 da.
11. $3900 at 12% for 90 da.
12. $7400 at 10% for 72 da.
13. $6900 at 9% for 160 da.
14. $7870 at 7½% for 96 da.
15. $8340 at 6% for 180 da.
16. $9870 at 5% for 216 da.
17. $8365 at 4% for 90 da.
18. $7182 at 3% for 240 da.
19. $6275 at 4½% for 80 da.
20. $3785 at 15% for 24 da.

SHORT METHODS

441. What is the interest at 6% on $485 for 60 da.?

What is the interest at 6% on $485 for 600 da.?

What is the interest at 6% on $485 for 6000 da.?

How does the interest for 600 da. compare with the interest for 60 da.?

How many places are pointed off in the principal to find the interest for 60 da.? to find the interest for 600 da.?

How does the interest for 6000 da. compare with the interest for 600 da.? How many places are pointed off to find the interest for 6000 da.?

442. Find the interest at 6% on $580 for 600 da.; for 6000 da.

$58.0 = int. for 600 da.
$580. = int. for 6000 da.

600 da. is 10 times 60 da.; hence, the interest for 600 da. is 10 times the interest for 60 da. Since the interest for 60 da. is found by pointing off two places in the principal, the interest for 600 da. is found by pointing off one place (Prin. 1, page 100).

Because 6000 da. is 10 times 600 da., the interest for 6000 da. is found by pointing off no places in the principal. That is, the interest at 6% equals the principal in 6000 da.

443. Summary of Principles:

1. *Pointing off* 3 *places in the principal gives the interest at* 6% *for* 6 *da.*

2. *Pointing off* 2 *places in the principal gives the interest at* 6% *for* 60 *da.*

3. *Pointing off* 1 *place in the principal gives the interest at* 6 % *for* 600 *da.*

4. *Pointing off no places in the principal gives the interest at* 6 % *for* 6000 *da.*

444. Find the interest at 6 % on:

1. $492 for 600 da.
2. $1486 for 1200 da.
3. $1590 for 1800 da.
4. $1840 for 6000 da.
5. $2460 for 3000 da.
6. $1230 for 1500 da.
7. $6280 for 4500 da.
8. $9684 for 2000 da.
9. $8346 for 900 da.
10. $9471 for 200 da.
11. $1760 for 3600 da.
12. $1840 for 4200 da.
13. $1932 for 4800 da.
14. $750.75 for 6000 da.
15. $916.73 for 900 da.
16. $813.25 for 12,000 da.
17. $532.37 for 6000 da.
18. $928.50 for 800 da.
19. $4328 for 300 da.
20. $7388 for 1500 da.

INTERCHANGING PRINCIPAL AND TIME

445. Time is rarely, if ever, expressed in days, except for parts of a year. Hence, the problems in the preceding exercise are not of direct practical importance. Indirectly the solutions are valuable. By the commutative law of multiplication, the interest on $6000 for 89 da. is the same as the interest on $89 for 6000 da., as shown in the following illustration.

Find the interest at 6 % on $6000 for 89 da.

(*a*)	$60	00 = int. for 60 da.	(*b*) $89 = int. for 6000 da.
	20	00 = int. for 20 da.	
	6	00 = int. for 6 da.	
	3	00 = int. for 3 da.	
	$89	00 = int. for 89 da.	

In solution (*a*) the direct solution is given by finding the interest first for 60 da., then for 20 da., 6 da., and 3 da., the sum of the several amounts being $89, for 89 da.

In solution (*b*) the principal, $6000, and the time, 89 da., are interchanged so that the problem would read "Find the interest on $89 for 6000 da." But the interest at 6 % on any principal for 6000 da. is equal to the principal. Therefore the interest is $89.

PROOF OF THE PRINCIPLE

Find the interest on $1800 for 47 da. at 6 %.

$1800 at 6 % int. for 47 da. = $47 at 6 % int. for 1800 da.

$$\frac{\$1800 \times .06 \times 47}{360} = \frac{\$47 \times .06 \times 1800}{360} = \$14.10.$$

By the cancellation method of finding interest the principal multiplied by the rate per annum, multiplied by the time expressed as years or a fraction of a year, gives the interest. In the illustration here given it is to be observed that both the principal and the number of days are above the line, — that is, they are each a part of a dividend to be divided by the number below the line. Since both principal and time (in days) are factors of the same product, their relative order in the list of factors is immaterial (Prin. 18, page 101). Therefore, the principal and the time in days can be interchanged without altering the amount of the interest.

446. By interchanging principal and time, find the interest at 6 % on:

1. $6000 for 29 da.
2. $3000 for 47 da.
3. $600 for 83 da.
4. $1200 for 19 da.
5. $1500 for 37 da.
6. $2000 for 81 da.
7. $12,000 for 43 da.
8. $9000 for 53 da.
9. $8000 for 79 da.
10. $1000 for 33 da.
11. $180 for 55 da.
12. $120 for 74 da.
13. $300 for 71 da.
14. $480 for 92 da.
15. $360 for 103 da.
16. $200 for 110 da.
17. $900 for 111 da.
18. $600 for 132 da.
19. $60 for 142 da.
20. $90 for 101 da.
21. $150 for 140 da.
22. $1500 for 64 da.
23. $2100 for 44 da.
24. $2700 for 14 da.
25. $6600 for 31 da.
26. $7200 for 25 da.
27. $7800 for 17 da.
28. $8400 for 13 da.
29. $540 for 38 da.
30. $630 for 41 da.
31. $72 for 35 da.
32. $30 for 80 da.
33. $240 for 70 da.
34. $15 for 100 da.
35. $15,000 for 160 da.
36. $12,600 for 131 da.

ACCURATE INTEREST

447. **Accurate interest** is computed for exact time in days, and on a basis of 365 da. to the year. It is used chiefly by the United States government, and by some bankers. Business men and bankers generally find the computation of accurate interest too inconvenient, though some merchants and bankers use it with the help of interest tables.*

448. Since by the "ordinary" interest methods, 360 da. are counted as a year, 360 days' interest by the ordinary method is greater than 360 days' interest by the exact interest method, by as much as $\frac{360}{360}$ is greater than $\frac{360}{365}$, or $\frac{1}{72}$. That is, for any given number of days, the ordinary interest is $\frac{1}{72}$ greater than the accurate interest at the same rate per cent. Hence, to change accurate interest to ordinary interest, add $\frac{1}{72}$ of the accurate interest to itself; and to change ordinary interest to accurate interest, deduct $\frac{1}{73}$ of the ordinary interest from itself.

449. In practice, however, accurate interest is computed as shown in the following illustrative example:

1. Find the accurate interest on \$790 at 5% for 140 da.

$$\frac{\$790 \times .05 \times \overset{28}{\cancel{140}}}{\underset{73}{\cancel{365}}} = \frac{1106}{73} = \$15.15, \text{ the accurate interest.}$$

Multiplying \$790 by 5% gives the interest for one year. Dividing that product by 365 gives the interest for 1 da., which multiplied by 140 gives the interest for 140 da.

450. Find the accurate interest on:

1. \$750 for 148 da. at 6%.
2. \$675 for 213 da. at 4%.
3. \$876 for 219 da. at 5%.
4. \$1314 for 58 da. at 7%.
5. \$1825 for 64 da. at 6½%.
6. \$1600 for 13 da. at 4½%.
7. \$1500 for 18 da. at 3%.
8. \$1700 for 38 da. at 5%.
9. \$4380 for 51 da. at 6%.
10. \$3825 for 47 da. at 5%.
11. \$4564 from Aug. 11, 1908, to May 15, 1909, at 5%.
12. \$3870 from June 1, 1909, to Dec. 30, 1909, at 6%.

*** Bankers and others use interest tables for both ordinary and accurate interest. The tables are too long to permit of illustration in this book.**

PROBLEMS

451. 1. A man buys 150 shares of stock for $18,000. He receives a semiannual dividend of $2.50 a share. How much better is this than 4% interest, if the interest is paid semiannually?

2. A savings bank pays 4% interest on its deposits. It loans on bond and mortgage at $5\frac{1}{2}$%. If its deposits amount to $16,475,000, and its loans are $14,850,000, what is its gross gain in interest per annum, interest in each case being paid semiannually?

3. A banker pays 3% interest on a deposit of $45,000. He loans it 3 mo. at 6%; 4 mo. at $5\frac{1}{2}$%; and for 48 da. at 7%. If it lies idle the remainder of the year, does he gain or lose, and how much?

4. A merchant bought a bill of goods amounting to $13,875; terms 3 mo., 5% cash. He accepted the cash terms. How much did he save if money was worth 6%?

5. A young man had $1600 saved. He purchased a house and lot for $4000, borrowing $2400 at a savings bank, giving as security a mortgage on the house and lot. The house rented for $37.50 a month. He paid $5\frac{1}{2}$% interest on the mortgage; $\frac{1}{8}$% premium on an insurance policy of $3000; 1.8% tax on a valuation of $3500; $14 water rent; and $50 for repairs. What rate of interest did he receive on his investment?

6. Aug. 1, 1909, a wholesaler sold a bill of goods amounting to $1875.28, terms net 30 da. If the bill was not paid until Jan. 8, 1910, what amount was due, interest being 6%? (Exact time.)

7. What amount can a man afford to pay for a house and lot that rents for $65 a month so that his investment shall net him 6% interest, if he pays for taxes, insurance, repairs, and all other expenses, $360 a year?

8. On July 8, 1909, a man borrowed money at 6% interest and bought 3000 bu. of wheat at $1.08 a bushel. He sold the wheat Aug. 17, 1909, at $1.21 a bushel and returned with interest the money he borrowed. What was his gain?

9. A piece of property valued at $8000 is sold on the following terms: 20% of the selling price is to be paid in cash; for the

remainder five notes falling due in one, two, three, four, and five years, respectively, with interest at 6 %, are given. If each note is paid at maturity, what is the total amount paid for the property?

10. What is the difference between the ordinary interest and the accurate interest at 6 % on $10,000 for 144 da.

11. A merchant's cash price of an article is $35; his price at 4 months' credit is $37.50. If money is worth 6 % per annum, how much extra profit does he make when selling on credit?

12. A real estate broker offers me a house and lot for $7500, guaranteeing an 8 % investment. The property is assessed for $5000 on which a tax of $1\frac{3}{4}$ % has to be paid. Insurance and other expenses will amount to $212.50 a year. At what price must the property rent per month to make good the broker's guarantee?

13. An invoice of merchandise was bought on the following terms: 4 mo. or 5 % cash. The cash discount is equivalent to what rate of interest per annum on the gross amount? on the net amount?

14. In what time will any sum of money double itself at 5 %? at 6 %? at 8 %? at 9 %?

15. At what rate per cent of interest will any sum of money double itself in 25 yr.? in $14\frac{2}{7}$ yr.? in 10 yr.? in 8 yr. 4 mo.?

16. A clerk is instructed to accept the best terms on all bills, allowing that money is worth 15 % to his employer. Under these conditions, which terms should the clerk accept in the following invoices?

$428.75, terms, 30 da., 2 % cash.
$769.50, terms, 90 da., 4/30, 5 % cash.
$1287.50, terms, 4 mo., 5/30, 6 % cash.
$1760.80, terms, 30 da., 1 % cash.

17. On an invoice of $4378.60, a merchant is offered 4 mo. credit or a discount of 5 % for cash. Not having the ready money, he accepts the credit terms. What rate per cent of interest does he pay on the net amount of the bill? How much would he have saved if he had borrowed the money at 6 % and paid cash?

18. A man bought $35,000 U.S. Panama Canal Bonds bearing 2 % interest. Forty-six days later he sold them. How much interest did they earn while he owned them?

BANK DISCOUNT

452. Bank discount is a deduction made from the amount due at maturity on notes and drafts, in consideration of their being cashed or bought before maturity.

453. If a merchant has received from one of his debtors a draft payable in 60 da., and does not wish to wait for payment till it matures, he can take it to a bank and sell it, or discount it, as it is called. The discount is determined by reckoning the simple interest on the amount of the draft from the day it is discounted to the day of maturity. The merchant will receive the **proceeds**, which is the difference between the amount due on the draft at maturity and the discount.

454. As a rule, banks charge interest, or discount, for the *exact number of days* from the day of discount to the day of maturity. This time is called the **term of discount**.

NOTES. (1) In a few states, viz. Ark., Del., Ky., La., Md., Mo. (in large cities), N.C., Pa., Utah, and Va., the day of discount, as well as the day of maturity, is included in the term of discount, thus adding one day to the time.

(2) In Ark., Del., Miss., and Tex., notes, acceptances, etc., falling due on Sunday or a legal holiday, are payable the preceding business day. In all other states they are payable the next succeeding business day.

(3) In Ariz., Ark., Cal., Del., Ga., Ky., Me., Minn., Miss., N.H., N.C., Okla., S.C., S.D., Tex., and Wis., notes, acceptances, etc., falling due on Saturday, are payable the same day. In all other states they are payable the next succeeding business day.

(4) In discounting paper falling due on Saturday, Sunday, or a legal holiday, bankers charge discount for the time till the "next succeeding business day" (except where payable on the same or the preceding day).

(5) Answers to problems in bank discount in this book are prepared in accordance with the most common method used by bankers. It is suggested that the student follow the custom of his own state.

455. Days of grace are three days allowed by law, for the payment of notes and drafts, beyond the expiration of the time specified in the paper.

NOTE. Days of grace are allowed on notes, acceptances, and sight drafts in Ark., Miss., Okla., S.C., S.D., and Tex., and on sight drafts only in Mass., Minn., N.H., and R.I.

456. The solution of problems in bank discount is based on the principles of simple interest, or of percentage. The *value* of the note at maturity corresponds to the *principal* in *interest*, or to the *base* in *percentage;* the *rate of discount* to the *rate per cent* in *interest* and in *percentage;* the *bank discount* to the *interest* or the *percentage;* and the *proceeds* to the *difference.*

457. To find the date of maturity and the term of discount.

1. Find the date of maturity and the term of discount of a 3 months' note dated Apr. 14, and discounted May 3.

Apr. 14 + 3 mo. = July 14, date of maturity.
From May 3 to July 14 = 72 da., term of discount.

The date of maturity is found by counting forward 3 mo. from Apr. 14, which gives July 14. The term of discount is found by counting all the days from the date of discount, May 3, to the date of maturity, July 14. There are 28 da. remaining in May, 30 da. in June, and 14 da. in July, the sum of which is 72 da. (See Art. 260, page 116.)

2. Find the date of maturity and the term of discount of a 60 days' sight draft, dated Aug. 18, accepted Aug. 21, and discounted Sept. 4.

Aug. 21 + 60 da. = Oct. 20, date of maturity.
From Sept. 4 to Oct. 20 = 46 da., term of discount.

A draft drawn at 60 days' sight does not mature until 60 da. after it is accepted, hence, the date of maturity in this draft is 60 da. after Aug. 21, which is Oct. 20. Exact days must be counted when the time is expressed in days. (See Art. 266, page 119.) The term of discount is the exact number of days from Sept. 4 to Oct. 20, which is 46 da.

458. Copy the following and fill in the dates of maturity and the terms of discount:

	DATE OF PAPER	TIME	DATE OF ACCEPTANCE	DUE DATE	DATE OF DISCOUNT	TERM OF DISCOUNT
1.	Jan. 15	4 months			Mar. 10	
2.	Feb. 23	90 days			Mar. 1	
3.	June 10	3 months			July 18	
4.	May 8	90 days after sight	May 14		May 26	
5.	Oct. 7	2 months			Oct. 21	
6.	Aug. 11	30 days after sight	Sept. 1		Sept. 10	
7.	July 8	60 days			Aug. 14	
8.	May 1	45 days			May 14	
9.	Apr. 3	2 months after date	May 1		May 2	
10.	Feb. 18	90 days			Mar. 31	

459. To find the bank discount and the proceeds of a note or draft.

Find the date of maturity, the term of discount, the bank discount, and the proceeds of a 90-da. note of $1500, dated May 1 and discounted May 16, at 6 %.

May 1 + 90 days = July 30, date of maturity.

From May 16 to July 30 = 75 days, term of discount.

$15|00 = discount for 60 days.
3|7 = discount for 15 days.
$18|75 = discount for 75 days.

$1500 – $18.75 = $1481.25, proceeds.

The date of maturity and the term of discount are found as in the preceding exercise. The bank discount is the simple interest on the value of the note for the term of discount, 75 days, which is $18.75. The proceeds is the difference between the bank discount and the value of the note; $1500 – $18.75 = $1481.25.

460. Find the date of maturity, the term of discount, the bank discount, and the proceeds of the following notes and drafts:

	Face	Time	Date of Paper	Date of Acceptance	Date of Discount	Rate of Discount
1.	$1800.00	4 months	Aug. 1		Aug. 11	6 %
2.	2000.00	3 months	July 11		Aug. 1	6 %
3.	1640.00	90 days	May 13		June 4	6 %
4.	1150.00	60 days after sight	Mar. 18	Mar. 24	Apr. 7	5 %
5.	1990.00	30 days' sight	Apr. 27	May 3	May 6	5 %
6.	2870.00	5 months	June 19		Aug. 1	6 %
7.	3675.00	2 months	July 17		Aug. 1	7 %
8.	9400.00	60 days' sight	Oct. 11	Oct. 18	Nov. 1	6 %
9.	8478.00	30 days' sight	May 28	June 4	June 11	4½ %
10.	7200.00	3 months	Aug. 13		Aug. 19	4 %
11.	5900.00	1 month	Jan. 31		Feb. 4	5½ %
12.	8212.00	2 months	July 1		Aug. 14	6 %
13.	4750.00	4 months	Oct. 6		Nov. 6	7½ %
14.	5125.00	6 months	June 30		Oct. 18	6 %
15.	6000.00	90 days after date	Sept. 5	Sept. 25	Oct. 11	6 %
16.	5400.00	60 days after sight	Aug. 4	Aug. 11	Aug. 23	8 %
17.	4500.00	30 days' sight	July 5	July 11	July 11	7½ %
18.	3100.00	5 mo. after date	Jan. 12	Mar. 11	Mar. 13	6 %
19.	412.50	3 months	Apr. 12		May 1	9 %
20.	368.75	90 days	May 10		June 11	10 %

461. Bankers often use a table like the following to find the number of days between two dates:

BANKERS' TIME TABLE

FROM ANY DAY OF	TO THE SAME DAY OF THE NEXT											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
Apr.	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

462. How to use the table.

The table gives at a glance the exact number of days from any day of any month to the corresponding day of any other month for periods of time not greater than one year. Thus, the number of days from the 18th of May to the 18th of the next January is found by taking the number to the right of the month of May in the "January column," which is 245 da. From the 9th of April to the 9th of December is 244 da., found on the "April line" and the December column.

From Mar. 15 to July 28 is found thus: Take from the table the number of days from Mar. 15 to July 15, which is 122 da., and add to it the number of days from July 15 to July 28. From July 15 to July 28 is 13 da., which, added to 122 da., equals 135 da.

For the time from Oct. 25 to Apr. 10, find the time from Oct. 25 to Apr. 25, which is 182 da., and deduct the number of days from Apr. 10 to Apr. 25, which is 15 da. 182 da. less 15 da. leaves 167 da., the time from Oct. 25 to Apr. 10.

463. Find, by using the table, the number of days from:

1. Aug. 4 to Mar. 4.
2. Jan. 8 to Dec. 8.
3. Apr. 11 to Aug. 11.
4. Mar. 3 to Jan. 3.
5. May 30 to Nov. 30.
6. July 1 to Jan. 1.

7. Jan. 1 to July 1.
8. Oct. 7 to May 12.
9. Aug. 13 to Dec. 25.
10. June 24 to Nov. 12.
11. Mar. 17 to Dec. 13.
12. Apr. 18 to Jan. 4.
13. May 29 to Apr. 1.
14. Jan. 14 to Dec. 20.
15. June 17 to Oct. 20.
16. May 11 to Dec. 18.
17. July 3 to Nov. 24.
18. Feb. 15 to Dec. 10.
19. Sept. 1 to Oct. 19.
20. Sept. 19 to June 10.

464. When a bank discounts a note or draft that is payable in some other town or city, an additional charge called **collection** is often made. In large cities having a clearing house the collection charge is uniform for all banks that are members of the clearing house. Banks that are not members of the clearing house, and banks in small cities, have their own regulations concerning collection charges. The charge may be either a per cent of the value of the paper collected, or it may be an arbitrary amount for each note or draft.

465. The following are the charges made for collecting out-of-town paper by banks that are members of the New York Clearing House.

PAR POINTS	$\frac{1}{10}$ OF 1 PER CENT		$\frac{1}{4}$ OF 1 PER CENT		
Greater New York . . .	* Conn.	* N. Y.	Ala.	Kan.	Okla.
Albany, N. Y.	Del.	N. H.	Ariz.	La.	Oregon
Baltimore, Md.	D. C.	* N. J.	Ark.	Minn.	S. C.
Bayonne, N. J.	Ill.	Ohio	Cal.	Miss.	S. Dak.
Boston, Mass.	Ind.	* Pa.	Colo.	Mont.	Tenn.
Hoboken, N. J.	Ky.	* R. I.	Fla.	Nebr.	Tex.
Newark, N. J.	Me.	Vt.	Ga.	Nev.	Utah
Philadelphia, Pa. . . .	* Md.	Va.	Idaho	N. Mex.	Wash.
Providence, R. I. . . .	* Mass.	W. Va.	Ind. Terr.	N. C.	Wyo.
Troy, N. Y.	Mich.	Wis.	Iowa	N. Dak.	Can.
	Mo.				

* Except cities and banks listed as par. Minimum charge, ten cents.

In addition to the above "par points" there is a specified list of banks and trust companies in about 175 cities and villages in Connecticut, Massachusetts, New Jersey, New York, and Rhode Island on which items will be received at par in New York City.

466. To find the proceeds of a note or draft when collection is charged.

Find the proceeds of a 90-da. note of $ 3800 dated June 3, 1910, if discounted July 1, 1910, at 6 %; collection $\frac{1}{10}$ %.

June 3 + 90 da. = Sept. 1, due date.

From July 1 to Sept. 1 = 62 da., term of discount.

$ 38|00 = discount for 60 da.
1|27 = discount for 2 da.
$ 39|27 = discount for 62 da.

$\frac{1}{10}$ % of $ 3800 = $ 3.80, collection.
$ 39.27 + $ 3.80 = $ 43.07, the bank's total charge.
$ 3800 − $ 43.07 = $ 3756.93, the proceeds.

The date of maturity, the term of discount, and the bank discount are found as previously explained (pages 215, 216). The collection charge is reckoned on the value of the note; $\frac{1}{10}$ % of $ 3800 is $ 3.80. The bank's total charge is the sum of the bank discount, $ 39.27, and the collection charge, $ 3.80, or $ 43.07. The proceeds is the difference between $ 3800 and $ 43.07, or $ 3756.93.

467. To find the proceeds of an interest-bearing note.

Find the proceeds of a 4 months' 6 % interest-bearing note of $ 3200, dated May 15, 1910, and discounted June 10, 1910 at 6 %; collection $\frac{1}{8}$ %.

May 15 + 4 mo. = Sept. 15, due date.

From June 10 to Sept. 15 = 97 da., term of discount.

$ 32|00 = int. for 2 mo.
64|00 = int. for 4 mo.

32|64 = dis. for 60 da.
16|32 = dis. for 30 da.
3|264 = dis. for 6 da.
|544 = dis. for 1 da.
$ 52|768 = dis. for 97 da.

$ 3200 + $ 64 = $ 3264, value of the note at maturity.

$\frac{1}{8}$ % of $ 3264 = $ 4.08, collection.
$ 52.77 + $ 4.08 = $ 56.85, bank's total charge.
$ 3264 − $ 56.85 = $ 3207.15, proceeds.

The due date and the term of discount are first found, as explained on page 215, Art. 457. Next the interest on the face of the note, $ 3200, is found for the *entire time* the note has to run, 4 mo. Then the interest, $ 64, is added to the face of the note, $ 3200, which gives $ 3264, the value of the note at maturity. The discount is then reckoned for 97 da. on the full value of the note, and amounts to $ 52.77. The collection charge is reckoned on the value of the note at maturity, $ 3264. $\frac{1}{8}$ % of $ 3264 equals $ 4.08. The bank's total charge is equal to the sum of the bank discount and the collection, which is $ 56.85. $ 3264 − $ 56.85 = $ 3207.15, proceeds of note.

468. DISCOUNT REGISTER

No.	For Whom Discounted	Rate	Indorser	Collateral	Date	Time
1.	A. C. Montgomery	6%			Jan. 10	4 mo.
2.	D. C. Tarbell	6%	A. G. Ward		Mar. 15	3 mo.
3.	M. L. Hess	6%	W. C. Dowd		June 8	90 da.
4.	G. H. Dexter	6%		$2500 U.S. 3s	Oct. 17	60 da.
5.	S. M. Hamilton . .	6%	G. O. Hall		Aug. 12	60 da.
6.	J. S. Lyman	5%			Oct. 11	6 mo.
7.	L. L. Holmes	4%			June 12	90 da.
8.	A. H. Beeman . . .	4½%		$5000 N.Y.C. 4s	Apr. 20	75 da.
9.	L. S. Pierce	6%	A. Sohmer		May 10	40 da.

Above and on the opposite page is shown a form of a bank discount register. Rule a similar form and fill in the missing items.

469. Find the proceeds of the following notes:

	Face	Time	Date	Interest	Date of Discount	Rate of Discount	Collection
1.	$1400	2 mo.	Aug. 1	6%	Aug. 10	6%	⅛%
2.	1550	3 mo.	May 15	5%	June 1	5%	
3.	2160	90 da.	Feb. 15	8%	Mar. 17	7%	⅛%
4.	560	60 da.	Dec. 10	6%	Dec. 20	8%	¼%
5.	940	5 mo.	July 1	6%	Sept. 10	5%	¼%
6.	1120	4 mo.	Oct. 20	5%	Dec. 1	6%	
7.	4382.75	90 da.	Nov. 17		Nov. 29	6%	$\frac{1}{10}$%
8.	526.80	2 mo.	Dec. 8		Dec. 13	5%	¼%
9.	2600	3 mo.	Mar. 8		Mar. 13	6%	
10.	557.68	2 mo.	Dec. 4		Dec. 13	5%	¼%
11.	943.75	4 mo.	Oct. 1		Oct. 2	6%	
12.	321.60	30 da.	Jan. 31		Feb. 4	7½%	$\frac{1}{10}$%
13.	416.50	60 da.	Mar. 31		Apr. 13	4½%	¼%
14.	531.60	3 mo.	Nov. 30		Jan. 3	5%	¼%

DISCOUNT REGISTER

FACE	RATE OF INT.	INT.	AMOUNT	DUE	DATE OF DIS.	DAYS	DIS.	RATE OF EXCH.	EXCH.	PROCEEDS
$5000					March 8			$\frac{1}{20}$%		
$4500					Apr. 13			$\frac{1}{10}$%		
$2500					July 1			$\frac{1}{8}$%		
$1600					Nov. 10			$\frac{1}{4}$%		
$1748					Sept. 1			—		
$1372					Nov. 4			$\frac{1}{4}$%		
$2875	6 %				June 27			$\frac{1}{10}$%		
$3125	7½ %				May 5			$\frac{1}{8}$%		
$4700	8 %				May 14			—		

15.

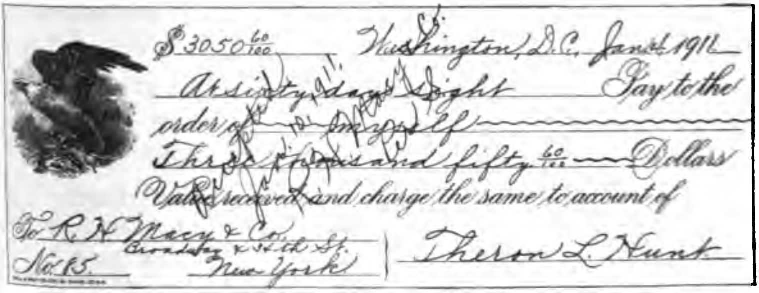
$4728.40 ROCHESTER, N. Y., June 3, 1910.

Two months after date we promise to pay to the order of C. J. HERMANS & Co. Forty-seven hundred twenty-eight $\frac{40}{100}$ Dollars, with interest at 6 %. Value received.

F. F. JONES & Co.

Discounted June 8 at 5 %; collection $\frac{1}{8}$ %.

16.



$3050 $\frac{60}{100}$ Washington, D.C. Jan. 4th 1912

At sixty days sight Pay to the order of myself

Three thousand fifty $\frac{60}{100}$ Dollars

Value received and charge the same to account of

To R. H. Macy & Co.
Broadway & 34th St.
No. 85. New York

Theron L. Hunt

Discounted Jan. 13 at 6 %; collection $\frac{1}{8}$ %.

TAXES

470. Who pays the salary of the governor, the legislators, the mayor, the policemen, and the teachers, in your state and city, or town?

How is the money raised for such purposes?

471. Money charged against persons and their property for the payment of public expenses is called a **tax**.

472. Taxes are of two kinds — direct and indirect.

473. A **direct tax** is a tax levied on a person, his property, or his business.

474. A tax on a person is called a **poll tax**.

475. A tax on property, real or personal, is called a **property tax**.

476. A tax on a person's business is called a **license fee**.

477. An **indirect tax** is a tax (called a duty) on imported goods or a tax (called excise or internal revenue) on the manufacture of liquors and tobacco products. The tax need not be paid on tobacco and liquors that are exported.

478. The persons appointed to make an estimate of the value of each person's property, and to apportion the taxes in proportion to the value of each person's property, are called **assessors**.

479. The assessors make what is called an **assessment roll**. It is a list showing the names of property owners, a brief description of the property owned, its assessed valuation, and the tax thereon.

480. The person who collects the tax from the property owners is called the **collector**.

481. Solutions of problems in taxes depend upon the principles of percentage. The **assessed valuation** is the *base.* The **tax rate** is the *rate.* The **tax** is the *percentage.*

482. The tax rate is frequently expressed as $1.80 per $100, instead of 1.8 %.

EXERCISES

483. 1. A tax of $.003 on a dollar is how much on $100? on $1000?

2. What is the tax on $4500 at the rate of $.65 per $100?

3. At $8.50 per $1000, what is the tax on $7800?

4. Find the tax on $15,400 at 1.365 %.

Find the tax on the following:

5. $13,500 at .0065.
6. $34,500 at .01467.
7. $44,800 at .00895.
8. $18,500 at .0102.
9. $125,000 at .00882.
10. $135,500 at .004.
11. $500,000 at .0115.
12. $240,000 at $1.64 per $100.
13. $15,900 at $1.845 per $100.
14. $32,600 at $13.26 per $1000.
15. $16,400 at $11.15 per $1000.
16. $69,500 at $.9875 per $100.
17. $51,600 at $1.065 per $100.
18. $12,600 at $12.853 per $1000.

Oftentimes the collector is allowed a commission on all taxes collected, instead of a salary.

484. 1. The town of Wellsville has an assessed valuation of $2,118,000. The total amount of tax to be collected is $18,917. There are 910 polls at $1 each, and $2200 highway tax not to be charged on the valuation. Find the rate of taxation and the amount of tax paid by a farmer whose property is assessed for $6400, who pays for 1 poll, and $3 highway tax.

$910 + $2200 = $3110, to be deducted from total tax.
$18,917 − $3110 = $15,807, tax levied on property.
$15,807 ÷ $2,118,000 = $.007463 = 7.463 mills on $1.
$6400 × .007463 = $47.76, property tax.
$47.76 + $1 + $3 = $51.76, total tax.

To find the amount of the tax to be levied on the property of the town, deduct the sum of the poll tax and the highway tax, which leaves $15,807. The tax on the property divided by the value of the property, gives the rate, which is 7.463

mills to the dollar. The property tax on a farm valued at $6400 is found by multiplying $6400 by .007463, which gives $47.76, to which is added $1, poll tax, and $3, highway tax, making a total tax of $51.76.

2. Find the total tax paid by a man whose property is assessed for $13,800, at 1.25 %, the collector's commission being 1 %.

1.25 % of $13,800 = $172.50, tax.
1 % of $172.50 = 1.73, collector's commission.
Total tax, $174.23.

The tax is 1¼ % of $13,800, or $172.50. The collector's commission of 1 % is reckoned on the amount of tax, and then added to the tax, and charged against the property owner.

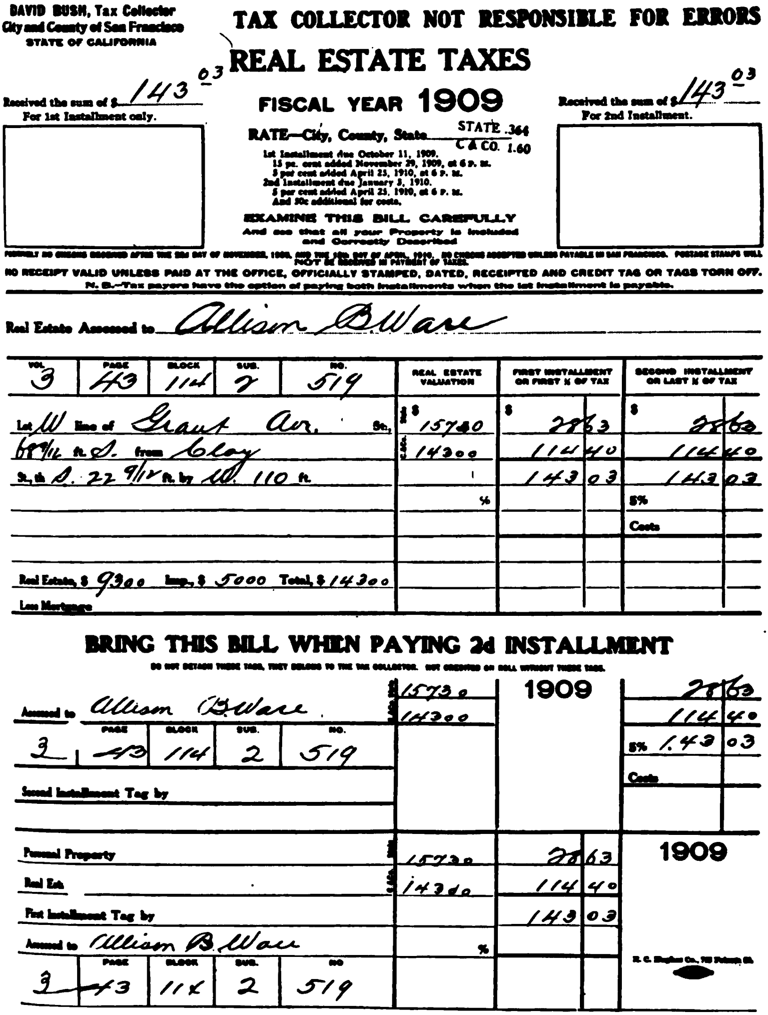
PROBLEMS

485. 1. The assessed valuation of the real and personal property in a certain town is $1,040,000. The total amount of tax to be collected is $7136. There are 206 polls at $1 each, and $1600, highway tax. Find the rate of taxation, and the amount of tax paid by a man whose property is assessed at $10,500, and who pays for 1 poll and a highway tax of $8.

In St. Louis, Mo., the tax rate for 1909 was, for state purposes, 17¢; for schools, 60¢; and for city purposes, $1.45 per $100. Taxes are due Sept. 1. A rebate of 8 % per annum from day of payment to Dec. 31 is allowed on city taxes only, if paid before Oct. 1. Taxes become delinquent after Dec. 31. The penalty on all delinquent taxes is 1 % a month from date of delinquency till paid, provided the taxes are paid before the Saturday preceding the first Monday in March. After said date all costs of collection are added.

Under the above conditions find the state, school, and city tax for the year 1909 on each of the following properties, if taxes were paid on the dates given:

	Description of Property	Valuation	Date Tax was Paid
2.	4-story apartment house . . .	$25,000	Sept. 25, 1909
3.	Lot 75′ × 100′	5,500	Feb. 18, 1910
4.	12-story office building . . .	1,200,000	Sept. 29, 1909
5.	2-story frame dwelling . . .	5,000	Dec. 30, 1909



DAVID BUSH, Tax Collector
City and County of San Francisco
STATE OF CALIFORNIA

TAX COLLECTOR NOT RESPONSIBLE FOR ERRORS

REAL ESTATE TAXES

FISCAL YEAR 1909

Received the sum of $ 143 03
For 1st Installment only.

Received the sum of $ 143 03
For 2nd Installment.

RATE—City, County, State STATE .364 C & CO. 1.60

1st Installment due October 11, 1909.
15 per cent added November 29, 1909, at 6 P. M.
5 per cent added April 25, 1910, at 6 P. M.
2nd Installment due January 3, 1910.
5 per cent added April 25, 1910, at 6 P. M.
And 50c additional for costs.

EXAMINE THIS BILL CAREFULLY
And see that all your Property is Included and Correctly Described

FORMERLY NO CHECKS RECEIVED AFTER THE 29th DAY OF NOVEMBER, 1909, AND THE 25th DAY OF APRIL, 1910. NO CHECKS ACCEPTED UNLESS PAYABLE IN SAN FRANCISCO. POSTAGE STAMPS WILL NOT BE RECEIVED IN PAYMENT OF TAXES.

NO RECEIPT VALID UNLESS PAID AT THE OFFICE, OFFICIALLY STAMPED, DATED, RECEIPTED AND CREDIT TAG OR TAGS TORN OFF.

N. B.—Tax payers have the option of paying both installments when the 1st installment is payable.

Real Estate Assessed to Allison B Ware

VOL.	PAGE	BLOCK	SUB.	NO.
3	43	114	2	519

	REAL ESTATE VALUATION	FIRST INSTALLMENT OR FIRST ½ OF TAX		SECOND INSTALLMENT OR LAST ½ OF TAX	
Lot W line of Grant Av. St.,	$ 15740	$ 28	63	$ 28	63
68 9/12 ft. S. from Clay	14300	114	40	114	40
St., th S. 22 9/12 ft. by W. 110 ft.		143	03	143	03
	%			5%	
				Costs	
Real Estate, $ 9300 Imp., $ 5000 Total, $ 14300					
Less Mortgage					

BRING THIS BILL WHEN PAYING 2d INSTALLMENT

DO NOT DETACH THESE TAGS, THEY BELONG TO THE TAX COLLECTOR. NOT CREDITED ON ROLL WITHOUT THESE TAGS.

Assessed to Allison B. Ware

	PAGE	BLOCK	SUB.	NO.
3	43	114	2	519

	$ 15730	1909	28	63
	14300		114	40
			5% 143	03
			Costs	
Second Installment Tag by				

Personal Property	$ 15730	28	63	1909
Real Est.	14300	114	40	
First Installment Tag by		143	03	
Assessed to Allison B. Ware	%			

	PAGE	BLOCK	SUB.	NO
3	43	114	2	519

Above is a form of tax bill used in San Francisco, Cal. Real estate valuation is higher for state purposes than for city and county, owing to a raise of 10% by the State Board of Equalization. Study this form carefully and verify the results given.

Using the information given in the tax bill on this page, find the tax paid in San Francisco on the following properties, the valuations being those given by the county assessor.

	Valuation	First Installment Paid	Second Installment Paid
6.	$1,960	Nov. 20, 1909	Apr. 10, 1910
7.	22,500	Apr. 20, 1910	Apr. 20, 1910
8.	13,600	June 1, 1910	June 1, 1910
9.	5,500	May 15, 1910	June 15, 1910
10.	3,200	Jan. 30, 1910	Feb. 8, 1910

The following table shows how the tax rate is apportioned in Chicago, Ill.:

Tax Rates for 1910

Town	State	County	City	School	Sanitary	Park	Town	Lake Shore Pro.	Blvd. and Park	Total
West Town	.30	.53	1.41	1.55	.34	.77			.05	4.95
South Town	.30	.53	1.41	1.55	.34	.51				4.64
North Town	.30	.53	1.41	1.55	.34	.54	.13	.08		4.88
Hyde Park	.30	.53	1.41	1.55	.34	.51				4.64
Lake	.30	.53	1.41	1.55	.34	.51				4.64
Lake View	.30	.53	1.41	1.55	.34	.76	.12			5.01
Jefferson	.30	.53	1.41	1.55	.34					4.13

The rates in the above table apply to the $100 assessed valuation, which is one third of the full value.

Taxes for 1910 are collectible in 1911.

On general taxes a penalty of 1% is charged after May 1, 2% after June 1, 3% after July 1, and so on, until paid. In addition to this, advertising and copying costs of 19¢ for each lot or part of a lot, and 29¢ for each tract of land, are added to unpaid general taxes and special assessments, early in May.

Using the rates and information given above, find the tax for each purpose and the total tax on the following properties:

	Description of Property	Assessed Valuation	Town	Date of Payment
11.	Office building	$150,000	North Town	April 20, 1911
12.	Three-story brown stone dwelling	90,000	Hyde Park	May 20, 1911
13.	Vacant lot 100′ × 100′ . .	15,000	Jefferson	July 30, 1911
14.	A tract of land containing 17 acres	17,000	Lake	June 25, 1911

In Denver, Col., taxes are paid in two payments of 50% each. Taxes are due Jan. 1 of each year. The first half is delinquent and draws interest at 1% a month on and after March 1. The second half is delinquent and draws interest at 1¼% a month on and after Aug. 1.

If the state tax is 21¢, and the city tax is $1.38 per $100, find the total tax paid on the following:

15. Store valued at $45,000, both installments paid July 20.

16. Dwelling valued at $12,500, first installment paid Feb. 27; second installment paid Dec. 1.

17. Hotel valued at $500,000, first installment paid July 20; second installment paid Oct. 10.

FIRE INSURANCE

486. **Insurance** is an agreement by one party, for a consideration, to indemnify another party for losses arising from certain stipulated causes.

487. **Fire insurance** treats of agreements to indemnify the insured for loss by fire.

488. "Loss by fire" is held to cover not only loss of property actually burned, but also loss or damage resulting from the use of chemicals or water used in extinguishing the fire, and from smoke. Fire caused by lightning comes within the meaning of the term "loss by fire."

489. The fire insurance policies of all companies in Connecticut, New Jersey, New York, and Pennsylvania, are uniform, and contain the "New York Standard (80%) Average Clause," which reads in part as follows: "This Company shall not be liable for a greater proportion of any loss or damage to the property described herein than the sum hereby insured bears to eighty per centum (80 %) of the actual cash value of said property at the time such loss shall happen." These policies also contain a "Waiver Clause," which reads: "In case of loss if the value of the property described herein does not exceed $ 2500, the 80 % average or co-insurance clause shall be waived."

In other states there is the ordinary policy, which stipulates the amount of loss for which the insurance company is liable. Under the ordinary policy the company pays any loss not exceeding the face of the policy.

490. In some states the policy contains an average clause which states that only such a part of the loss will be paid as the face of the policy bears to the value of the property insured.

491. If more than one company insures the same property, each company pays only its pro rata share of any loss.

492. In the **valued policy**, the value is stipulated and the company agrees to pay on that specified amount. The **open policy** is used to cover goods in storage and elsewhere, the amount of the policy varying as the quantity of goods is increased or diminished by additions or withdrawals. The addition and withdrawal of goods are recorded in an "Open Policy Book" retained by the company. Upon each receipt of goods, their nature and value are recorded in the policy book, and the premium charged is based on the annual rate.

In case of withdrawal in less than 1 yr. the company returns the unearned premium.

493. A policy may be canceled at any time, either by the insurer or by the insured, on 5 days' notice. If the insurance company cancels the policy, it will return to the insured such a part of the premium as the unexpired time of the policy is of the entire term of the policy. If, however, the insured cancels the policy, the company will return to him only the amount by which the premium paid is in excess of the premium reckoned at the short rate. The short rate is a correspondingly higher rate for a short period of time.

494. The principles of percentage apply in insurance.

The **value of the policy** is the *base.*

The **rate of premium** is the *rate.*

The **premium** is the *percentage.*

495. To find the premium.

If property is insured for $18,000 at 20¢ per $100 per annum, what is the annual premium?

$$\$18{,}000 = 180 \text{ hundreds of dollars.}$$
$$180 \times \$.20 = \$36, \text{ the annual premium.}$$

Since the premium rate is on $100, first find the number of hundreds of dollars by pointing off 2 places in the $18,000, which gives 180 hundreds dollars. If the premium on $100 is $.20, on 180 hundreds it is 180 times $.20, or $36.

PROBLEMS

496. Find the premium on each of the following policies:

	Face of Policy	Rate of Insurance		Face of Policy	Rate of Insurance
1.	$13,200	22¢ per $100	8.	24,600	$1\frac{1}{8}$%, less 10 %
2.	9,600	$37\frac{1}{2}$¢ per $100	9.	55,000	$\frac{7}{8}$%, less 10 % and 5 %
3.	16,400	45¢ per $100	10.	48,000	55¢ per $100, less 10 %
4.	27,500	65¢ per $100	11.	20,000	$\frac{3}{8}$%
5.	8,500	$\frac{1}{4}$%	12.	24,500	$27\frac{1}{2}$¢ per $100
6.	11,500	$\frac{5}{8}$%, less 10 %	13.	38,000	$\frac{4}{5}$%, less 20 %
7.	17,800	$\frac{3}{4}$%, less 15 %	14.	75,000	44¢ per $100, less 10 %

497. To find the amount paid by the insurer.

1. Property valued at $25,000 is insured for $15,000 at $\frac{3}{8}$% per annum. Fire and water cause a loss of $12,000. What amount would be paid by the insurance company (*a*) under an ordinary policy? (*b*) under a co-insurance clause policy? (*c*) under the New York Standard (80 %) Average Clause Policy?

(*a*) $12,000, amount paid under an ordinary policy.

(*b*) $\frac{15000}{25000} = \frac{3}{5}$.
$\frac{3}{5}$ of $12,000 = $7200, amount paid under a co-insurance policy.

(*c*) 80% of \$25,000 = \$20,000.
$\frac{15000}{20000} = \frac{3}{4}$.
$\frac{3}{4}$ of \$12,000 = \$9000, amount paid under New York policy.

(*a*) Under an ordinary policy the company pays the full amount of loss up to the amount of the policy; hence, \$12,000 would be paid in this case.

(*b*) Under a co-insurance, or average clause policy, the part of the loss paid is the ratio of the face of the policy, \$15,000, to the value of the property \$25,000, or $\frac{15000}{25000}$, which is equal to $\frac{3}{5}$. Hence in this case the company would pay $\frac{3}{5}$ of \$12,000, or \$7200.

(*c*) Under the 80% average clause policy, the company pays such a part of the loss as the ratio of the face of the policy, \$15,000, is to 80% of the value of the property, 80% of \$25,000 = \$20,000. The ratio of \$15,000 to \$20,000 is $\frac{15000}{20000} = \frac{3}{4}$. Hence the company would pay $\frac{3}{4}$ of \$12,000, or \$9000.

2. A stock of merchandise is insured in Company A for \$5000, in Company B for \$7000, and in Company C for \$8000. In case of damage amounting to \$10,000, how much will each company pay?

\$5000 + \$7000 + \$8000 = \$20,000, total amount of insurance.
$\frac{5000}{20000} = \frac{1}{4}$; $\frac{1}{4}$ of \$10,000 = \$2500, paid by Company A.
$\frac{7000}{20000} = \frac{7}{20}$; $\frac{7}{20}$ of \$10,000 = \$3500, paid by Company B.
$\frac{8000}{20000} = \frac{2}{5}$; $\frac{2}{5}$ of \$10,000 = \$4000, paid by Company C.

The total insurance is \$20,000. Each company pays such part of the loss as its risk is of the total risk. Hence, Company A pays $\frac{1}{4}$ of the loss, or \$2500; Company B pays $\frac{7}{20}$ of the loss, or \$3500; and Company C pays $\frac{2}{5}$ of the loss, or \$4000.

PROBLEMS

498. **1.** A dwelling valued at \$9600 was insured for $\frac{2}{3}$ of its value at $\frac{3}{4}$% and contents valued at \$3200, for $\frac{3}{4}$ of its value at $\frac{4}{5}$%. Fire causes a total loss of the building and a loss of \$1400 on the contents. Under an ordinary policy, how much will the insurance company pay?

2. Property insured under an ordinary policy for \$15,000 at 60¢ per \$100 per term of 3 yr. is damaged by fire to the amount of \$8000. What is the insurance company's net loss if they have held the insurance 15 yr., interest 6%?

3. A store and its contents were insured in Company X for \$22,500 at 60¢ per \$100; in Company Y for \$30,000 at $\frac{5}{8}$%; and in Company Z for \$40,000 at 65¢ per \$100. The property was damaged by fire and water to the amount of $40,000. What was each company's net loss, they having held the insurance 7 yr., if money is worth 5 %?

4. A man has his furniture insured for $\frac{2}{3}$ of its value, in a policy containing an average clause. In case of a loss of $2400, how much should the company pay?

5. A house valued at $15,000, and contents valued at $7500, are insured for $\frac{2}{3}$ of their value. Fire causes a loss of $4000 on the house and $1500 on the contents. Find the amount payable under an 80 % average policy.

499. The following tables show the short rates adopted by the "Western Union":

Standard Short Rate Scale for computing Premiums for Terms less than One Year, as adopted by the "Western Union"

RULE. Take the percentage indicated in scale opposite the number of days risk is to run, on the premium for one year at given rate, and the result will be the premium earned in case of cancellation, or to be charged in case of short risks.

1 Day	2 per cent annual prem.	55 Days	29 per ct. an'l prem.
2 Days	4 per cent annual prem.	60 Days	30 per ct. an'l prem.
3 Days	5 per cent annual prem.	65 Days	33 per ct. an'l prem.
4 Days	6 per cent annual prem.	70 Days	36 per ct. an'l prem.
5 Days	7 per cent annual prem.	75 Days	37 per ct. an'l prem.
6 Days	8 per cent annual prem.	80 Days	38 per ct. an'l prem.
7 Days	9 per cent annual prem.	85 Days	39 per ct. an'l prem.
8 Days	9 per cent annual prem.	90 Days or 3 mo.	40 per ct. an'l prem.
9 Days	10 per cent annual prem.	105 Days	45 per ct. an'l prem.
10 Days	10 per cent annual prem.	120 Days or 4 mo.	50 per ct. an'l prem.
11 Days	11 per cent annual prem.	135 Days	55 per ct. an'l prem.
12 Days	12 per cent annual prem.	150 Days or 5 mo.	60 per ct. an'l prem.
13 Days	13 per cent annual prem.	165 Days	65 per ct. an'l prem.
14 Days	13 per cent annual prem.	180 Days or 6 mo.	70 per ct. an'l prem.
15 Days	14 per cent annual prem.	195 Days	73 per ct. an'l prem.
16 Days	14 per cent annual prem.	210 Days or 7 mo.	75 per ct. an'l prem.
17 Days	15 per cent annual prem.	225 Days	78 per ct. an'l prem.
18 Days	16 per cent annual prem.	240 Days or 8 mo.	80 per ct. an'l prem.
19 Days	16 per cent annual prem.	255 Days	83 per ct. an'l prem.
20 Days	17 per cent annual prem.	270 Days or 9 mo.	85 per ct. an'l prem.
25 Days	19 per cent annual prem.	285 Days	88 per ct. an'l prem.
30 Days	20 per cent annual prem.	300 Days or 10 mo.	90 per ct. an'l prem.
35 Days	23 per cent annual prem.	315 Days	93 per ct. an'l prem.
40 Days	26 per cent annual prem.	330 Days or 11 mo.	95 per ct. an'l prem.
45 Days	27 per cent annual prem.	360 Days or 12 mo.	100 per ct. an'l prem.
50 Days	28 per cent annual prem.		

500. To find the cost of insurance at short rates, or the amount of premium to be returned if policy is canceled.

1. Find the cost of insuring a stock of merchandise for $15,000 for 3 mo. if the annual rate is $\frac{7}{8}$%.

$\frac{7}{8}$% of $15,000 = $131.25, annual premium.
40% of $131.25 = $52.50, premium for 3 mo.

By the short rate scale, page 325, the premium for 3 mo. is 40% of the annual premium. The annual premium is $131.25; hence, the premium for 3 mo. is 40% of $131.25, or $52.50.

2. Property valued at $24,000 is insured for $\frac{3}{4}$ of its value for 1 yr. at 60¢ per $100. How much of the premium should be returned if the policy is canceled at the expiration of 8 mo. (*a*) by the insured? (*b*) by the insurance company?

(*a*) $\frac{3}{4}$ of $24,000 = $18,000.
180 × $.60 = $108, annual premium.
20% of $108 = $21.60, amount to be returned if insured cancels the policy.

(*b*) 4 mo. : 12 mo. : : $? : $108.
$\frac{4 \times \$108}{12}$ = $36, amount to be returned if insurer cancels the policy.

(*a*) By the short rate scale the rate for 8 mo. is 80% of the annual rate. That is, the company will return 20% of the premium paid. The annual premium is $108; hence, the amount returned is 20% of $108, or $21.60.

(*b*) When the insurer cancels the policy, the amount returned bears the same ratio to the entire premium paid as the unexpired time bears to the entire time; hence, the proportion 4 mo. : 12 mo. : : $? : $108, the solution of which gives $36, the amount to be returned.

PROBLEMS

501. 1. A policy of insurance for $10,000 at $\frac{3}{4}$% per annum was dated Apr. 7, 1910. 5 mo. later it was canceled by the insured. How much of the premium was returned?

2. May 10, 1910, I took out a policy of insurance on my furniture for $2400 at 40¢ per $100 per annum. Feb. 19, 1911, I canceled the policy. How much of the premium should be returned to me?

3. A stock of merchandise was insured for 1 yr. for $25,000 at $\frac{7}{8}$ %. At the end of 7 mo. the insurance company canceled the policy. How much of the premium was returned? (Solve by proportion.)

4. A merchant insured his store and contents for 1 yr. for $16,500 at $\frac{3}{5}$ %. At the end of 5 mo. and 15 da. the insurer canceled the policy. What was the return premium? (Solve by proportion.)

5. A policy of $32,000 at $\frac{5}{8}$ % for 1 yr. was canceled by the insured at the expiration of 8 mo. Find the amount of the return premium. If the company had canceled the policy, what amount of premium would have been returned?

6. The Ætna Insurance Company insured an office building for $45,000 at $\frac{1}{2}$ % for 1 yr. Find the return premium if the policy was canceled at the end of 6 mo. (*a*) by the company; (*b*) by the insured.

7. Find the cost of insuring a stock of goods for $12,000 for 6 mo. at 75¢ per $100 per annum.

REVIEW PROBLEMS

502. 1. Find the cost of insuring a house for $8000 for 3 yr. if the annual premium is $\frac{1}{4}$ %.

2. At 60¢ per $100 per annum, what is the cost of insuring a shipment of merchandise for $6500 for 30 da.?

3. Property worth $75,000 is insured as follows: In the Hartford Insurance Company for $20,000 at $1\frac{1}{4}$ %, and in the Albany Insurance Company for $30,000 at $1\frac{1}{8}$ %. There is a $30,000 loss by fire during the first term of insurance. Find each company's net loss under (*a*) an ordinary policy; (*b*) an average clause policy.

Find the amount to be paid in each of the following cases, supposing each policy contains the Standard 80 % Average Clause:

	Value of Property	Face of Policy	Damage by Fire	Paid by Insurance Co.
4.	$20,000	$12,000	$ 8,000	?
5.	28,000	15,000	15,000	?
6.	12,000	5,000	12,000	?
7.	16,500	10,000	3,000	?
8.	45,000	20,000	25,000	?

9. An open policy of insurance was issued on merchandise stored, or to be stored, in a warehouse, the premium to be 80¢ per $100 per annum. Goods withdrawn inside of 1 yr. were to be charged the short rate. Find the total premium paid, and the total return premium on the following receipts and withdrawals:

Receipts		Withdrawals	
Date	Amount	Date	Amount
Apr. 1, 1910	Silk, $ 8,400	Oct. 1, 1910	Woolens, $5000
Apr. 15, 1910	Furs, 12,800	Oct. 10, 1910	Furs, 9000
May 10, 1910	Woolens, 11,500	Oct. 20, 1910	Silk, 8400
June 1, 1910	Gloves, 6,500	Nov. 10, 1910	Gloves, 4500
June 20, 1910	Hosiery, 5,600	Dec. 1, 1910	Woolens, 6500

POSTAL SAVINGS BANKS

503. The **postal savings** system was established in the United States January 3, 1911, at 48 second-class post offices, one in each state and territory. By Jan. 31, 1913, there were 12,823 offices where patrons of the system might deposit their funds.

504. Deposits are evidenced by postal-savings certificates issued in fixed denominations of $1, $2, $5, $10, $20, and $50, each bearing the name of the depositor, the number of his account, the date of issue, the name of the depository office, and the date on which interest begins.

505. Interest is allowed on all deposits at the rate of 2 per cent per annum, computed on each savings certificate separately, and payable annually. No interest is paid on money that remains on deposit for a fraction of a year only.

506. Deposits bear interest from the first day of the month next following that in which they are made.

507. Compound interest is not allowed on an outstanding certificate, but a depositor may withdraw interest when due and include it in a new deposit, which will bear interest at the regular rate.

508. A depositor is permitted to exchange the whole or any part of his deposits in sums of $20, $40, $60, $80, $100, or multiples of $100 up to and including $500, for United States registered or coupon bonds bearing interest at the rate of 2½ per cent per annum, payable semiannually, and redeemable at the pleasure of the United States after one year from date of issue, both principal and interest payable 20 years from such date in United States gold coin. Such exchange may be made under date of Jan. 1 and July 1 of each year, provided such bonds are then available.

PROBLEMS

509. 1. On Feb. 28, 1911 (after 2 months' operation), the total number of depositors was 3664, and the amount on deposit was $133,869. What is the average for each post office? for each depositor? At the same rate, what would be the total amount on deposit at the expiration of one year?

2. At Globe, Ariz., 138 depositors had the largest average balance, $70.63; at Clifton Forge, Va., 27 depositors had the smallest average balance, $9.07. Find the average for each depositor at these two towns, and compare with the average for all depositors.

3. The population of the 48 towns having postal savings banks Feb. 28, 1911, was 372,000, and the population of the United States is 91,000,000. If postal savings banks were established at every post office in the United States, and were patronized in the same proportion as the 48 banks then in operation, what would be the amount on deposit at the end of one year?

4. A man deposits $10 each month in a postal savings bank for five years. What amount will he then have to his credit?

EXAMINATIONS

SPEED TEST

510. Minimum time thirty minutes; maximum, one hour. Deduct one credit for each minute beyond minimum time.

1. Find the interest in the following cases:

$500 for 2 yr. 2 mo. 6 da. at 6 %.
$300 for 5 yr. 6 mo. 18 da. at 6 %.
$930 for 6 yr. 8 mo. at 5 %.
$750 for 4 yr. 6 mo. at 3½ %.

2. Find the proceeds of the following notes discounted at a bank:

$350	90 da.	6 %.
$750	75 da.	6 %.
$960	45 da.	4 %.
$150	72 da.	5 %.

3. Find amount due July 1, 1914, on the following:

Face of note, $3000; date, June 1, 1912; interest, 6%.

4. Find the accurate interest in the following cases:

$3650 for 125 da. at 5%.
$7300 for 116 da. at 6%.
$1095 for 17 da. at 4%.
$ 550 for 146 da. at 3½%.

5. Find the proceeds of each of the following notes:

$ 515 due in 90 da. at 6%.
$2020 due in 72 da. at 5%.
$ 858.50 due in 60 da. at 6%.
$1540.25 due in 90 da. at 4%.

6. Find the total insurance on the following properties:

Store worth $25,000 at ⅘% premium.
Mdse. worth $40,000 at 1¼% premium.
House worth $15,000 at 60¢ per $100 premium.
Furniture worth $5000 at 80¢ per $100 premium.

7. Find the total tax on the following:

Farm valued at $2400 at .0175.
House and lot valued at $18,000 at .01875.
City lot valued at $5500 at $1.60 per $100.
Store valued at $30,000 at $1.7246 per $100.

8. At what price each shall these articles be sold to gain the rates given?

Cost per Dozen	Rate of Gain
$18	20%
$24	40%
$15	33⅓%
$14.40	25%

9–10. Add vertically and horizontally and find grand total:

16832	21265	17234	18875
15293	17294	16872	5570
13846	17286	9237	18765
16450	18642	27436	18778
8287	18643	21299	17694

WRITTEN TEST

511. 1. In a certain county containing 25,482 taxable inhabitants, a tax of $103,294.60 is assessed for town, county, and state purposes: a part of this sum is raised by a tax of 30¢ on each poll; the entire valuation of property on the assessment roll is $38,260,000. What is the per cent of tax, and what is a person's tax who pays for 1 poll and whose property is valued at $9470?

2. Find the interest on $738.72 for 1 yr. 3 mo. 22 da., at 5 %.

3. I bought a horse for $360 and sold it for a note at 60 da. for $418. I discounted the note at 6 % the day it was made. What was my gain?

4. A store was insured for $12,000 at ¾ % and the goods for $15,000 at 1¼ %. What was the entire premium?

5. A grocer sold 50 bbl. of wine, each containing 31 gal. 2 qt., at $2.40 a gallon, receiving in payment a note at 90 da., with interest at 6 %. What would be the proceeds of this note when discounted at a bank at 7½ % per annum?

6. In a given town the real estate is assessed at $1,250,000, and the personal property at $150,000. The tax to be raised therefrom amounts to $19,250. Find the tax rate. How much tax does a man pay whose property is valued at $12,000, if it is assessed at 80 % of its value?

7. A man deposited $15,000 in a bank June 1, 1910, receiving a certificate of deposit which stated that 3 % interest would be paid if the money was left on deposit 6 mo. or more. The bank loaned the money at 5 %. Dec. 20, 1910, the man asked for his money and interest. If the bank paid exact interest and charged ordinary interest, find the gain to the bank, the money having been loaned all the time.

8. A merchant bought a bill of goods invoiced at $5475.76, less 10 % and 5 %, terms net 60 da. or 5 % 10 da. Not having the ready money, he borrowed it at 6 % interest and accepted the 10-da. terms. Find his gain by so doing.

9. A stock of household furniture valued at $1800 was insured for 3 years from April 1, 1913, at 70¢ per $100. July 1, 1914, the company canceled the policy. Find the amount of the return premium.

10. A factory was insured under a policy containing the standard 80 % average clause, as follows: Building for $22,000 at 1¼ %; machinery for $25,000 at 1⅛ %; and stock for $15,000 at 1⅜ %. The building was worth $30,000, the machinery $35,000, and the stock $20,000. Fire broke out causing a damage as follows: to the building, $8000; machinery, $6000; and to the stock, $12,000. Find the amounts paid by the company.

STOCKS

512. **Stock, or capital stock,** is the name given to the capital of a corporation.

513. A **corporation** is a person composed of several real persons. It is formed only by the consent of the state, and hence, has only those rights and powers that are granted to it by the state as set forth in its charter. The list of powers, rights, and duties are stated in writing and issued to the corporation. This written instrument is called a **charter.**

514. The capital stock of a corporation is divided into shares, usually $ 100 each, though shares may be of various values. Reading Railroad shares are $ 50 par value. Mining stocks are at all prices from 10¢ to $ 100.

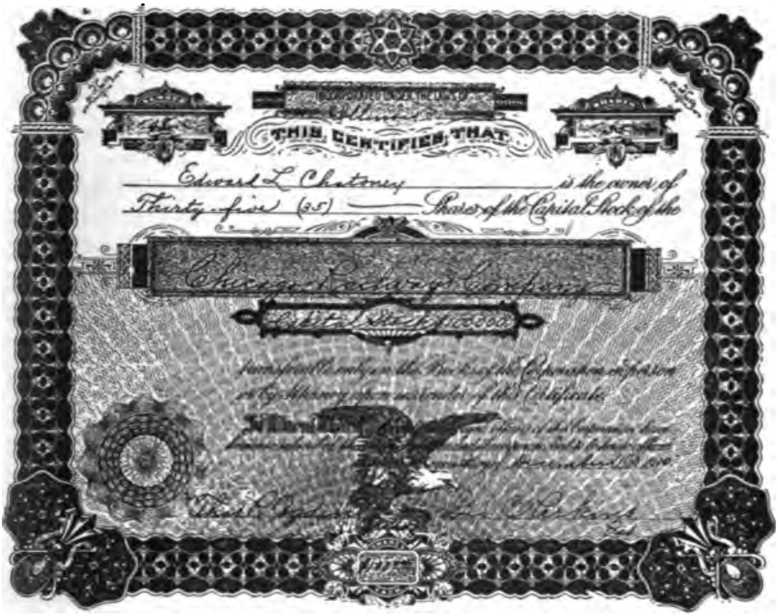
515. A **stockholder** is any person who owns one or more shares of stock. As evidence of ownership of stock, each stockholder receives a written statement called a **stock certificate,** showing the number of shares he owns and their par value.

516. Stock is either common or preferred. **Common stock** is the kind most usually issued by a corporation, and carries no guaranty of a specified dividend. **Preferred stock** carries with it a guaranty of a definite dividend even to the exclusion of dividends on the common stock. Preferred stock is often issued in times of emergency to provide working capital or to pay debts.

517. The **par value** of stock is the value stated on the stock certificate. The **market value** is the price for which the stock can be sold.

518. The market value of stock depends chiefly upon the rate of dividends paid or earned by the company. If the dividends are small, the market value is below par (less than face value, or at a discount), and if the dividends are large, the market value is above par (or at a premium).

519. ILLUSTRATION OF A STOCK CERTIFICATE



Incorporated under the laws of Illinois

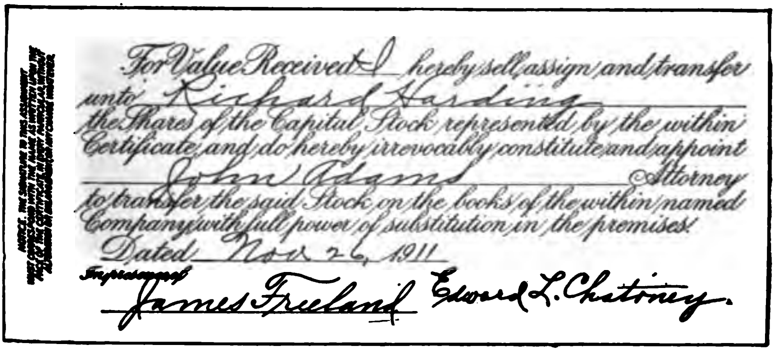
THIS CERTIFIES THAT

Edward L. Chatoney is the owner of

Thirty-five (35) Shares of the Capital Stock of the

520. Shares of stock are transferred by assignment. The assignee, then, has all the rights and privileges of the original stockholder.

521. FORM OF ASSIGNMENT



NOTICE. THE SIGNATURE TO THIS ASSIGNMENT MUST CORRESPOND WITH THE NAME AS WRITTEN UPON THE FACE OF THE CERTIFICATE IN EVERY PARTICULAR WITHOUT ALTERATION OR ENLARGEMENT OR ANY CHANGE WHATEVER.

For Value Received I hereby sell, assign and transfer unto Richard Harding the Shares of the Capital Stock represented by the within Certificate and do hereby irrevocably constitute and appoint John Adams Attorney to transfer the said Stock on the books of the within named Company with full power of substitution in the premises.

Dated Nov 26, 1911

In presence of

James Freeland Edward L. Chatoney.

522. The following prices of certain stocks were printed in the *New York Journal of Commerce*, Jan. 7, 1911.

NEW YORK STOCK EXCHANGE

STOCK EXCHANGE SALES AND QUOTATIONS JAN. 6, 1911

Sales		Open	High	Low	Close	Net Chg.
15,600	Amal Copper	63⅝	64⅞	63⅝	64⅞	+ 1⅜
600	Am Cotton Oil	59	59	58⅜	58⅜	− ⅝
300	Am Locomotive	39½	39⅝	39½	39⅝	+ ½
500	Am Sugar	114	114	113	113	− 2
900	Am Tel & Tel	141½	141½	141⅜	141⅜	..
2,900	Atchison	102¼	102¾	102½	102½	+ ¼
2,100	Ches & Ohio	82¼	82¼	81⅝	82	− ⅛
3,600	C M & St P	124¾	125¼	124¾	125	+ ¼
500	Chi & N W	142¾	142¾	142½	142½	..
250	Col & South 1st pfd . . .	76	76	75¾	75¾	+ ¼
6,600	Consol Gas	140¼	140½	139¾	140½	+ ¼
200	Del & Hudson	166	166½	166	166½	− ⅛
500	Den & R G	29¼	29¾	29¼	29⅝	+ ⅜
600	Erie	28	28	27⅝	27⅝	+ ⅛
100	do 1st pfd	46⅛	46⅛	46⅛	46⅛	− ⅜
200	Illinois Central	132⅞	132⅞	132½	132½	..
300	Louis & Nash	145	145¼	145	145	..
100	Mer Marine pfd	16	16	16	16	+ ⅜
400	Missouri Pacific	48	48	47⅝	48	..
200	National Biscuit	118⅛	118⅛	118	118	− 1
100	New York Central	112	112	111½	111½	− ½
100	Northern Pacific	118¼	118¼	117¾	118	..
800	Pacific Tel & Tel	45	45⅛	44¾	45⅛	+ ⅛
3,700	Pennsylvania R R	129¼	129⅝	129	129⅛	− ⅛
86,100	Reading	153⅝	154⅛	153¼	154	+ ⅝
4,900	Rock Island	30¼	30⅞	30½	30½	+ ½
3,300	Southern Pacific	116⅜	116⅝	116⅛	116⅛	− ⅜
900	Tenn Copper	34⅞	35¾	34⅞	35¾	+ 1¼
38,000	Union Pacific	173⅝	174⅛	173¼	173¾	+ ⅜
48,100	U S Steel	74	74⅜	73⅞	74⅛	+ ⅛
1,100	do pfd	117½	118	117½	118	+ ½
1,200	Wisconsin Central	59	60⅜	59	60	+ 1½

523. The profits of a corporation, after deductions are made for surplus, reserve for bad debts, sinking fund, etc., are divided among the stockholders as follows: To the preferred stockholders (if any) is paid the dividend guaranteed on their stock; to the common stockholders, the balance in proportion to the number of shares they own.

NOTES. (1) In some cases, however, the preferred stockholders share equally with the common stockholders if the profits are sufficient to pay on all stock more than the guaranteed rate on the preferred stock.

(2) Dividends are always reckoned as a certain rate per cent of the par value of the issued stock of the company.

524. An **assessment** is a certain per cent of the par value of the stock levied against the stockholders to pay losses, etc.

525. Stocks are bought and sold by **stockbrokers**, who make a business of buying and selling stocks for others. For their services they charge a commission, or brokerage, usually $\frac{1}{8}$ % of the par value of the stocks handled.

526. **Watered stock** is stock based on no real value. Certificates of stock are issued which do not represent capital paid in. The chief purpose of issuing watered stock is to conceal from the public the true rate of profit earned by the company.

527. Stocks are bought and sold under various conditions, some of which are:

1. "Cash," meaning that the stock is to be delivered the day of the sale.
2. "Regular way," which means that the stock is to be delivered the day after the sale.
3. "Buyer 3, 10, 20, 30, 60," in which case the buyer has the right to call for the delivery of the stock any time within the number of days specified. If the buyer's option is for more than 3 da., he is charged interest on the purchase price of the stock unless "flat" (without interest) is specified.
4. "Seller 3, 10, 20, 30, 60," sold deliverable at the seller's option within the number of days specified. Interest is charged the buyer, if the option is for more than 3 da., unless sold "flat."

528. EXPLANATION OF STOCK EXCHANGE TERMS

1. **Bear.** A broker who operates for declining prices. He has sold "short" (see 6), and wishes to cover his short sale at a lower price.
2. **Bull.** A broker who operates for rising prices. He has bought stocks (is "long"—see 5) and wishes to sell them at higher prices.
3. **Collaterals.** Stocks or bonds deposited as security for a loan.
4. **Ex-dividend.** A sale ex-dividend is a sale in which the dividend just declared or about to be declared does not go to the purchaser. Sales are usually made ex-dividend from the day the transfer books of a corporation are closed to the day they are opened.

5. **Long.** A trader who buys stocks for a rise in price is long.

6. **Short.** When a trader has sold more stock than he owns, he is said to be short.

7. **Margin.** A deposit made with a broker by a person who wishes to buy more stock than he can pay for, with the expectation of a rise. The broker furnishes or borrows the balance of the cost of the stocks. The margin protects the broker in case of a decline.

529. **To find the cost of a purchase, or the proceeds of a sale, of stock.**

1. Find the cost, including brokerage, of 250 shares of Amalgamated Copper at the lowest price quoted on page 240.

$$\$63\tfrac{3}{8} + \$\tfrac{1}{8} = \$63\tfrac{1}{2}, \text{ gross cost of 1 share.}$$
$$250 \times \$63\tfrac{1}{2} = \$15{,}875, \text{ cost.}$$

The quotation is $\$63\frac{3}{8}$ per share. The brokerage is $\frac{1}{8}\%$ of the par value of the stock, but in all cases in which the par value of the stock is $100, the brokerage is equal to $\$\frac{1}{8}$ a share, and hence increases the cost of a share by $\$\frac{1}{8}$. 250 shares cost 250 times $\$63\frac{1}{2}$, or $15,875.

2. Find the proceeds of 400 shares of Canadian Pacific at 199.

$$\$199 - \$\tfrac{1}{8} = \$198\tfrac{7}{8}, \text{ proceeds of 1 share.}$$
$$400 \times \$198\tfrac{7}{8} = \$79{,}550, \text{ proceeds.}$$

The brokerage is deducted from the quotation, giving $\$198\frac{7}{8}$ as the proceeds of 1 share. The proceeds of 400 shares is 400 times $\$198\frac{7}{8}$, or $79,550.

530. Using the quotations on page 240, find the cost, including brokerage, at the lowest price, and the proceeds, including brokerage, at the highest price, on the following:

Find the cost of:

1. 350 Am Locomotive.
2. 125 Am Tel & Tel.
3. 50 Chi & N W.
4. 175 Consol Gas.
5. 225 Del & Hudson.
6. 325 Louis & Nash.
7. 500 Mer Marine pfd.
8. 750 New York Central.
9. 600 Northern Pacific.
10. 800 U S Steel.

Find the proceeds of:

11. 200 Illinois Central.
12. 275 Missouri Pacific.
13. 175 National Biscuit.
14. 450 Rock Island.
15. 375 Southern Pacific.
16. 650 Tenn Copper.

17. 500 Atchison.
18. 400 Pacific Tel and Tel.
19. 1250 Union Pacific.
20. 200 Wisconsin Central.

Find the gain or loss on the following stocks if bought at the lowest and sold at the highest price quoted, allowing for brokerage each way:

21. 150 Am Cotton Oil.
22. 200 Am Sugar.
23. 300 Ches & Ohio.
24. 400 C M & St. P.
25. 250 Col and South. 1st pfd.
26. 350 Den & R G.
27. 500 Erie 1st pfd.
28. 450 Pennsylvania R R.
29. 800 Reading.*
30. 1000 U S Steel pfd.

531. To find the number of shares.

How many shares of Manhattan Elevated at 140½ can be bought for $ 33,750, brokerage ⅛ % ?

$$\$140\tfrac{1}{2} + \$\tfrac{1}{8} = \$140\tfrac{5}{8}\text{, gross cost 1 share.}$$

$$\$33{,}750 \div \$140\tfrac{5}{8} = 240\text{, no. shares.}$$

The quotation, $ 140½, plus the brokerage, $ ⅛, gives $ 140⅝, the gross cost of 1 share. $ 33,750 will buy as many shares as $ 140⅝ is contained times in $ 33,750, or 240 times.

PROBLEMS

532. 1. At 35, how many shares of Texas and Pacific can be bought for $ 9835, including brokerage?

2. Western Union Telegraph is quoted at 76¼. How many shares can be bought for $ 12,260, usual brokerage?

3. I sold stock through a broker at 112½, and received therefrom $ 13,485. How many shares were sold?

4. To meet a debt of $ 18,550, a man sold through his broker Pacific Telephone and Telegraph stock at 46½. How many shares did he have to sell?

5. A broker bought on his own account 1000 shares of Pressed Steel Car stock at 49¾. When the price advanced to 51¼, he sold enough of the stock to gain $ 1200. How many shares did he sell?

---

* Par value $ 50 a share.

533. To find the dividend on stocks.

1. A man invested $27,720 in American Locomotive pfd. at 115⅜. If the stock paid 6 % dividends, what was his annual income?

$115⅜ + $ ⅛ = $115½, gross cost of 1 share.
$27,720 ÷ $115½ = 240 no. shares.
Value of 240 shares = $24,000, par value of the stock.
6 % of $24,000 = $1440, annual income.

Since the dividends are always based on the par value of the stock, it is necessary, first, to find the number of shares of stock (see art. 531), which is 240. The par value of the stock is, then, $24,000 (240 × $100). 6 % of the par value gives the annual income, $1440.

2. The capital stock of a corporation is $5,000,000. The gross earnings are $1,250,000; the operating expenses are $900,000. $50,000 is carried to the surplus fund. What per cent dividend can be declared? If A owns 250 shares, how much will he receive?

$900,000 + $50,000 = $950,000, to deduct from gross earnings.
$1,250,000 − $950,000 = $300,000, total dividend.
$300,000 ÷ $5,000,000 = .06 = 6 %, rate of dividend.
Value of 250 shares = $25,000, par value of A's stock.
6 % of $25,000 = $1500, A's dividend.

From the gross earnings there are deducted the operating expenses, and the amount set aside for surplus fund, which leaves $300,000 as the total dividend. Dividing by the capital stock, $5,000,000, gives 6 %, the rate of dividend. The par value of A's stock is $25,000. 6 % of $25,000 gives $1500, A's dividend.

PROBLEMS

534. 1. An investment of $39,183.75 was made, through a broker, in Louisville and Nashville at 145. The stock pays regular dividends of 8 %. Find the dividend.

2. A gentleman drew $44,550 from a bank paying 3½ % interest annually, and invested it in U. S. Steel at 74⅛, brokerage ⅛ %, paying 4 % dividends. Find the increase or decrease in his income.

3. A man sells 25 shares of Union Pacific stock, paying 8 %, at 199½, brokerage ⅛ %, and deposits the proceeds in savings banks paying 4 % interest. Does he gain or lose in his annual income?

4. The Delaware & Hudson Railroad Company has a capital stock of $42,400,000. The gross income for 1 yr. was $44,829,555.20; the total expenses were $38,363,381.79. Find the largest whole per cent dividend that could be declared, and the balance left over for the reserve fund. What amount of dividend would be received by a man who owned 1500 shares of the stock?

535. To find the investment required to produce a given income.

How much must be invested in Amalgamated Copper at the last price (page 240) to yield an annual income of $1500 if the stock pays 5 %?

5% of $100 = $5, income from one share.
$1500 ÷ $5 = 300, no. shares.
300 × $65 = $19,500, investment.

5% stock pays $5 a share in dividends (when the par value is $100). To produce $1500, requires 300 shares. One share costs $$64\frac{7}{8}$ + $$\frac{1}{8}$, or $65. 300 shares cost 300 times $65, or $10,500.

536. 1. What investment in American Smelting & Refining pfd. at $111\frac{5}{8}$ will yield $1680 annual income, if the stock pays 6 %?

2. If Chicago, Milwaukee & St. Paul pfd. pays 8 % dividends, what investment at $173\frac{1}{2}$ will yield $1800 annually?

3. When Illinois Central stock paying 6 % sells at 147, what investment will yield $2250 per annum?

4. If Rock Island Company pfd. costs $79\frac{7}{8}$, when it pays 5 % what investment will produce a semiannual return of $750?

537. To find what per cent an investment pays.

If Denver & Rio Grande pfd. pays 5 %, and sells at 84, what rate per cent does the stock pay on the investment?

5 % of 100 = $5, income from 1 share.
$84 + $$\frac{1}{8}$ = $$84\frac{1}{8}$, cost of 1 share.
$5 ÷ $$84\frac{1}{8}$ = .0594 = 5.94 %, rate on the investment.

Since the income from one share is $5, and the cost of a share is $$84\frac{1}{8}$, the rate of the investment is found by dividing the income by the cost, which gives 5.94 %.

NOTE. The above example stated in its simplest type form would be, "What % of $$84\frac{1}{8}$ = $5?" Hence, $5 ÷ $$84\frac{1}{8}$ equals the rate per cent required (Prin. 14, page 101).

538. 1. If 6 % stock is bought at 120, no brokerage, what per cent on the investment does the stock pay?

2. When a stock costs 210, and pays quarterly dividends of 3 %, what rate does it pay on the investment (no brokerage)?

3. Which is better, and how much per cent, to buy stock paying semiannual dividends of 4 % at $159\frac{7}{8}$, or to buy a $4\frac{1}{2}$ % stock at $87\frac{7}{8}$, allowing for brokerage in each case?

4. An investor is offered 5 % stock at $89\frac{7}{8}$, and $3\frac{1}{2}$ % semiannual dividend stock at $124\frac{7}{8}$. Which is the better investment, and how much per cent, usual brokerage being allowed?

5. A broker owns 600 shares of Manhattan Elevated stock, which pays annual dividends of 7 %. He sells it at $140\frac{1}{2}$, and buys with the proceeds Atchison, Topeka & Santa Fe pfd., paying 6 %, at $105\frac{3}{8}$. Is his income increased or diminished, and what per cent?

539. To find the investment to yield a given rate per cent.

At what price will a 4 % stock pay 5 % on the investment?

$$4\,\%\text{ of }\$100 = \$4\text{, income from 1 share.}$$
$$\$4 \div .05 = \$80\text{, cost of 1 share.}$$

Since this is to be a 5 % investment, the dividend, $4, must be equal to 5 % of the cost of the share. Hence, $4 ÷ .05 gives $80, the cost of 1 share.

NOTE. In its simplest type form, this problem is: 5 % of what sum equals $4? (Prin. 14, page 101.)

540. 1. A man desires a 5 % investment. What price can he afford to pay for 6 % stock?

2. A 7 % stock pays only 4 % on the investment. What is the market value (usual brokerage)?

3. What is the market value of stock paying dividends of $4\frac{1}{2}$ %, if an investment in them, including brokerage, pays 5 %?

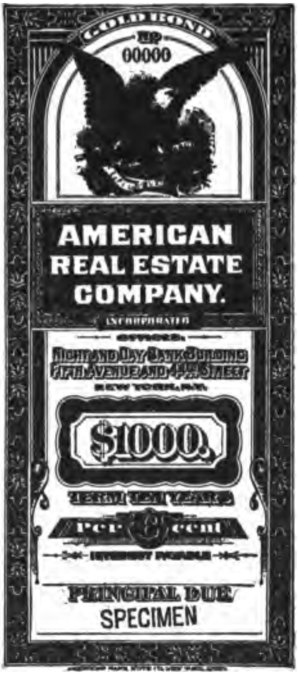
4. Find the cost, including brokerage, of 350 shares of stock paying 6 % dividends, if the rate on the investment is $4\frac{4}{5}$ %.

5. A broker has an option on 500 shares of 5 % stock at a price to yield 8 % on the investment, or a sufficient quantity of stock paying 2 % semiannually at 50, to require the same investment. He accepts the latter stock. Is his income more or less, and how much, than it would have been on the other stock, if money is worth 6 % per annum?

BONDS

541. A **bond** is a secured obligation, promising to pay, for value received, a sum of money at a certain time, with interest, payable at stated intervals and at a fixed rate.

542. Bonds are known by the name of the government, municipality, or corporation issuing them, by the rate of interest they bear, the date of maturity, the purpose for which they were issued, and by the type of security back of them. For example, U. S. 3's, 1908–1918, means United States bonds paying 3 % interest, due in 1918, payment of the bond being optional on any interest date after 1908. New York City 4½'s, 1957, means New York City bonds bearing 4½ % interest, due in 1957. Chicago, Milwaukee & St. Paul General Mortgage 4's, 1989, means bonds bearing 4 % interest, issued by the Chicago, Milwaukee & St. Paul Railway, secured by a general mortgage on the property of the company, and due in 1989.

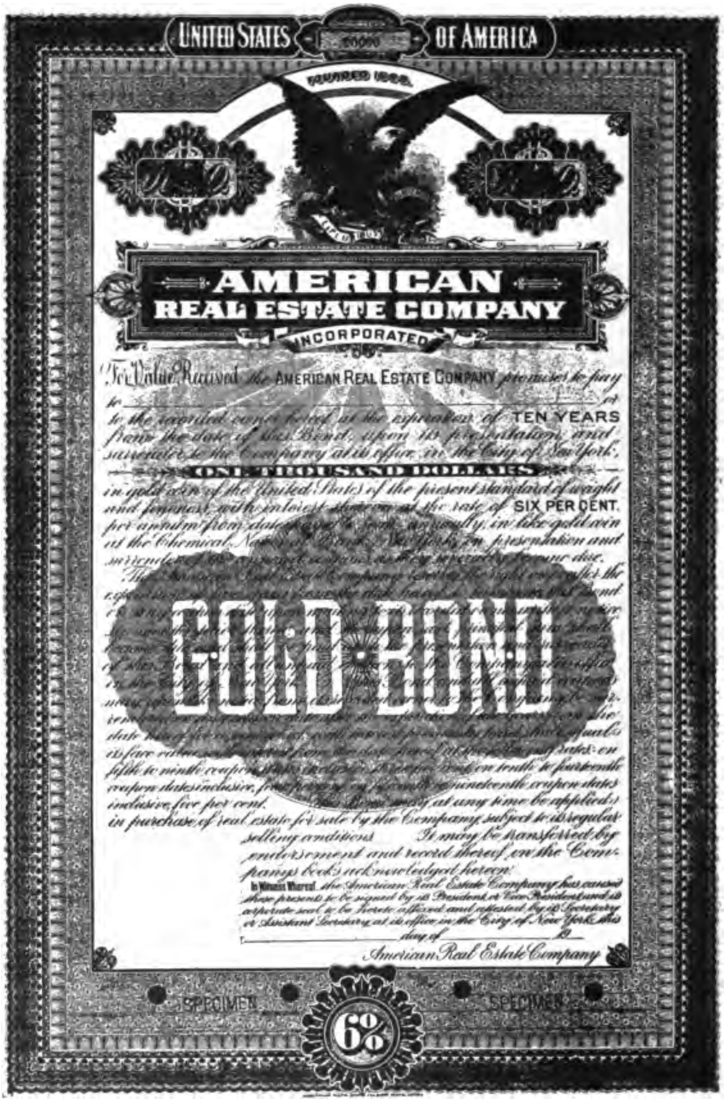


GOLD BOND
No 00000
AMERICAN REAL ESTATE COMPANY.
INCORPORATED
NIGHT AND DAY BANK BUILDING
FIFTH AVENUE AND 44TH STREET
$1000.
TERM TEN YEARS
Per 6 cent
PRINCIPAL DUE
SPECIMEN

Bonds have many names, some of which are First Mortgage 5's, General Mortgage 4's, Prior Lien 4½'s, Refunding and Extensions 5's, etc. If issued by a private corporation, they are generally secured by mortgage on the property of the company.

English Consols (consolidated stock) are government bonds of England bearing 2½ % interest payable quarterly, Jan. 5, April 5, July 5, and Oct. 5. These bonds have no definite date of ma-

543. ILLUSTRATION OF A BOND



UNITED STATES OF AMERICA

AMERICAN REAL ESTATE COMPANY

INCORPORATED

For Value Received the AMERICAN REAL ESTATE COMPANY promises to pay to ____ or to the recorded owner hereof at the expiration of TEN YEARS from the date of this Bond, upon its presentation and surrender to the Company at its office in the City of New York:

ONE THOUSAND DOLLARS

in gold coin of the United States of the present standard of weight and fineness, with interest thereon at the rate of SIX PER CENT. per annum from date hereof [illegible] in like gold coin at the Chemical National Bank, New York, on presentation and surrender of the [illegible] as they severally become due.

[illegible]

GOLD BOND

[illegible] its face value [illegible] from the date hereof [illegible] dates: on fifth to ninth coupon dates inclusive, [illegible] on tenth to fourteenth coupon dates inclusive, [illegible] on fifteenth to nineteenth coupon dates inclusive, five per cent. This Bond may at any time be applied in purchase of real estate for sale by the Company, subject to its regular selling conditions. It may be transferred by endorsement and record thereof on the Company's books acknowledged hereon.

In Witness Whereof, the American Real Estate Company has caused these presents to be signed by its President or Vice President and its corporate seal to be hereto affixed and attested by its Secretary or Assistant Secretary at its office in the City of New York this ____ day of ____ 19__

American Real Estate Company

SPECIMEN SPECIMEN

6%

turity, but cannot be paid before April 5, 1923. They are one of the so-called perpetual loans.

French Rentes are government bonds of France, and bear interest at 3 % payable quarterly. These loans are also perpetual loans.

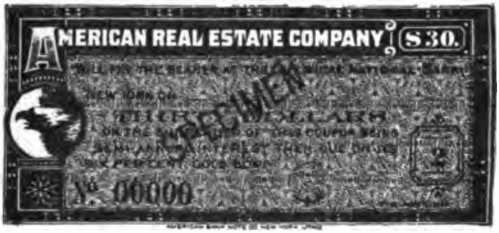
These bonds are issued in small denominations in order that persons with small means may purchase them.

544. There are two general classes of bonds — coupon and registered.

545. **Coupon bonds** have interest coupons attached. The coupons are made payable to bearer, and are dated at regular intervals (usually semiannually) and are, in effect, negotiable notes promising to pay the interest on the bond as it becomes due.

546. **Registered bonds** take their name from the fact that the name of the owner is registered with an agent of the maker of the bond. On "fully registered" ("registered as to principal and interest") bonds, interest is paid by check or draft mailed to the registered holders. Bonds "registered as to principal only" carry interest coupons like an ordinary coupon bond.

547. ILLUSTRATION OF A COUPON



AMERICAN REAL ESTATE COMPANY $30.

SPECIMEN

No 00000

548. The terms, *par value, market value, above par, premium, below par,* and *discount,* have the same signification as they have when applied to stocks.

549. STOCKS AND BONDS COMPARED

STOCKS	BONDS
1. Owner is part owner of corporation.	1. Owner is a creditor of the corporation.
2. Owner generally has a vote in the election of directors.	2. Owner generally has no vote.

Stocks	Bonds
3. Dividends not due until declared by the directors.	3. Dividends (interest) are payable regularly.
4. Dividends depend on earning power of the corporation.	4. Interest is fixed.
5. Purchased, many times, for speculation.	5. Purchased, generally, for investment.

550. Bond quotations are not subject to sudden fluctuations, as are stock quotations. The price of bonds depend chiefly upon four things: 1st, the security back of the bonds; 2d, the time the bond has to run; 3d, interest rates of money; and 4th, the earning power of the issuing corporation.

Notes. Bonds are redeemable at maturity at face value unless otherwise specified.

If interest payments on bonds are not made when due the company is said to default in the interest. To default in the bonds is to fail to redeem them at maturity.

551. Following is a partial list of bond sales on the New York Stock Exchange, Friday, Jan. 6, 1911. Note the variations in price of bonds paying the same rate of interest.

Sales of Bonds

Adams Ex 4s
500...... 90¼
Allis Chal 5s
1,000...... 76½
Am Tob 4s
14,000...... 80⅜
13,000...... ½
2,000...... ¾
Am Tob 6s
9,000......105¾
2,000...... ⅝
17,000......106
Cen Pac 1st 4s
5,000...... 96¾

C M & St P 4s
5,000...... 99½
8,000...... ⅜
1,000...... ½
Del & Hud fd
12,000......100⅜
4,000......101½
Ga & Ala ref 5s
2,000......104½
5,000......114
Mo Pacific 4s
21,000...... 77½
Mo Pac con 6s
1,000..... 109½

N. Ry Mex 4½s
50,000...... 84⅝
N Y City 4s 1958
15,000...... 99¾
N Y H H & H cv
3½s 12,500...100⅛
Nor Pac 3s
3,000...... 70¾
1,000...... ⅝
Nor Pac pr 4s
500......100¼
Rio G & W 4s
1,000...... 91

So Pac ref 4s
6,000...... 94¾
17,000...... ⅝
5,000...... ½
U S Rubber 6s
5,000......103⅜
1,000...... ¼
9,000......103
U S Stl s f 5s
6,000......103⅞
3,000...... ¾
3,000...... ⅞
West Un 4½s
10,000......95¾

552. To find the cost of bonds.

1. What is the cost of $25,000 New York City 4½'s, 1957, at 109½ (no brokerage)? 1.09½ × $25,000 = $27,375.

The quantity of bonds is expressed in dollars par value. The quotation is to be understood as "per cent of the par value." Hence, the cost is 109½% of $25,000, or $27,375.

2. Find the cost of $10,000, Baltimore & Ohio 4's, on Feb. 14, 1914, at 102 and interest.

102 % of $10,000 = $10,200, cost, flat.
Jan. 1 to Feb. 14 = 1 mo. 13 da.
Int. on $10,000 for 1 mo. 13 da. at 4 % = $47.78.
$10,200 + $47.78 = $10,247.78, cost.

At 102 flat, the bonds would cost $10,200. Interest days are Jan. 1 and July 1; hence, interest at 4% has been accumulating on these bonds from Jan. 1 to Feb. 14, a period of 1 mo. 13 da., and at the rate of interest the bonds pay, amounts to $47.78, which, added to $10,200, gives the cost as $10,247.78.

553. Using the prices quoted on page 250, find the cost, including interest, of the following bonds:

	Amount and Name	Int. Dates	Date of Purchase
1.	$6,000 Adams Express 4's	Mar.–Sept.*	Oct. 18
2.	8,000 American Tobacco 4's	Feb.–Aug.	Sept. 20
3.	4,000 Western Union 4½'s	May–Nov.	Nov. 15
4.	12,000 Missouri Pacific con. 6's	May–Nov.	Jan. 4
5.	25,000 Missouri Pacific 4's	Jan.–July	Apr. 3
6.	15,000 U. S. Rubber 6's	June–Dec.	Aug. 25
7.	20,000 Chicago, Milwaukee & St. Paul General 4's	Jan.–July	June 10
8.	30,000 N. Y., N. H. & H. con. deb. 3½'s	Jan.–July	June 30
9.	40,000 American Tobacco 6's	Apr.–Oct.	July 18
10.	35,000 So. Pac. Railroad first refunding 4's	Jan.–July	Aug. 11

BOND TABLES

554. Bankers use bond tables to determine the rate of interest an investment in bonds will pay, or the price to pay to secure a given rate on the investment. The following is a reproduction of one page from the "Bond Tables" published by N. W. Harris & Co., Bankers, New York and Boston, and is printed with their permission:

The tables cover periods of time from ½ yr. to 100 yr., and show the net annual rates of interest yielded by bonds paying 7 %, 6 %, 5 %, 4½ %, 4 %, 3½ %, and 3 %, respectively, for 64 different quotations on bonds bearing the above rates of interest.

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* Interest is due on the first day of the months named unless otherwise specified.

HOW TO USE BOND TABLES

555. 1. If a 4% bond maturing in 20 yr. costs 80.64, how much will it net the purchaser?

Find the 4% column in the table and follow down the column until 80.64 is reached. Then follow the horizontal line in which 80.64 is found to the extreme left-hand column, and there find 5⅝ in the column marked "net per annum." The bond will net 5⅝% on the cost.

2. If a man wishes a 4½% investment, what price can he afford to pay for a 6% bond maturing in 20 yr.?

In the left-hand column find 4½, and follow the horizontal line in which 4½ is, to the right until the column having 6% at its top is reached. The number found in both the horizontal line and in the vertical column, viz., 119.65 is the price he can pay.

20 YEARS—Interest Payable Semi-annually.

Net per Annum.	BONDS BEARING INTEREST AT THE RATE OF						
	7%	6%	5%	4½%	4%	3½%	3%
4	141.03	127.36	113.68	106.84	100.00	93.16	86.32
4.10	139.32	125.76	112.20	105.42	98.64	91.86	85.09
4⅛	138.90	125.37	111.84	105.07	98.31	91.54	84.78
4.20	137.63	124.19	110.75	104.03	97.31	90.59	83.87
4¼	136.80	123.42	110.04	103.35	96.65	89.96	83.27
4.30	135.98	122.65	109.33	102.66	96.00	89.34	82.68
4⅜	134.75	121.51	108.27	101.65	95.04	88.42	81.80
4.40	134.35	121.14	107.93	101.32	94.72	88.11	81.51
4½	132.74	119.65	106.55	100.00	93.45	86.90	30.35
4.60	131.16	118.18	105.19	98.70	92.21	85.72	79.22
4⅝	130.77	117.82	104.86	98.38	91.90	85.42	78.94
4.70	129.61	116.74	103.86	97.43	90.99	84.55	78.11
4¾	128.84	116.02	103.20	96.80	90.39	83.98	77.57
4.80	128.08	115.32	102.55	96.17	89.79	83.40	77.02
4⅞	126.95	114.27	101.59	95.24	88.90	82.56	76.22
4.90	126.58	113.92	101.27	94.94	88.61	82.28	75.95
5	125.10	112.55	100.00	93.72	87.45	81.17	74.90
5.10	123.65	111.20	98.76	92.53	86.31	80.09	73.86
5⅛	123.29	110.87	98.45	92.24	86.03	79.82	73.61
5.20	122.22	109.87	97.53	91.36	85.19	79.02	72.85
5¼	121.51	109.22	96.93	90.78	84.64	78.49	72.34
5.30	120.81	108.57	96.33	90.21	84.09	77.97	71.85
5⅜	119.77	107.60	95.44	89.36	83.27	77.19	71.11
5.40	119.42	107.28	95.14	89.07	83.01	76.94	70.87
5½	118.06	106.02	93.98	87.96	81.94	75.92	69.90
5⅝	116.38	104.47	92.55	86.59	80.64	74.68	68.72
5¾	114.74	102.95	91.16	85.26	79.36	73.46	67.57
5⅞	113.13	101.46	89.78	83.95	78.11	72.27	66.43
6	111.56	100.00	88.44	82.66	76.89	71.11	65.33
6¼	108.50	97.17	85.84	80.18	74.51	68.85	63.19
6½	105.55	94.45	83.34	77.79	72.24	66.69	61.14
6¾	102.72	91.83	80.95	75.50	70.06	64.62	59.17
7	100.00	89.32	78.64	73.31	67.97	62.63	57.29

PROBLEMS

556. Using the table on this page, find the price at which bonds maturing in 20 yr. can be bought to produce as follows:

1. 5% bond to yield 4%.
2. 4½% bond to yield 5%.
3. 6% bond to yield 4.6%.
4. 3½% bond to yield 4½%.
5. 5% bond to yield 5½%.
6. 7% bond to yield 5⅝%.

Find the rate on the investment on bonds maturing in 20 yr. and bought as follows:

7. 4½% bonds bought at 96.80.
8. 4% bonds bought at 88.90.
9. 6% bonds bought at 117.82.
10. 5% bonds bought at 101.27.
11. 3½% bonds bought at 71.11.
12. 7% bonds bought at 126.95.

EXCHANGE

557. The system by which merchants in different places discharge their debts to each other without the transmission of money is called **exchange.**

558. Exchange is of two kinds, viz.: Inland or Domestic, and Foreign.

DOMESTIC EXCHANGE

559. Domestic exchange treats of the payment of debts between merchants in the same country, without the sending of money.

For instance, if a merchant of New York owes a manufacturer of St. Louis for a bill of merchandise, he can pay his debt without sending money, in any one of the following ways:

1. By his personal check.
2. By a bank draft.
3. By a commercial draft on one of his debtors.
4. By a postal money order.
5. By an express money order.
6. By a telegraphic money order.

If he sends his check, the manufacturer in St. Louis will deposit it in his bank. In due time the check will be returned to the bank on which it is drawn, whereupon the amount will be charged against the merchant's account.

If the merchant sends a bank draft, he must buy and pay cash in advance for it. The draft is then sent in the same manner as a check.

Note. Bankers in New York City seldom draw drafts on other cities. They sell cashier's or manager's checks, drawn on their own bank.

If the merchant draws and sends a draft on one of his debtors, the manufacturer must present the draft to the drawee for payment, or he must have his bank collect it for him, and place the proceeds to his credit. Or, if it is a time draft, the bank will buy (discount) it, after acceptance by the drawee, and credit him at once with the proceeds. The bank will then collect the amount of the draft from the drawee at maturity, to reimburse itself for the credit given the indorser.

560. New York, Chicago, and San Francisco are the principal **exchange centers** in the United States.

561. An exchange center is a recognized money center. In it are located the correspondent banks of the banks in the surrounding cities and towns.

562. A **postal money order** is a written order given by the postmaster in one place, to the postmaster in another place, to pay a specified sum of money to a certain person or to his order.

563. To obtain a postal money order, one has to make a written application stating:

1. The amount of the order wanted.
2. The name and address of the payee.
3. The name and address of the purchaser of the money order.

564. The largest amount for which a single postal money order can be obtained is $100. The rates charged are as follows:

$2.50 or less	3¢	$30.01 to $40.00	15¢
2.51 to $5.00	5¢	40.01 to 50.00	18¢
5.01 to 10.00	8¢	50.01 to 60.00	20¢
10.01 to 20.00	10¢	60.01 to 75.00	25¢
20.01 to 30.00	12¢	75.01 to 100.00	30¢

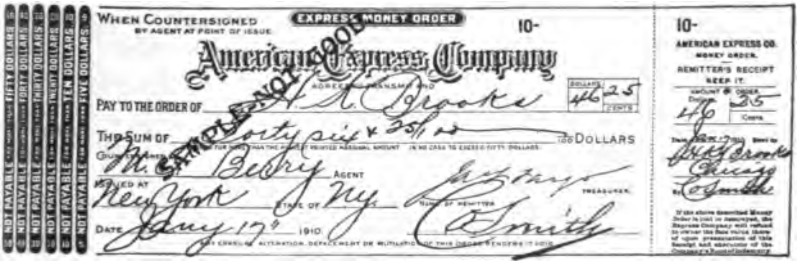
565. More than one indorsement on a postal money order is prohibited by law, except indorsements by banks.

566. Postal money orders may be presented for payment at the post office on which they are drawn, or at any bank. If not presented for payment within one year from their date, or if lost, the Post Office Department at Washington will upon request and presentation of evidence of loss, issue a duplicate.

567. An **express money order** is a written order of an agent of an express company directing another agent of the company to pay a certain sum of money to a specified person or to his order.

568. Express money orders are similar to postal money orders. The rates charged for them are the same. The largest amount that can be purchased in a single order is $ 50. Express money orders may be indorsed and transferred any number of times, in the same manner as checks and drafts.

FORM OF EXPRESS MONEY ORDER



NOT PAYABLE FIFTY DOLLARS
NOT PAYABLE FORTY DOLLARS
NOT PAYABLE THIRTY DOLLARS
NOT PAYABLE TWENTY DOLLARS
NOT PAYABLE TEN DOLLARS
NOT PAYABLE FIVE DOLLARS

WHEN COUNTERSIGNED BY AGENT AT POINT OF ISSUE
EXPRESS MONEY ORDER
10-
American Express Company
SAMPLE NOT GOOD
PAY TO THE ORDER OF H. A. Brooke DOLLARS 46 CENTS 25
THE SUM OF Forty six & 25/100 DOLLARS
M. C. Berry AGENT
ISSUED AT New York STATE OF N.Y.
TREASURER
NAME OF REMITTER C. Smith
DATE Jany 17th 1910

10-
AMERICAN EXPRESS CO.
MONEY ORDER.
REMITTER'S RECEIPT
KEEP IT.
AMOUNT OF ORDER Dollars 46 Cents 35
H. A. Brooke
Chicago
By C. Smith

569. A **telegraph money order** is an order by telegraph sent by an agent of a telegraph company at one place to a telegraph agent at another place directing him to pay a specified sum of money to the person named in the telegram with or without identification as directed by the sender.

570. The cost of sending a telegraphic money order is as follows:

1. Twenty-five cents for twenty-five dollars ($ 25.00) or less. For any amount from twenty-five dollars ($ 25.00) to one thousand dollars ($ 1000.00) one per cent. For any amount over one thousand dollars ($ 1000.00) one per cent is charged on the first thousand dollars, and one half of one per cent on all amounts over one thousand dollars.

2. In addition to the charges noted above, twice the regular rate of a ten-word message is charged.

NOTE. A telegraphic order between the depositing and paying agents is a message in cipher, the interpretation of which is known only to the company and its agents.

571. Domestic rates of exchange fluctuate slightly, depending on the conditions of supply and demand. If in New York there is a large demand for drafts on Chicago without an equivalent demand in Chicago for drafts on New York, the supply (that is, the funds of

New York banks on deposit in banks in Chicago) will become exhausted. To check or remedy this condition of affairs, the price of exchange on Chicago rises. There will be a corresponding drop in Chicago in the price of exchange on New York.

572. The following rates were published in the New York *Wall Street Journal*, Jan. 7, 1911. All quotations are in cents per $ 1000 premium or discount.

Boston — New York Exchange at par.
Chicago — New York Exchange 10 cents premium.
Montreal — New York Exchange 31¼ cents premium.
San Francisco — New York Exchange 80 cents premium.
St. Louis — New York Exchange 15 cents premium.
St. Paul — New York Exchange 70 cents premium.

573. To find the cost of a draft or money order.

1. At $\frac{1}{2}\%$ premium, find the cost of a draft on Chicago for $4000.

$\frac{1}{2}\%$ of $4000 = $20, premium.
$4000 + $20 = $4020, cost.

Since exchange is selling at a premium of ½%, the premium amounts to ½% of $4000, or $20. The cost is the sum of $4000 and $20, or $4020.

2. Find the cost of a draft of $3500, when exchange is selling at 50¢ discount.

50¢ discount means 50¢ discount on $1000.
3.500 × $.50 = $1.75, discount on $3500.
$3500 − $1.75 = $3498.25, cost.

Since the discount on $1000 is 50¢, on $3500 it is 3½ times $.50, or $1.75. $3500 less $1.75 gives $3498.25, the cost.

PROBLEMS

574. Using the rates quoted above, find the cost of drafts on New York as follows:

1. At Chicago for $1500.
2. At Montreal for $3875.
3. At San Francisco for $4580.
4. At St. Paul for $547.50.
5. At Montreal for $12,875.
6. At St. Louis for $1350.
7. At Chicago for $15,728.70.

8. Find the cost of postal money orders for $17.20; $21.60; $32.85; and $63.10.

9. A man desires to make a payment of $78.70 by means of express money orders. Find the cost.

10. A traveling salesman telegraphs for expense money. His principal sends a telegraphic money order for $150. The regular charge on the telegram being $.40, find the cost of the order.

11. Compare the costs of sending $175 by postal money order, express money order, and by telegraphic money order, if a ten-word telegram costs 25¢.

12. When exchange is quoted at $1.005, which is the cheaper way to make a payment of $500, by draft, or by a telegraphic money order, if the regular rate of a ten-word telegram is 35¢?

13. A merchant pays the following bills: $39.75 by express money order; $78.50 by postal money order; $382.50 by telegraphic money order (telegram 50¢); and $1837.50 by draft purchased at $.9975. Find the total cost of making settlement.

14. A man wishes to remit $1890 in settlement of a debt. Which is better, and how much, to send his check or to send a bank draft purchased at $.9985?

575. To find the proceeds of a draft.

1. Find the proceeds of a sight draft of $2840, collection and exchange being $\frac{1}{8}$ %.

$\frac{1}{8}$ % of $2840 = $3.55, collection and exchange.
$2840 − $3.55 = $2836.45, proceeds.

Since the bank charges $\frac{1}{8}$ % for collection, the charge amounts to $\frac{1}{8}$ % of $2840, or $3.55. The proceeds is the difference between $2840 and $3.55, or $2836.45.

2. Find the proceeds of a 60-da. commercial draft of $1500, if sold on its date at $\frac{1}{2}$ % discount, money being worth 6 %.

$15.00 = the interest for 60 da.
$\frac{1}{2}$ % of $1500 = $7.50, discount.
$15 + $7.50 = $22.50, total discount.
$1500 − $22.50 = $1477.50, proceeds.

This solution is identical with that of bank discount. The interest (bank discount) is found for the unexpired time of the draft (term of discount). The $\frac{1}{2}$ % discount is equivalent to a collection charge, and is reckoned on the face value of the draft. The total discount is the sum of the two discounts, $15 and $7.50, or $22.50. The proceeds is the face of the draft less the total discount.

576. Find the proceeds of the following sight drafts:

	Face	Collection and Exchange		Face	Collection and Exchange
1.	$ 784.00	½%	11.	$3981.43	¼%
2.	1260.00	¼%	12.	4289.64	⅛%
3.	2500.00	⅛%	13.	1562.50	¼%
4.	3675.00	$\frac{1}{10}$%	14.	5283.25	$\frac{1}{10}$%
5.	3875.00	½%	15.	6874.90	$\frac{1}{20}$%
6.	4272.50	¼%	16.	9743.85	$\frac{1}{20}$%
7.	1761.40	⅛%	17.	8498.75	$\frac{1}{10}$%
8.	2364.82	¼%	18.	4964.80	⅛%
9.	4152.87	$\frac{1}{20}$%	19.	3829.34	¼%
10.	7387.95	⅛%	20.	4382.86	½%

21. Find the proceeds of a 90-da. draft of $968.40 sold at ¼% discount, money being worth 6%.

22. A jobber purchased a bill of goods invoiced at $6483.28, less 5% for cash. To make immediate payment, he discounted a draft of $1864.50 due in 48 da. at ½% discount, and 6% interest; a note of $2487.25 due in 33 da., at ¼% discount and 6% interest. He sent a bank draft bought at $1.0025 for the balance. How much did he save by discounting the invoice and paying cash?

EXAMINATIONS

SPEED TEST

577. Minimum time thirty minutes; maximum, one hour. Deduct one credit for each minute required beyond the minimum time.

1. Find the market value in each of the following cases:

Par Value	Market Price	Market Value
$7,500	110	——
7,900	90	——
67,200	62½	——
475,200	131¼	——

2. What must be the asking price of a watch costing $24 in order to make a gain of 33⅓%, and allow the purchaser 20% discount?

3. Goods are invoiced at $ 800, less 20 %, 5 %, and 2½ %. Find the net cost.

4. Find the cost of the following bill of lumber at $ 24 per M:

24 planks 2″ × 8″ × 14′
25 boards 1″ × 9″ × 16′
17 pieces 4″ × 6″ × 14′
66 boards 1″ × 8″ × 12′

5. Find cost of each, and the total cost of the following drafts:

FACE	RATE	COST
$ 1,500	50 ¢ premium	——
1,875	$ 1.00 discount	——
3,500	$ 1.50 premium	——
5,000	10 ¢ discount	——
	Total . .	

6. What is the market value of a 6 % stock paying 5 % on the investment, brokerage ⅛ %?

7. Find the par value of bonds necessary to produce income as follows:

BONDS	INCOME	PAR VALUE
New York City 4½'s	$ 1,800	——
United States 3's	2,400	——
C. B. & Q. 4's	2,000	——
Sea. Air Line 5's	3,000	——

8. Add vertically, horizontally, and prove by finding the sum of the totals:

$ 16,157	$ 11,273	$ 10,559	——
48,287	2,380	16,272	——
473,892	8,721	145,468	——

9. Find gain or loss and per cent of gain or loss:

COST	SELLING PRICE
$ 2.50	$ 3.00
3.60	4.50
5.00	6.50
120.00	110.40

10. Find total interest on the following:

$ 1200 for 60 days at 4½ %.
1275 for 90 days at 6 %.
2500 for 72 days at 5 %.
3600 for 13 days at 7 %.

WRITTEN TEST

578. 1. A sight draft on Chicago for $ 1771 is purchased at $\frac{5}{8}$ % premium. Find the cost.

2. How many yards of carpet 27 in. wide will be required to carpet the floor of a room 24 ft. long and 18 ft. wide, if the strips run the long way of the room, and 6 in. are allowed on each strip for loss in matching?

3. What must be the list price of goods that cost $ 410 in order to make a profit of 20 %, if they are to be sold at a discount of 20 % and 10 % from the list price?

4. On May 2, 1910, J. Williams bought the following goods less a discount of 5 %, terms 2/10, n/60; 5 pieces velveteen, 42, 44, 43, 42, 45 yd. at 25¢ a yard. Williams sold the goods May 3 at 35¢ a yard, terms 1/10, n/30. He received cash for his sale on May 9, and paid for his purchase on May 10. Find his profit.

5. My broker buys for me 200 shares C. M. & St. P. at 121$\frac{3}{4}$, brokerage $\frac{1}{8}$ %. Thirty days later, I instruct him to sell at 122. Allowing $\frac{1}{8}$ % for selling, find my gain or loss.

6. If the cost of erecting a $ 28,000 building is assessed on $ 450,000 capital stock, how much must a stockholder pay who owns 100 shares at $ 100 each?

7. On an asking price of $ 400, which is the more favorable for the buyer, and how much: a discount of 30 %, 20 %, and 12$\frac{1}{2}$ %, or a discount of 40 %, 12$\frac{1}{2}$ %, and 10 %?

8. On the Panama Canal Bonds, interest is payable quarterly — on the 1st of February, May, August, and November, the annual rate being 2 %. Find the cost, including interest, of $ 50,000 of these bonds on the 10th of June at 99$\frac{3}{4}$. (Use accurate interest.)

9. A 4 months' note for $ 875, without interest, dated May 17, 1914, was discounted at a bank July 11, 1914, at 6 %. Find the discount and the proceeds.

10. I sold 2978 bu. wheat at $ 1.05 a bushel, and invested the proceeds in sugar, reserving my commission of 5 % for selling and 1$\frac{1}{2}$ % for buying, and the expenses of shipping, $ 53.57. How much did I invest in sugar?

APPENDIX

INVOLUTION AND EVOLUTION

INVOLUTION

579. A **power** of a number is the product obtained by multiplying the number by itself. 16 is a power of 4, because 4 × 4 = 16.

580. Powers are named from the number of times a given factor is used to produce the power.

Thus, 36 is the *second* power (called *square*) of 6, because 6 is used twice as a factor to produce 36.

27 is the *third* power (called *cube*) of 3, because 3 is used three times as a factor to produce 27.

256 is the *fourth* power of 4, because 4 is used four times as a factor to product 256, etc.

581. The number of times a given factor is to be used is shown by a small figure, called an **exponent**, written to the right of the factor.

Thus in 5^2, 2 is the exponent and shows that 5 is to be used twice as a factor;

$$5^2 = 5 \times 5 = 25.$$

In the same manner

$$4^3 = 4 \times 4 \times 4 = 64.$$

$$5^4 = 5 \times 5 \times 5 \times 5 = 625, \text{ etc.}$$

582. **Involution** is the process of finding the power of a number.

PROBLEMS

583. 1–20. Find the square, the cube, and the fourth power of all the numbers from 1 to 20 inclusive.

21–40. Find the square, the cube, and the fourth power of all the two-place decimals .01 to .20 inclusive.

Find the square, the cube, and the fourth power of:

41. $\frac{1}{2}$

42. $\frac{1}{3}$

43. $\frac{2}{3}$

44. $\frac{1}{4}$

45. $\frac{3}{4}$

46. $\frac{1}{5}$

47. $\frac{2}{5}$

48. $\frac{3}{5}$

49. $\frac{4}{5}$

50. $\frac{5}{6}$

51. $\frac{7}{8}$

52. $\frac{5}{9}$

53. $\frac{9}{16}$

54. $\frac{5}{11}$

55. $\frac{4}{7}$

EVOLUTION

584. 1. What number multiplied by itself will produce 9? 16? 25? 36? 49? 64? 81?

2. What number used three times as a factor will produce 8? 27? 64? 125?

585. The **root** of a number is one of the equal factors of that number. Thus, 4 is a root of 16; 5 is a root of 125; etc.

586. The **square root** of a number is one of the *two* equal factors which produce the number. 5 is the square root of 25.

587. The **cube root** of a number is one of the *three* equal factors which produce the number. 6 is the cube root of 216.

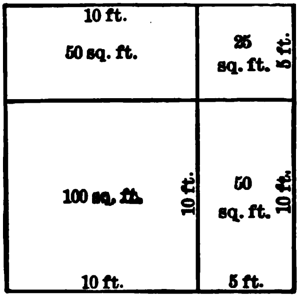
588. **Evolution** is the process of finding the root of a number.

589. The **radical sign** ($\sqrt{\ }$) denotes that a root of a number is to be found.

$\sqrt{625}$ indicates that the square root is to be found.

$\sqrt[3]{343}$ indicates that the cube root is to be found.

590. The following diagram will illustrate the principle involved in finding the square root of a number.



The square is 15 ft. on a side, and is divided into four parts as follows:

1 square 10 ft. on a side

$= (10 \text{ ft.})^2 = 100$ sq. ft.

2 rectangles 10 ft. long and 5 ft. wide

$= 2 \times (10 \times 5)$ sq. ft. $= 100$ sq. ft.

1 square 5 ft. on a side

$= (5 \text{ ft.})^2 = 25$ sq. ft.

Adding the several results gives

$$[10^2 + 2 \times (10 \times 5) + 5^2] \text{ sq. ft.} = 225 \text{ sq. ft.}$$

This result may be expressed as follows:

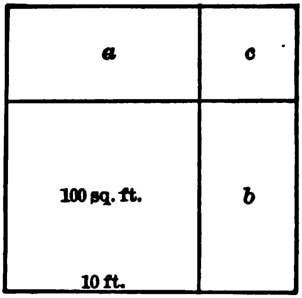
The square of a number of two figures is equal to the square of the tens plus twice the product of the tens by the units plus the square of the units.

By careful inspection and application of this principle, the square root of any number may be found.

591. 1. Find the square root of 225.

$$\begin{array}{lr}
 & \sqrt{2'25} = 10 + 5 = 15. \\
2 \times 10 = 20, \text{ trial divisor.} & \underline{1,00} \\
20 + 5 = 25, \text{ complete divisor.} & 1,25 \\
 & \underline{1,25}
\end{array}$$

Beginning at units, divide the number into periods of two figures each. The largest perfect square in the left-hand period is 1. The square root of 1 is 1. Write, as part of the square root, 1 with a cipher to represent the remaining period in the number 225. The square of 10 is 100, which subtract from 225, leaving 125 as the new dividend.



The accompanying diagram illustrates the condition of the problem at this point.

The remaining 125 sq. ft. is represented by the two rectangles *a* and *b*, and by the square *c*. Each rectangle is 10 ft. long. Hence, both rectangles are 2 × 10 ft., or 20 ft. long. 20 ft. is the trial divisor as shown in the solution. 125 ÷ 20 = 6, but 6 is too large because, when it is added to the 20 to complete the divisor, it makes the divisor too large. Try 5 as the quotient figure. Complete the divisor by adding the 5 to the 20, making 25. The 5 is the width of the rectangles *a* and *b*, and is the length of one side of the square *c*. The complete divisor represents the total length of both rectangles *a* and *b*, and of the square *c*.

Now multiply the total length, 25, by the width, 5, and the result, 125, is just equal to the remaining area of the large square. Hence the square root of 225 is 15.

NOTE. The student should observe that

$$2 \times (10 \times 5) + 5^2 = [(2 \times 10) + 5] \times 5.$$

2. Find the square root of 1489.96.

$$\begin{array}{rr}
 & \sqrt{14'89'.96} = 38.6 \\
 & \underline{9} \\
2 \times 3 = 6^0 & 5\,89 \\
\underline{60 + 8 = 68} & \underline{5\,44} \\
2 \times 38 = 76^0 & 45\;96 \\
760 + 6 = 766 & \underline{45\;96}
\end{array}$$

The preceding solution and explanation were made in full that the principle involved might be thoroughly understood. The actual work of finding the square root is reduced to the minimum in this solution.

3. Find the square root of $\frac{169}{256}$.

$$\sqrt{169}=13 \qquad \sqrt{\tfrac{169}{256}}=\tfrac{13}{16}.$$
$$\sqrt{256}=16$$

To find the square root of a common fraction, find the square root of the numerator for the numerator of the root, and the square root of the denominator for the denominator of the root.

4. Find the square root of $\frac{3}{8}$.

$$\tfrac{3}{8}=.375 \qquad \sqrt{.37'50'00}=.612$$

$$\begin{array}{rl} & 36 \\ 2\times 6 = 12^0 & \\ 120+1=121 & 1\ 50 \\ & 1\ 21 \\ 2\times 61 = 122^0 & 29\ 00 \\ 1220+2=1222 & 24\ 44 \end{array}$$

When the numerator and denominator of the fraction are not perfect squares, reduce the fraction to its decimal form and find the square root of the resulting decimal. The degree of accuracy of the result is determined by the number of places the result is carried out.

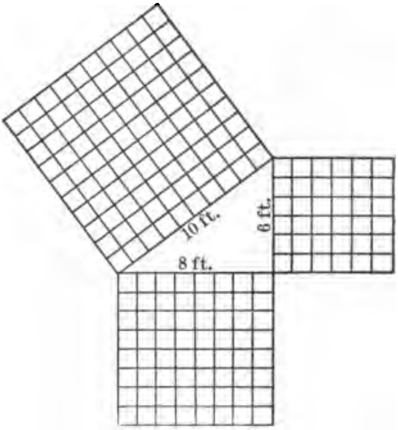
PROBLEMS

592. Find the square root of the following:

1. 289
2. 961
3. 1764
4. 15.452
5. 18.215
6. 18.0625
7. .144
8. .576
9. .8925
10. $\frac{3}{4}$
11. $\frac{576}{961}$
12. $\frac{529}{625}$

13. Find the hypotenuse of a right-angled triangle whose base and altitude are 8 ft. and 6 ft., respectively.

$8^2=64$
$6^2=36$
$8^2+6^2=64+36=100$
$\sqrt{100}=10$, no. ft., hypotenuse.



The square on the base plus the square on the perpendicular equals the square on the hypotenuse. To find the side of a square area, extract the square root of the area. Hence, to find the hypotenuse of a right-angled triangle extract the square root of the sum of the squares of the base and the perpendicular.

For problems requiring square root see pages 126, 127, and 136.

DENOMINATE NUMBER TABLES

Linear Measure

593. Linear measure is used for measuring distance.

TABLE

12 inches = 1 foot
3 feet = 1 yard
5½ yards = 1 rod
320 rods = 1 mile

The unit of length is the **yard**.

Square Measure

594. Square measure is used to measure the areas of surfaces.

TABLE

144 square inches = 1 square foot
9 square feet = 1 square yard
30¼ square yards = 1 square rod
160 square rods = 1 acre
640 acres = 1 square mile

A **square** is 100 sq. ft.

Other Linear Measures

1 size, ⅓ in. Used by shoemakers.
1 hand, 4 in. Used in measuring the height of horses.
1 fathom, 6 ft. Used in measuring depths at sea.
1 knot, nautical or geographical mile — 1.152⅔ mi. or 6086 ft.

The knot is used in measuring distances at sea. It is equivalent to 1 min. of longitude at the equator.

Cubic Measure

595. Cubic measure is used to measure the volume of solids and the contents or capacity of hollow bodies.

TABLE

1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard
24¾ cubic feet = 1 perch
128 cubic feet = 1 cord
1 cubic yard (of earth) = 1 load

A **perch** of stone or masonry is 16½ ft. (1 rd.) long, 1½ ft. wide, and 1 ft. high.
A **cubic foot** of water weighs 62½ lb. (Avoir.).
A ton of timothy hay in a well-settled mow is about 450 cu. ft.
A ton of clover hay in a well-settled mow is about 550 cu. ft.
A ton of stove coal is about 34½ cu. ft.

Liquid Measure

596. Liquid measure is used for measuring liquids, and in estimating the capacity of cisterns, tanks, reservoirs, etc.

TABLE

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

Apothecaries' Fluid Measure

597. Apothecaries' fluid measure is used by druggists in prescribing and compounding liquid medicines.

TABLE

60 minims (m.) = 1 fluid drachm (f ʒ)
8 fluid drachms = 1 fluid ounce (f ℥)
16 fluid ounces = 1 pint (O)
8 pints = 1 gallon (Cong.)

The gallon of this measure is the same as the wine gallon.

Technically, the barrel contains 31½ gal. In practice, barrels are of various sizes, as are also hogsheads, pipes, butts, etc. The capacity of each is marked upon it.

The unit of liquid measure is the wine gallon of 231 cu. in. A gallon of water weighs about 8⅓ lb. (Avoir.).

Dry Measure

598. Dry measure is used for measuring grain, fruit, vegetables, etc.

TABLE

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel

Liquid and Dry Measures Compared

	GALLON	QUART	PINT
Liquid .	231 cu. in.	57¾ cu. in.	28⅞ cu. in.
Dry . .	268⅘ cu. in. (½ peck)	67⅕ cu. in.	33⅗ cu. in.

The unit of dry measure is the **Winchester bushel**, which contains 2150.42 cu. in. It is a hollow cylinder 18½ in. in diameter and 8 in. deep.

The Winchester bushel is used for measuring grain, sand, etc.

The heaped bushel of 2747.71 cu. in. is used for measuring fruits, vegetables, and other coarse articles.

Weight

599. Weight is the measure of the earth's attraction of matter on its surface.

600. The **Troy pound** is the standard unit of weight in the United States.

Troy Weight

601. Troy weight is used for weighing gold, silver, diamonds, and other precious minerals.

Table

24 grains = 1 pennyweight
20 pennyweights = 1 ounce
12 ounces = 1 pound

Apothecaries' Weight

602. Apothecaries' weight is used by druggists and physicians in compounding and prescribing medicines.

Table

20 grains = 1 scruple
3 scruples = 1 dram
8 drams = 1 ounce
12 ounces = 1 pound

The term "carat" (or karat) has two meanings. In weighing diamonds, it is a denomination of weight, and is equal to 3.168 gr. Its second use is to denote the fineness of gold, and means $\frac{1}{24}$ part. Gold marked 18 K (18 carats) is $\frac{18}{24}$ by weight pure gold, and $\frac{6}{24}$ alloy.

Drugs and chemicals are bought and sold wholesale by avoirdupois weight.

Avoirdupois Weight

603. Avoirdupois weight is used in weighing all sorts of coarse, heavy articles.

Table

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
20 hundredweight = 1 ton (T.)

Long Ton

604. The long ton (or gross ton) contains 2240 lb. It is used in the United States customhouse in determining the duty on merchandise taxed by the ton. Coal and iron are sold wholesale at the mines by the long ton.

Table

16 ounces = 1 pound
28 pounds = 1 quarter
4 quarters = 1 hundredweight
20 hundredweight = 1 ton

Comparison of Weights

	Pound	Ounce	Grain
Troy	5760 gr.	480 gr.	1 gr.
Apothecaries'	5760 gr.	480 gr.	1 gr.
Avoirdupois	7000 gr.	437½ gr.	1 gr.

Other Common Measures

		Pounds			Pounds
Beef,	Barrel	200	Nails,	Keg	100
Butter,	Firkin	56	Pork,	Barrel	200
Flour,	Barrel	196	Salt,	Barrel	280

605. TABLE SHOWING WEIGHTS IN POUNDS PER BUSHEL LEGALLY ESTABLISHED FOR CERTAIN PRODUCTS BY THE SEVERAL STATES AND (FOR CUSTOMS PURPOSES) BY CONGRESS

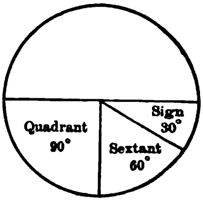
	U.S.	STATES	EXCEPTIONS
Barley	48	48	Ala., Ga., Ky., Pa., 47 ; Ariz., 45 ; Cal., 50.
Beans	60	60	Ariz., 55 ; N. H., Vt., 62.
Buckwheat	48	52	Cal., 40 ; Conn., Me., Mass., Mich., Miss., N.Y., Pa., R.I., Vt., 48 ; Idaho, N. Dak., Okla., Ore., S. Dak., Tex., Wash., 42 ; Ind., Kan., Minn., N. J., N. C., Ohio, Tenn., Wis., 50.
Clover seed	60	60	N. J., 64.
Corn (in ear)	70	70	Miss., 72 ; Ohio, 68.
Corn (shelled) . . .	56	56	Mass., 50.
Corn meal	48	50	Ark., Fla., Ga., Ill., Miss., S. C., 48.
Oats	32	32	N. J., Va., 30.
Onions	57	57	Conn., Me., Mass., Minn., N. Dak., Okla., S. Dak., Vt., 52 ; Fla., Tenn., 56 ; Ind., 48 ; Mich., 54 ; Ohio, 55 ; Pa., R. I., 50.
Peas	60	60	
Potatoes	60	60	Md., Pa., 56.
Rye	56	56	Cal., 54 ; Me., 50.
Timothy seed	45	45	Ark., 60 ; Okla., S. Dak., 42.
Wheat	60	60	

Circular or Angular Measure

606. Circular or angular measure is used for measuring angles or arcs of circles as applied to surveying, civil engineering, astronomical calculations, latitude, longitude, etc.

TABLE

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circle (cir.)



In astronomical calculations the circle is sometimes divided into signs, sextants, and quadrants.

In a circle there are : 12 signs of 30° each.
6 sextants of 60° each.
4 quadrants of 90° each.

Time

607. Time is the measure of duration.

Table

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
30 days = 1 month (mo.)
52 weeks = 1 year (yr.)
12 months = 1 year.
365 days = 1 common year
366 days = 1 leap year
100 years = 1 century

The length of the year is determined by the time required for the earth to make one complete revolution around the sun. The exact time is 365 da. 5 hr., 48 min. 46 sec. (very nearly $365\frac{1}{4}$ da.).

It is because of this extra $\frac{1}{4}$ da. each year that we have a leap year once in 4 yr. But by allowing $\frac{1}{4}$ of a day each year in making a leap year each 4 yr., a little too much time is allowed, as the true year lacks a little of being $365\frac{1}{4}$ da. To correct the error, all centennial years (1800, 1900, 2100) not divisible by 400 are not leap years. All other years divisible by 4 are leap years.

English Money

608. English money is the legal currency of Great Britain. There are gold, silver, and copper coins, and bills.

Table

4 farthings (far.) = 1 penny (*d.*)
12 pence = 1 shilling (*s.*)
20 shillings = 1 pound, or sovereign (£)

The unit of English monentary value is the **sovereign.** £1 = $3.8665.

French Money

609. French money is the legal currency of France.

Table

10 millimes (m.) = 1 centime (c.)
10 centimes = 1 decime (dc.)
10 decimes = 1 franc (F.)

The unit of French money is the **franc**, and is equal to $.193.

German Money

610. German money is the legal currency of the German Empire.

Table

100 pfennig (pf.) = 1 mark (M.) The unit is the **mark**. Its value is $.238.

COMMON BUSINESS ABBREVIATIONS

A.	acre
acct. $^{a}/_{c}$.	account
bbl.	barrel; barrels
bdl.	bundle; bundles
bl.	bale; bales
bot.	bought
bu.	bushel; bushels
bx.	box; boxes
cd.	cord; cords
ck.	check
C. O. D.	collect on delivery
coll.	collection
Cr.	credit; creditor
cs.	case; cases
ct. ¢.	cent; cents
cu. ft.	cubic foot; cubic feet
cu. in.	cubic inch; cubic inches
cu. yd.	cubic yard; cubic yards
cwt.	hundredweight
d.	pence
da.	day; days
do.	ditto (the same)
doz.	dozen; dozens
Dr.	debit; debtor
ea.	each
e.g.	exempli gratia (for example)
E.O.E.	errors and omissions excepted
etc.	et cetera (and so forth)
f.o.b.	free on board
F.	franc; francs
ft.	foot; feet
gal.	gallon; gallons
gi.	gill; gills
gr.	grain; grains
gro.	gross
hhd.	hogshead; hogsheads
hr.	hour; hours.
i.e.	id est (that is)
in.	inch; inches
inst.	instant (the present month)
kg.	keg; kegs
lb.	pound; pounds
M.	mark; marks
mdse.	merchandise
mi.	mile; miles
min.	minute; minutes
mo.	month; months
No.	number
oz.	ounce; ounces
p. pp.	page; pages
per	by; by the
per cent	per centum (by the hundred)
pc.	piece; pieces
pk.	peck; pecks
pkg.	package; packages
pr.	pair; pairs
prox.	proximo (next month)
pt.	pint; pints
pwt.	pennyweight; pennyweights
qr.	quire; quires
qt.	quart; quarts
rd.	rod; rods
rm.	ream; reams
s.	shilling; shillings
sec.	second; seconds
sq. in.	square inch; square inches
sq. ft.	square foot; square feet
sq. mi.	square mile; square miles
sq. yd.	square yard; square yards
T.	ton; tons
ult.	ultimo (last month)
via.	by way of
viz.	videlicet (namely)
wk.	week; weeks
wt.	weight
yd.	yard; yards
yr.	year; years

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